Analysis of Reversible and Nonreversible Queueing Networks with Rejection Blocking

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ABSTRACT

Queueing networks which contain finite capacities have proved useful in modeling actual computer systems and communication networks. The finite capacity of stations introduces blocking events which should be considered in the performance evaluation. Specifically, we shall examine the effects of rejection blocking upon queueing networks. Rejection blocking is defined in the following manner. Upon completion of its service of a particular station's server, a job attempts to proceed to its next station. If, at that moment, its destination station is full, the job is rejected. The job goes back to the server of the source station and immediately receives a new service. This is repeated until the next station releases a job and a place becomes available. In the first part of this work the well known exact product form solution for the equilibrium state probabilities is presented for closed rejection blocking networks which have reversible routing. An algorithm is given for computation of performance measures in reversible networks with rejection blocking. In the second part nonreversible networks with rejection blocking are analyzed. The analysis is based on the transformation of the state space of a blocking queueing network into an equivalent state space of a nonblocking network with infinite station capacities. It is shown that the state spaces of both systems are isomorphic under a given condition. Markov processes describing the evolution of both networks over time have the same structure. This leads to the product form solution for blocking networks. Based on product form solution new formulae are given for the exact computation of performance measures.

"Key Words:" Performance Evaluation, Queueing Networks, Blocking, Equilibrium State Probabilities

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1. Introduction

Queueing networks have received a special attention in the last fifteen years in performance evaluation and analysis of complex multiprocess, multiresource computer systems. A queueing network is composed of a collection of stations (devices with queues) in which jobs/processes proceed from one station to another in order to satisfy their service requirements. The basic results of queueing network theory were given by Jackson and Gordon/Newell [JACK63, GORD67a]. They showed that open and closed queueing networks with a single job class, with exponential arrival and service time distributions, and FCFS queueing disciplines at each station have a *product form solution*. The product form solution states that the equilibrium state probabilities consist of a product of terms where each term represents a state of the queues. Their result implies that the individual stations behave as if they were separate queueing systems. Baskett, Chandy, Muntz and Palacios [BCMP75] extended the results of [JACK63, GORD67a] to obtain product form solutions for open, closed and mixed queueing networks with different job classes, non-exponential service time distributions and different queueing disciplines such as FCFS, Processor Sharing (PS) and Last Come First Served Pre-emptive Resume (LCFS-PR). Several algorithms have been introduced for effective computation of performance measures for queueing networks [BUZE71, CHAN75, REIS75, REIS80, SAUE81].

Product form networks (also known as BCMP or separable networks) have proved invaluable for the modeling of a variety of computer and communication systems. They are sufficiently flexible as to adequately represent the features arising in such applications. They have not, however, been able to provide proper insight into the phenomenon of *blocking*. This is because product form networks assume that each station in the network has an infinite capacity. Since in actual systems the resources have a finite capacity, queueing networks with blocking must be used for performance analysis. Blocking arises because of the limitations imposed by the capacity of these stations.

In recent years there has been a growing interest in the development of computational methods to analyze queueing networks with blocking. Researchers from various areas such as Computer Performance Analysis, Operations Research, and Electrical Engineering Telecommunication Systems have studied blocking networks. Several papers have been published dealing with various types of blocking types.

Formally, we distinguish between three types of blocking: *"Transfer Blocking"*, *"Service Blocking"* and *"Rejection Blocking"*.

In the *"Transfer Blocking"* case, the blocking event occurs when a job completing service at station i cannot proceed to station j because station j is full. The job resides in station i's server, which stops processing until station j releases a job. This type of blocking has been used to model systems such as production systems and disk I/O subsystems. [AKYL87a,b,c,d, PERR81, PERR86, ONVU87, PERR87, SURI86, TAKA80].

In the *"Service Blocking"* case, blocking occurs when a job in front of queue at station i declares its destination station j before it starts its service in station i's server. If the destination station j is full, the i-th server becomes blocked, i.e., it can not serve jobs. When a departure occurs from destination station j, the i-th server becomes unblocked and the job begins receiving service. This blocking type has been used to model systems such as telecommunication systems and production systems. [BOXM81, GORD67b, SURI84].

In the *"Rejection Blocking"* case, blocking occurs when a job completes service at station i's server and wants to join station j, whose capacity is full. The job is rejected by station j. That job goes back with a certain probability (rejection probability) to station i's server and receives a new service with the same mean service time. This activity is repeated until station j releases a job, and a place becomes available.

The "rejection blocking" type has been used to model systems such as communication networks, computer systems with limited multiprogramming, production lines and flexible manufacturing systems. Most of the previous work was done on the "rejection" blocking in both open and closed queueing networks. Within this category, the studies fall into two groups. The first group provides exact results for both open queueing networks [KONH76, KONH77] and closed queueing networks [BALS83, HORD81, PITT79, YAO85]. Konheim/Reiser [KONH76, KONH77] propose an algorithm for the solution of an open network with two single server stations exhibiting exponential service time distributions. It also permits feedback by allowing some departures from the second station to proceed back to the first station's queue. Balsamo/Iazeoalla [BALS83], Hordijk/VanDijk [HORD81], Pittel [PITT79] and Yao/Buzacott [YAO85] have shown the existence of product form solutions for closed queueing networks which satisfy one of the following three conditions:

i) The network routing matrix is *reversible.*

ii) The probability of blocking is constant; that is, independent of the number of jobs in the station causing the blocking event.

iii) The service rate of each station is constant provided that it is impossible to have empty station at any time.

The second group of studies is characterized by the specific solution method [AKYL85, CASE79, LABE80, SURI84]. Cascau/Pujolle [CASE79] studied a blocking queueing network consisting of two or more stations in tandem in an effort to obtain an approximate expression of the maximum throughput.

Since the "service blocking" is identical to "rejection blocking" in case of tandem networks, we consider the work of Suri/Diehl [SURI84] also in the group of studies for rejection blocking networks. The Suri/Diehl [SURI84] study examined closed tandem queueing networks with finite station capacities in which the first queue has a capacity greater than the number of jobs in the system. By an application of Norton's Theorem [CHAN75], they reduce each two-station to a single station with a variable size queue capacity that is easily analyzed. An approximation algorithm is derived for the total mean residence time of the network, assuming exponentially distributed service times. The major disadvantage to this technique is that only the total throughput and the total mean residence time of a network can be determined. Performance measures for individual stations are impossible to compute. Another disadvantage is that it is restricted to networks with small populations (computation of marginal probabilities) and with serially switched stations. In addition the capacity of the first station must be infinite.

Several other investigators in recent years have published results on queueing networks with rejection as well as transfer blocking. A bibliography of studies about queueing network models with all types of blocking is given by Perros [PERR84].

In this work we present a computational algorithm for analyzing closed queueing networks with rejection blocking and reversible routing. As mentioned before, an exact product form solution exists for closed rejection blocking networks which have a reversible routing. However, no algorithm has been proposed for the computation of the normalization constant. In order to compute the equilibrium state probabilities in the product form solution, the normalization constant must be determined for closed networks. A naive technique to compute the normalization constant is to enumerate all states and compute their relative probabilities. Absolute probabilities can then be determined from the relative probabilities by normalizing their sum to one. This is of course feasible only for small networks, because for larger networks, the number of states grows rapidly.

We consider closed queueing networks with N stations and K jobs which constitute a single job class. Each station has single server and each server has an exponentially distributed service time with mean value $1/\mu_i$ (for $i = 1,2,...,N$). Each station has a fixed finite capacity, M_i, where M_i = *queue capacity* + 1. A job which is serviced by station i proceeds to station j with the transition probability p_{ij} (for $i,j = 1,2,...,N$), if station j is not full. In other words, the number of jobs in station j, k_j, is less than M_j. Otherwise, the job will be rejected from the station j, and it will return to the server of station i and receive another round of service. This is repeated until a place is available in station j. Furthermore, we assume that

$$K < \sum_{i=1}^{N} M_i$$

which means that the total number of jobs, K, in the network may not exceed the total capacity of the entire network. The service discipline of each station is First-Come-First-Served.

Section 2 explores the product form solution for reversible networks given by Hordijk and van Dijk [HORD81]. In section 3 we introduce an algorithm for the computation of $G(K)$ and derive formulae for performance measures for reversible networks. Section 4 and 5 contain the analysis of nonreversible networks.

2. Product Form Solution for Reversible Networks with Rejection Blocking

A queueing network is "reversible" [KELL79, MELA82], if the following condition is satisfied:

$$e_i\, p_{ij} = e_j\, p_{ji} \quad \text{for } all\ \ i,j = 1,2,...,N. \tag{1}$$

This equation states that the rate at which jobs arrive at station j from station i equals the rate at which jobs leave station j to return to station i. Simple examples for reversible networks are two-station networks and central server models.

It is well known that there exists a positive solution for e_i:

$$e_i = \sum_{j=1}^{N} e_j\, p_{ji} \qquad \text{for } i,j = 1,2,...,N \tag{2}$$

The relative utilization (also called loadings) of station i is denoted by x_i and is computed by

$$x_i = e_i / \mu_i \qquad \text{for } i = 1, 2, \cdots, N \tag{3}$$

We define the vector, $\underline{k} = (k_1, k_2, \ldots, k_N)$, as a state of the system where k_i denotes the number of jobs in the station i. We say that a state, $\underline{k} = (k_1, k_2, \ldots, k_N)$, is "feasible" if all k_i's are less than or equal to their respective M_i's, the capacity of the station i. Otherwise, the state is said to be "infeasible". In an infeasible state, at least one of the stations in the system violates its capacity restrictions. The following global balance equation is derived for the proposed model.

$$\left[\sum_{i=1}^{N}\sum_{j=1}^{N} \mu_i \; p_{ij} \;\; \delta_i(k_i)\right] \; p(k_1, k_2, \ldots, k_N) = \tag{4}$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} \mu_j \; p_{ji} \;\; \delta_j(k_j - 1) \;\; p(k_1, \cdots, k_i + 1, \cdots, k_j - 1, \ldots, k_N)$$

Informally, the left-hand side of equation (4) denotes the stream out of the state $(k_1, k_2, \cdots, k_N)$, and the right-hand side denotes the stream into that state.

The binary function, δ_i, eliminates the infeasible states.

$$\delta_i(k_i) = \begin{cases} 0 & \text{if} \quad k_i > M_i \\ 1 & \text{if} \quad \textit{otherwise} \end{cases}$$

Hordijk and VanDijk [HORD81] have shown that in a closed queueing network with rejection blocking that satisfies the reversibility condition (1), the equilibrium state probability solution for equation (4) for a feasible state $(k_1, k_2, \cdots, k_N)$ is represented by the following product form of marginal probabilities:

$$p(k_1, k_2, \cdots, k_N) = \frac{1}{G(K)} \prod_{i=1}^{N} x_i^{k_i} \;\; \delta_i(k_i) \tag{5}$$

where

$$\sum_{i=1}^{N} k_i = K$$

$G(K)$ is the normalization constant, which adjusts all the probabilities of "feasible" states so that they sum to one. For formal proof of the theorem, see [HORD81].

Although Hordijk/VanDijk [HORD81] have shown that a product form solution exists for networks with rejection blocking, they do not provide an algorithm for an efficient computation of the normalization constant, $G(K)$, as well as other performance measures. In the next section we give a convolution algorithm for the computation of the normalization constant and as well as formulae for performance measures in such queueing networks.

3. Performance Measures for Reversible Networks with Rejection Blocking

In order to compute the normalization constant for "rejection" blocking networks, we use the "convolution" algorithm. The normalization constant, G, is computed by the convolution of N vectors, G_i:

$$G = G_1 \otimes G_2 \otimes \dots\dots \otimes G_N$$

where G_i (for $i = 1,2,...N$) is a $(K+1)$ dimensional vector with

$$G_i = \begin{bmatrix} g_i(0) \\ g_i(1) \\ g_i(2) \\ \dots \\ g_i(K) \end{bmatrix} \tag{6}$$

and

$$g_i(k) = \begin{cases} 1 & \text{if} \quad k = 0 \\ \dfrac{e_i \; g_i(k-1)}{\mu_i} & \text{if} \quad k \le M_i \\ 0 & \text{if} \quad k > M_i \end{cases} \tag{7}$$

with $\otimes$ as the convolution operation

Informally, if the number of jobs in the station i exceeds its capacity, M_i, the component $g_i(k)$ will be set to 0. This eliminates all infeasible states. Once $G(K)$ is computed, the other performance measures can easily be obtained using the formulae which are given in the following.

The marginal probability, $p_i(n)$, which denotes the probability that there are n jobs in the station i, is obtained using the following equation:

$$p_i(n) = \sum_{\sum_{j}^{k} k_j = K \;\&\; k_i = n} p(\underline{k}) \tag{8}$$

By substituting the solution for $p(\underline{k})$ in equation (5) into equation (8) we get

$$p_i(n) = \frac{g_i(n)}{G(K)} \; G_{i^{-}}(K-n) \quad \text{for} \quad i = 1,2,...,N \;\text{ and }\; n = 1,2,...,M_i \tag{9}$$

where $g_i(n)$ is computed by equation (7) and $G_{i^{-}}$ is the normalization constant calculated without considering station i.

$$G_{i^{-}} = G_1 \otimes G_2 \otimes \dots \otimes G_{(i-1)} \otimes G_{(i+1)} \otimes \dots\dots \otimes G_N$$

The *mean number of jobs* in each station is computed using the following formula:

$$\bar{k}_i(K) = \sum_{n=1}^{M_i} n \quad p_i(n)$$

Substituting the value for marginal probability, $p_i(n)$, equation (9) into this formula, we obtain

$$\bar{k}_i(K) = \sum_{n=1}^{M_i} n \quad \frac{g_i(n)}{G(K)} \quad G_{i^-}(K-n) \tag{10}$$

The *utilization* of each station is determined by the following equation:

$$\rho_i(K) = \sum_{n=1}^{M_i} p_i(n) \tag{11}$$

By substituting the values for $p_i(n)$ in equation (11) we can find a direct solution (i.e., using the normalization constant) for utilization.

$$\rho_i(K) = \sum_{n=1}^{M_i} \frac{g_i(n)}{G(K)} \quad G_{i^-}(K-n) \tag{12}$$

4. Product Form Solution for Nonreversible Networks with Rejection Blocking

Our concept is based on finding an equivalent non-blocking network which has the same number of states and the same state space structure as the blocking network. To solve this problem we use the concept of "holes" (as introduced by Gordon/Newell [GORD67b]). Note that Gordon/Newell [GORD67b] investigated closed networks with serially connected stations and *service blocking* where the blocked job stays at the head of the queue and resides there until a space becomes available in the destination station. A "hole" is the number of available places in the non-blocking network. We assume that the "holes" in the nonblocking network are moving in the opposite direction of the jobs in the blocking network. For the sake of simplicity in the following we will use the notations Γ for the blocking network, Φ for the nonblocking network, "holes" as jobs in Φ.

We assume that each station in Γ must have capacity equal or larger than n, the number of "holes" in Φ, such that the state spaces of both networks Γ and Φ are isomorphic.

$$M_i \geq n \quad \text{for all } i = 1, 2, \cdots, N. \tag{13}$$

where n is computed by:

$$n = \sum_{i=1}^{N} M_i - K \tag{14}$$

Theorem. A closed queueing network with rejection blocking satisfying the condition, equation (13), has the following product form solution for the equilibrium probability distribution of feasible states:

$$p(\underline{k}) = \frac{1}{G(n)} \prod_{i=1}^{N} x'_i{}^{(M_i - k_i)} \tag{15}$$

where

$G(n)$ represents the normalization constant with n jobs computed by equation (14). The normalization constant $G(n)$ can be obtained by the convolution algorithm [BUZE71, CHAN75] or mean value analysis [REIS80].

$x'_i = e'_i / \mu'_i$ is the relative utilization of the i-th station in Φ.

e'_i is computed by equation (2). Note that the transition probabilities of "holes" in Φ are computed by considering the fact that the "holes" move in the opposite direction in Φ than in Γ:

$$p'_{ij} = \frac{\mu_j p_{ji}}{\sum_{1 \le t \le N} \mu_t p_{ti}} \tag{17}$$

μ'_i is the service rate of the i-th station in Φ and is computed by:

$$\mu'_i = \sum_{1 \le j \le N} \mu_j p_{ji} \tag{18}$$

Note also that equation (18) is derived from the fact that the job flow in Φ is in the opposite direction than in Γ.

Proof.

The network Φ has the same number of stations as Γ, $N' = N$. The difference is that the station capacities are unlimited, hence no blocking occurs and the total number of "holes" in Φ is $n \ne K$. Another difference is that the "holes" in Φ move in the opposite direction than in Γ as mentioned above. The service times in Φ are also exponentially distributed with rates μ'_i, computed by equation (18) and the transition probabilities in Φ are determined by equation (17).

The behavior of Γ can be modeled by a Markov process $X(t)$. The transition structure of $X(t)$ can be described by the global balance equation for Γ:

$$\{\sum_{i=1}^{N} \sum_{j=1}^{N} \mu_i \, p_{ij} \, \varepsilon_i(k_i) \, \delta_j(k_j)\} \, p(\underline{k}) = \sum_{i=1}^{N} \sum_{j=1}^{N} \mu_j \, p_{ji} \, \varepsilon_j(k_j) \, \delta_i(k_i) \, p(k_1, \ldots k_i + 1, \cdots, k_j - 1, \cdots, k_N)$$

where the binary functions ε and δ express the impossibility of jobs departing from a station that is empty and entering a station that is full:

$$\varepsilon_i(k_i) = \begin{cases} 0 & \text{if} \quad k_i = 0 \\ 1 & \text{if} \quad k_i > 0 \end{cases}$$

$$\delta_i(k_i) = \begin{cases} 0 & \text{if} \quad k_i > M_i \\ 1 & \text{if} \quad otherwise \end{cases}$$

The Markov process $X(t)$ has the following state space:

$$S = \{ \underline{k} \ / \ \forall \ i \ \ (0 \le k_i \le M_i) \ \& \ \sum_{i=1}^{N} k_i = K \ \}$$

Similarly, we define a Markov process $X'(t)$ whose transition structure is described by the following global balance equation for Φ :

$$\{\sum_{i=1}^{N} \sum_{j=1}^{N} \mu'_i p'_{ij} \ \ \varepsilon_i(k'_i)\} \ p'(\underline{k}') = \sum_{i=1}^{N} \sum_{j=1}^{N} \mu'_j p'_{ji} \ \ \varepsilon_{(j)}(k'_j) \ p'(k'_1, ...k'_i+1, \cdots, k'_j-1, \cdots, k'_N)$$

The Markov process $X'(t)$ has the following state space:

$$S' = \{\underline{k}' \ / \forall \ i \ \ (0 \le k'_i \le n \ ) \ \& \ \sum_{i=1}^{N} k'_i = n\}$$

We assert that $S = S'$

i) The number of jobs in Φ is defined in the following range:

$$0 \le k'_i \le n$$

Replacing the values for n we get

$$0 \le k'_i \le \sum_{i=1}^{N} M_i - K$$

Subsituting $k'_i = (M_i - k_i)$ and considering $K = \sum_{i=1}^{N} k_i$ we get

$$0 \le (M_i - k_i) \le \sum_{i=1}^{N} M_i - \sum_{i=1}^{N} k_i$$

Rewriting

$$0 \le \sum_{i=1}^{N} k_i - k_i \le \sum_{i=1}^{N} M_i - M_i$$

we obtain

$$0 \le \sum_{i=2}^{N} k_i \le \sum_{i=2}^{N} M_i$$

which provides

$$0 \le k_i \le M_i$$

ii) Substituting the value of n we obtain

$$\sum_{i=1}^{N} k'_i = \sum_{i=1}^{N} M_i - K$$

Rewriting

$$\sum_{i=1}^{N} (M_i - k'_i) = K$$

and substituting $k_i = (M_i - k'_i)$ we get

$$\sum_{i=1}^{N} k_i = K$$

This implies that the equilibrium state probability $p(\underline{k})$ of Γ is equivalent to the equilibrium state probability $p'(\underline{k}')$ of Φ :

$$p(\underline{k}) = p'(\underline{k}')$$

Substituting the value $\underline{k}' = (\underline{M} - \underline{k})$ we obtain

$$p(\underline{k}) = p'(\underline{M} - \underline{k})$$

Since Φ has product form solution, the $p'(\underline{M} - \underline{k})$ values are obtained from the Gordon/Newell Theorem [GORD67a] which provides equation (15).

Remark. As mentioned in the introduction Balsamo/Iazeoalla [BALS83] and Hordijk/VanDijk [HORD81] investigate queueing networks with rejection blocking. However, they have the following conditions which are not required in our concept:

i) The capacity of each station must be equal to the total number of jobs with one less job divided by the total number of stations with one less station, $M_i = \dfrac{K-1}{N-1}$ for all i.

ii) The total number of jobs must be greater than the total number of stations, $K > N$. This implies that no station is allowed to be empty.

5. Performance Measures for Nonreversible Networks with Rejection Blocking

Corollary. Since the Markov processes in Γ with K jobs have the same structure as the Markov processes in Φ with n "holes" computed by equation (14), the throughput of Γ with K jobs satisfying the condition, equation (13), is equal to the throughput of Φ with n jobs:

$$\lambda^{\Gamma}(K) = \lambda^{\Phi}(n) \tag{19}$$

Since Φ with n jobs has product form solution, any exact algorithm such as mean value analysis [REIS80] can be applied for the computation of $\lambda^{\Phi}(n)$.

Each station's throughput in Γ is then computed by:

$$\lambda_i(K) = e'_i \cdot \lambda^{\Gamma}(K) \qquad \text{for } i = 1, \cdots, N. \tag{20}$$

The *mean number of jobs in the i-th station of* Γ is computed by:

$$\bar{k}_i(K) = \sum_{\min(b) \text{ in station } i}^{M_i} b \;\; p_i(b) \qquad \text{for } i = 1, \cdots, N. \tag{21}$$

where $p_i(b)$ is the *marginal probability* that there are b jobs in the i-th station which are obtained from the equilibrium state probabilities, equation (15):

$$p_i(k) = \sum_{feasible \;\; \underline{k}} p(\underline{k}) \qquad \text{for } i = 1, \cdots, N. \tag{22}$$

Using Little's law the *mean residence time* of jobs at the i-th station in Γ is determined by:

$$\bar{t}_i(k) = \frac{\bar{k}_i(k)}{\lambda_i(k)} \qquad \text{for } \;\; i = 1, \cdots, N. \tag{23}$$

6. Conclusion

We have presented an algorithm for the computation of the normalization constant and formulae for other performance measures in queueing networks with rejection blocking and reversibility. This permits effective analysis of such networks. As generally known the set of reversible networks is contained in the set of non-reversible networks. However, the exact analysis of non-reversible networks is possible under the condition, equation (13), which weakens the set of the non-reversible networks. We are in the process of finding an approximate solution for cases where the condition, equation (13), is not satisfied. It is also interesting to investigate the cases where the stations have generally distributed service times and FCFS scheduling disciplines. H. von Brand [VONB87] gives an exact product form solution for reversible open, closed and mixed queueing networks with rejection blocking. He also introduces an algorithm for the computation of performance measures.

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