Coding for Energy Efficient Wireless Embedded Networks

Heikki Karvonen, Zach Shelby, Carlos Pomalaza-Ráez

Centre for Wireless Communications, University of Oulu, Finland

***Abstract*—This paper studies the effect of coding on the energy consumption in wireless embedded networks. An analytical model of the radio energy consumption is developed to study how different DC balanced codes affect the energy consumption for the one-hop case. A Rayleigh fading channel is assumed. The analysis is extended to include multihop scenarios in order to study the tradeoff between coding overhead and energy consumption. The numerical results obtained show that energy efficiencies of the codes used in a multihop routing scenario are strongly dependent on the channel conditions and on the number of hops used.**

***Index Terms*—Minimum energy transmission, multihop networks, energy efficient codes, sensor networks**

I. Introduction

Recent developments in wireless communication and electronics have made possible the construction of low-cost wireless embedded networks (WENs). WEN devices typically have a limited supply of energy and thus it is important to develop energy consumption models when designing physical, link, network, and transport protocols. This paper addresses the effect of coding on the energy consumption of single hop and multihop WENs. The energy consumption of multihop transmissions is studied for various common channel codes.

II. The Channel

In this paper, binary orthogonal non-coherent frequency shift keying (NCFSK) modulation with a frequency non-selective, Rayleigh fading channel is assumed. This is a realistic assumption for the Industrial, Scientific, and Medical (ISM) narrowband transceivers commonly used for WENs. The bit-error-probability P_b in this case is given by [1]:

$$p_{FSK} = \frac{1}{2+\bar{\gamma}_b} \quad (1)$$

where $\bar{\gamma}_b$ is the average received bit signal to noise ratio, which depends on the receiver characteristics and the distance between the receiver and transmitter. As an example [2], $\bar{\gamma}_b$ can be calculated for the RFM-TR1000 transceiver with fixed transmission power as:

$$\bar{\gamma}_b = 62.4 - 10\beta\log(d) \quad (2)$$

where β is the path loss exponent and d is the distance between the transmitter and receiver. For a variable transmission power scenario the radio can dynamically adjust its power so that the desired $\bar{\gamma}_b$ is guaranteed at the receiver.

III. Radio Model

Fig. 1 shows the energy consumption model for a one hop wireless link [3, 4].

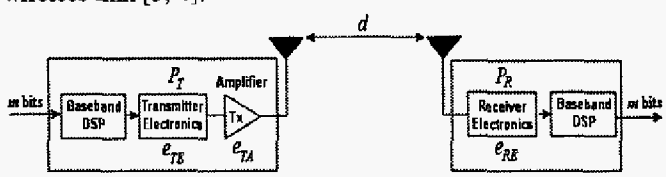

Figure 1. Energy Consumption Model.

The energy consumed when sending a packet of m bits over a one hop wireless link of distance d, can be expressed as:

$$E_L(m,d) = \left(E_T(m,d) + P_T T_{st} + E_{enc}\right) + \left(E_R(m) + P_R T_{st} + E_{dec}\right) \quad (3)$$

where,

E_T = energy used by the transmitter circuitry and power amplifier
E_R = energy used by the receiver circuitry
P_T = power consumption of the transmitter circuitry
P_R = power consumption of the receiver circuitry
T_{st} = startup time of the transceiver
E_{enc} = energy used to encode
E_{dec} = energy used to decode

Using this basic radio model, two analyses have been carried out to study how coding affects the energy consumption in a WEN.

A. *Single Hop Case*

For the single hop case the energy efficiencies of different coding techniques have been calculated. This study takes into account the effect of start-up energies and the reliability of the codes on energy consumption. A fixed transmission power mode is assumed. The packet format used is shown in Fig. 2.

Figure 2. Packet format. Length $m = \alpha + \iota + \tau$

Using the model of Fig. 1, an expression for the power consumption per bit e_b can be derived as [5]:

$$e_b = e_{tx} + e_{rx} + \frac{E_{dec}}{\iota} \quad (4)$$

where e_{tx} and e_{rx} are the transmitter and receiver power consumption per bit respectively and ι is the payload length in bits. The encoding energy is assumed to be negligible. Equation (4) can be expanded using the parameters e_t and e_r, representing the transmitter and receiver power consumption per bit and e_{st} and e_{sr}, representing the transceiver and receiver start-up power consumption per frame:

$$e_b = (e_t + e_r)\left(1 + \frac{(\alpha + \tau)}{\iota}\right) + \frac{(e_{st} + e_{sr} + E_{dec})}{\iota} \quad (5)$$

where

$$e_t = \frac{P_{TE} + P_{TA}}{R} \qquad e_r = \frac{P_R}{R} \quad (6)$$

$$e_{st} = P_T T_{st} \qquad e_{sr} = P_R T_{st}$$

P_{TE} and P_{TA} are the power consumption of the transmitter circuitry and the amplifier, respectively, and R is the data rate. The above parameters are constant for a given radio transceiver. The energy spent in the transmission of an information bit is $e_t + e_r$ whereas $e_{st} + e_{sr}$ represents the start-up energy consumption. For a typical low-power, wireless transceiver, the contribution of this last component is significant, reinforcing the effect of packet size on the overall power consumption. The energy efficiency measure used to calculate the efficiencies of different coding schemes is the same as the one described in [5]:

$$\eta = \eta_\tau r = \frac{\iota(e_t + e_r)}{m(e_t + e_r) + e_{st} + e_{sr} + E_{dec}}(1 - PER) \quad (7)$$

where $(1\text{-}PER)$ is the packet acceptance rate r. The packet acceptance rate can be improved with the use of Forward Error Detection (FEC) and depends on the coding scheme. This measure can be used to calculate the energy efficiency of the communication between two neighboring nodes.

This paper expands the results reported in [5] by using different balanced channel codes. DC-balancing of the channel codes has not been considered in previous studies. The reliabilities of different coding schemes are discussed in section IV.

B. Multihop case

As in the case of other studies [6, 7], the start-up energy component is not taken into account for the multihop scenario. The effect of start-up energy depends strongly on the MAC protocol used. The objective of this paper is to highlight the effect of coding on energy consumption. In this case, Equation 3 can be simplified as:

$$E_L(m,d) = E_T(m,d) + E_R(m) \quad (7)$$

The encoding/decoding energies are also assumed to be negligible. The transmitter energy consumption can now be expressed as:

$$E_T(m,d) = E_{TE}(m) + E_{TA}(m,d) \quad (8)$$

where E_{TE} is the energy used by the transmitter circuitry and E_{TA} is the energy required by the transmitter amplifier to achieve an acceptable signal to noise ratio at the receiver. Assuming a linear relationship for the energy spent per bit by the transmitter and receiver circuitry the equations above can be further simplified as:

$$E_T(m,d) = m(e_{TE} + e_{TA} d^\beta) \quad (9)$$

$$E_R(k) = m e_{RE} \quad (10)$$

where e_{TE}, e_{RE} and e_{TA} are hardware dependent constants. An explicit expression for e_{TA} can be derived from [6] as:

$$e_{TA} = \frac{\left(\frac{S}{N}\right)_r (NF_{Rx})(N_0)(BW)\left(\frac{4\pi}{\lambda}\right)^\beta}{(G_{ant})(\eta_{amp})(R_{bit})} \quad (11)$$

where $(S/N)_r$ is the desired signal to noise ratio at the receiver's demodulator, NF_{Rx} is the receiver noise figure, N_0 is the thermal noise floor in a 1 Hz bandwidth, BW is the channel noise bandwidth, λ is the wavelength in meters, β is the path loss exponent, G_{ant} is the antenna gain, η_{amp} is the transmitter power efficiency and R_{bit} is the raw bit rate in bits per second. This expression for e_{TA} can be used for those cases where a particular hardware configuration is being considered. Equation 11 highlights the implicit relationship between e_{TA} and the probability of bit error p, which depends on $(S/N)_r$.

Consider now a linear sensor array model as shown in Fig. 3, which has also been used in similar studies [6, 7]:

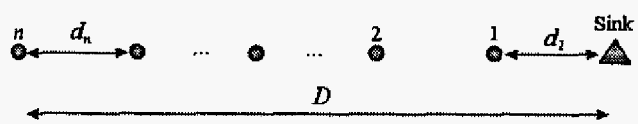

Figure 3. Linear sensor array model

For link i assume that the probability of bit error is p_i. Assume a packet length of m bits. For the analysis below it is assumed that an FEC mechanism is being used. Define $p_{link}(i)$ as the probability of receiving a packet with uncorrectable errors.

The conventional use of FEC is that at each hop the packet is accepted and delivered to the next stage, which in this case is forwarding it to the next node downstream. Assuming a variable transmission power mode, the energy consumed in sending a packet from the n_{th} node to the sink using a multihop routing that uses the downstream neighbor as a relay node is:.

$$E_{linear} = m\left[e_{TE} + e_{TA}(d_1)^{\beta} + \sum_{i=2}^{n}\left[e_{TE} + e_{RE} + e_{TA}(d_i)^{\beta}\right]\right] \quad (12)$$

The probability of the packet arriving at the nth sink node with no errors is:

$$P_c = \prod_{i=1}^{n}(1 - p_{link}(i)) \quad (13)$$

It can be easily shown that E_{linear} is minimized when all the distances are the same, i.e. $d_i = D/n$, and thus,

$$E_{linear}^{min} = m\left[n(e_{TE} + e_{RE}) - e_{RE} + \frac{e_{TA}D^{\beta}}{n^{\beta-1}}\right] \quad (14)$$

where n is the number of hops.

The optimal number of hops can be calculated as [6]:

$$n_{opt} = \left\lfloor \frac{D}{d_{char}} \right\rceil \quad (15)$$

where the characteristic distance d_{char} is defined as:

$$d_{char} = \sqrt[\beta]{\frac{e_{TE} + e_{RE}}{e_{TA}(\beta - 1)}} \quad (16)$$

where β is the path loss exponent of the channel which is typically between 2 and 4. In this paper $\beta = 3$ is used.

IV. Coding Techniques

In this study DC-balanced coding techniques are used for the single hop model. The codes used with the multihop model are not DC-balanced but they can easily become balanced by adding Manchester coding.

A. *Codes for the Single Hop Model*

In the Manchester code, "0" is encoded to "01" and "1" is encoded to "10" [7]. The packet length is doubled after this type of coding which increases the transmission energy. The Manchester code does not correct, so the reliability of a Manchester coded packet of length m is $r = (1-p)^m$ where p is the bit error probability of the channel. The encoding and decoding energies of Manchester are assumed to be negligible in this paper.

The properties of binary BCH (Bose-Chaudry-Hocquehen) codes with hard decision decoding are discussed next. The encoder at the transmitter adds τ parity bits to the ι payload and α header bits. In the (m, k) representation, $(m = \iota + \alpha + \tau)$ is the packet length and $(k = \iota + \alpha)$ is the message length. The reliability r of a *t-error* correcting BCH code word is given as [5]:

$$r = (1 - PER) = \sum_{j=0}^{t}\binom{m}{j}p^{j}(1-p)^{m-j} \quad (17)$$

where p is bit error probability of the channel. Note that this equation is valid only under the assumption of independent bit errors or when a suitable interleaving strategy is employed for burst error conditions. BCH is not DC-balanced and so a Manchester coding is added to the coded packet. Manchester coding increases the bit error probability, e.g. p in Equation 17 becomes $1-(1-p_{old})^2$ because there is error in a BCH code word when one of the Manchester bits is in error. Decoding energy must also be considered in this case because of the complexity of the code. Encoding energy is negligible.

The Golay code is a perfect block code (23,12) [8] which can correct tree errors. The efficiency of combined Golay and Manchester coding is also calculated. The ι information bits and the α header bits are first encoded with a Golay (23, 12) code and after that a Manchester encoding is added to the word to make it balanced. The reliability of the Golay code can be calculated using Equation 17 and the bit error probability for this case can be calculated as combined Manchester and BCH coding. The code can correct three errors and so the reliability increases but the energy needed for communication also increases because of the larger overhead.

The SEC/DED (24, 8) (Single Error Correction/Double Error Detection) code can correct one error after each received byte and it can be implemented so that it is DC balanced [7]. The code rate of the SEC/DED code is 1/3. The implementation of this code is so simple that encoding/decoding energy is negligible. The reliability of the code can be calculated using Equation 17.

B. *Linear Codes for Multihop Model*

Common codes are used in this study so that the trade-off between coding overhead and energy consumption per information bit can be clearly illustrated. The codes are linear block codes of the form (m, k, d_{min}), where m is the length of the code word, k is the number of information bits and d_{min} is the minimum distance of the code. Parameters for the studied codes are shown in Table 1.

Table 1.
Code parameters, t is the error correction capability.

Code	m	k	d_{min}	Code rate	t
Hamming	7	4	3	0.57	1
Golay	23	12	7	0.52	3
Shortened Hamming	6	3	3	0.5	1
Extended Golay	24	12	8	0.5	3

Probabilities of code word errors for the different codes depend on the channel bit error rate and the properties of the codes. Since a variable transmission power mode is being assumed the probability of the bit error for each link is fixed and thus $P_c = (1 - p_{link})^n$. The value of p_{link} depends on the received signal to noise ratio as well as on the modulation and coding method used. For FSK-modulation with non-coherent detection and assuming ideal interleaving the probability of a linear (m, k, d_{min}) code word being in error is bounded by [4]:

$$P_M < \frac{\sum_{i=2}^{M} \binom{2w_i - 1}{w_i}}{\left(2 + \overline{\gamma_b}\right)^{d_{\min}}} \quad (18)$$

where w_i is the weight of the ith code word and M=2^k. A simpler bound is:

$$P_M < (M-1)[4 p_{FSK}(1 - p_{FSK})]^{d_{\min}} \quad (19)$$

In the case discussed here p_{link} = P_M and the probability of packet error for the multihop scenario can be written as:

$$\begin{aligned} P_e &= 1 - P_c = 1 - (1 - p_{link})^n = 1 - (1 - P_M)^n \\ &< 1 - \{1 - (2^k - 1)[4 p_{FSK}(1 - p_{FSK})]^{d_{\min}}\}^n \end{aligned} \quad (20)$$

The probability of a successful transmission of a single code word is then:

$$P_{success} = (1 - P_e) \quad (21)$$

The expected energy consumption per information bit is defined as:

$$E_{i\text{-}bit} = \frac{E_{linear}^{\min}}{k\ (P_{success})} \quad (22)$$

V. Results

Energy efficiencies of the different channel codes introduced in Section IV have been calculated. The radio transceiver parameters e_t + e_r and e_{st} + e_{sr} for the RFM-TR1000 were calculated to be 2.41μJ/bit and 27.5μJ/bit respectively. Numerical calculations were made in MATLAB based on the model used in this paper. Calculations were made with different payload lengths t between 10 to 1000 bits, α=16 and bit error probability ranges of the channel between 10^{-1} and 10^{-4}. Bit error probabilities were approximated to consider normal channel conditions in the 433 MHz ISM band channel.

Fig. 4 shows the energy efficiency of single hop communication as a function of bit error probability (BEP) with a fixed packet length of 384 bits (48 bytes) using Equation 7.

The BCH code which has an error correction property of t = 2 seems to be the most energy efficient with a BEP between 10^{-4} and 10^{-3}. When the BEP is between 10^{-3} and $7*10^{-3}$, a BCH code with t = 6 is the best. Golay code appears to be the best choice in very bad channel conditions where the BEP is over $7*10^{-2}$. Obviously, when the packet length gets shorter and the bit error probability decreases there is less need for error correction and vice versa.

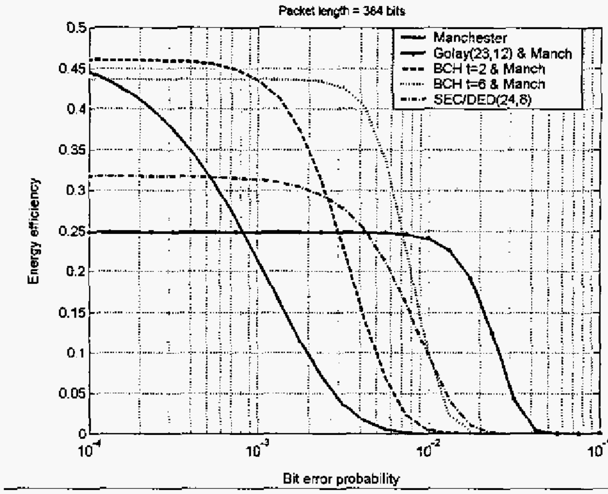

Figure 4. Energy efficiency as a function of bit error probability over a single hop

The following numerical analysis was then made to study the effects of coding on the energy consumption of multihop WENs. For these calculations, the size D, of the linear array is assumed to be 1000 meters. The energy consumption per information bit for different codes, using different number of hops, was calculated using the equations from the previous section and with the radio parameters shown in Table 2.

Table 2.
Radio Parameters.

Parameter	Value
NF_{Rx}	10dB
N_0	-173.8 dBm/Hz or 4.17 * 10^{-21} J
R_{bit}	115.2 Kbits
λ	0.3 m
G_{ant}	-10dB or 0.1
η_{amp}	0.2
β	3
BW	Depends on the modulation method. For FSK-modulation, it can be assumed to be the same as R_{bit}
e_{TC}	50nJ/bit
e_{RC}	50nJ/bit

For these parameters d_{char} is between 30 and 37 meters depending on the bit error probability. Fig. 5 shows E_{i-bit} for the different codes when the number of nodes is 10. In this case the distance of one hop is 100 meters which is about three times longer than the optimal distance between two neighbor nodes. The simplest (7, 4, 3) Hamming code appears to be the most energy efficient for low BER values. The Golay codes start to be the most energy efficient when the BER is over

0.02. It is apparent that the coding overhead unnecessarily increases the energy consumption when p_{FSK} is low. The error correction properties of the stronger codes are useful only when p_{FSK} starts increasing.

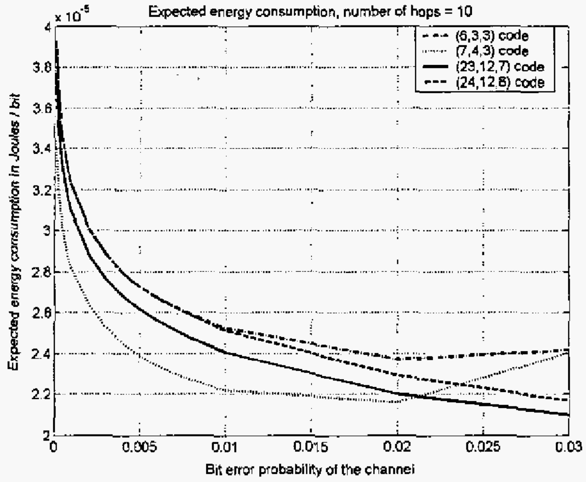

Figure 5. Expected energy consumption per information bit, n=10.

Fig. 6 shows the energy consumption when n=30. This makes the distance of one hop to be near the optimal value. In this case the energy efficiency of the various codes has increased. The energy efficiency of the simpler codes start to decrease when p_{FSK} is above 0.01. Here the large number of hops increases the probability of code word error.

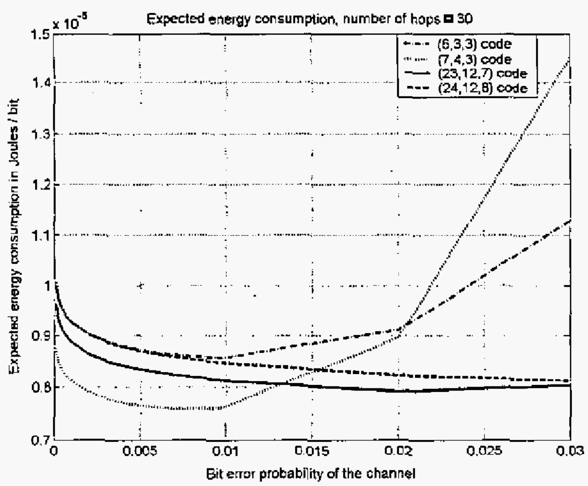

Figure 6. Expected energy consumption per information bit, n=30.

This effect is more clearly seen in Fig. 7 where the number of hops is 60. For p_{FSK} equal to 0.01, the Golay codes are better because they have a stronger error correction capability. Fig. 7 also shows that energy efficiency of all the codes decreases when compared to the 30-hop case because the length of a single hop is below the optimal distance.

VI. Conclusion

This paper extends previous work on energy consumption models to more accurately analyze realistic coding techniques for wireless embedded and sensor networks. The model is extended to study the effect of different channel codes on the energy consumption in the case of multihop scenarios. The results show that, depending on the channel conditions, the efficiencies of the codes vary. For good channel conditions, simpler codes work better. Error control is more useful when the channel conditions get worse. As the number of hops increases, the importance of error correction control also increases. The results also show that it is better to keep the number of hops above rather than below the optimal number of hops when reliable coding is used.

This work was performed with funding from the Finnish Defense Forces and TEKES through the WIRSU project.

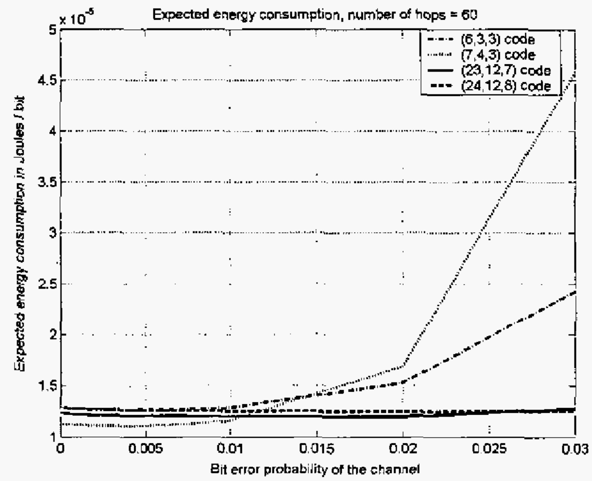

Figure 7. Expected energy consumption per information bit, n=60.

References

[1] J.G. Proakis, *Digital Communications*, McGraw-Hill, New York, Fourth Edition, 2001.

[2] RF Monolithics, Inc., "ASH Transceiver Designer's Guide," 2002, Url: http://www.rfm.com

[3] W. Heinzelman, A. Chandrakasan, and H. Balakrishnan, "Energy-Efficient Communications Protocols for Wireless Microsensor Networks," Proceedings of the 33rd International Conference on System Sciences, January 2000.

[4] A.P. Chandrakasan, R. Min, M. Bhardwaj, S. Cho, and A. Wang, "Power Aware Wireless Microsensor Systems," Keynote Paper ESSCIRC, Florence, Italy, September 2002, pp. 1-8.

[5] Y. Sankarasubramaniam, I.F. Akyildiz, and S.W. McLaughlin, "Energy Efficiency Based Packet Size Optimization in Wireless Sensor Networks," in Proceedings of Sensor Network Protocols and Applications, 2003.

[6] P. Chen, B. O'Dea, and E. Callaway, "Energy Efficient System Design with Optimum Transmission Range for Wireless Ad Hoc Networks," IEEE International Conference on Comm. (ICC 2002), 28 April -2 May 2002, Vol. 2, pp. 945-952.

[7] W. Ye, J. Heidemann, and D. Estrin, "A Flexible and Reliable Radio Communication Stack on Motes," USC/Information Sciences Institute, Tech. Rep. ISI-TR-565, September 2002. [Online]. Available: http://www.isi.edu/ johnh/PAPERS/Ye02b.html

[8] J.S. Lin and D.J. Costello, Error Control Coding: Fundamentals and Applications. Prentice Hall, 1983.