

On the Stability of Dynamic Spectrum Access Networks in the Presence of Heavy Tails

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Abstract—The heavy-tailed nature in dynamic spectrum access networks challenges the applicability of conventional network stability criterion. To counter this problem, a new stability criterion, namely moment stability, is introduced, which requires that all secondary users (SUs) with light-tailed traffic have bounded queueing delay with finite mean and variance. This stability criterion can prevent heavy-tailed traffic, e.g., video and Internet traffic, from degrading the QoS performance of light-tailed ones, e.g., email deliveries and audio/voice traffic. The critical conditions for the existence of a scheduling policy to achieve moment stability are derived. The network stability region yielded from these conditions is shown to be directly related to the primary and secondary user activities, the number of SUs, and the total number of primary user channels available to SUs. Moreover, it is shown that the maximum-weight- α scheduling algorithm, which makes the scheduling decision based on the queue lengths raised to the α -th power, is throughput optimal with respect to moment stability. It is proven that its throughput optimality holds independent of the stochastic properties of SU traffic including its marginal distribution (i.e., heavy-tailed or light-tailed distribution) and its time correlation structure (i.e., long range or short range dependence).

Index Terms—Network stability, dynamic spectrum access, throughput optimal.

I. INTRODUCTION

DYNAMIC SPECTRUM ACCESS (DSA) is an emerging technique that allows the secondary users (SUs) to share the spectrum in an opportunistic manner [1]. More specifically, in DSA networks, the SUs are empowered to access the unoccupied spectrum during idle periods of the primary users (PUs), and stop transmissions when the PU channels become busy. DSA has been widely envisioned as a key technology to solve the spectrum scarcity problem. What is more important, its opportunistic spectrum access nature will support high bandwidth multimedia applications, allow wireless sensor devices to avoid the interference from coexisting networks,

enable military networks to be set up in foreign lands, and promise greater wireless coverage in the areas lacking wireless infrastructure. Despite the great advantages of DSA networks, the achievable Quality of Service (QoS) performance of SUs is significantly affected by the dynamically changing PU traffic and the resource allocation policies used by the SUs.

Heavy-tailed traffic has been widely observed in a variety of computer and communication networks such as Ethernet, WLAN, mobile ad-hoc networks, and cellular networks. Such heavy-tailed network traffic can be either caused by the inherent heavy-tailed distribution in the traffic source such as the file size on the Internet servers, the web access pattern, the scene length of VBR (variable bit rate) video and MPEG streams [15], or caused by the network protocols themselves such as retransmissions and random access schemes [17]. Different from the conventional light-tailed traffic (i.e., Markovian or Poisson traffic), heavy-tailed traffic exhibits high burstiness or dependence over a long range of time scale. Such highly bursty nature can induce significant performance degradations including the considerably reduced network throughput, queue stability, and system scalability.

Currently, the majority of research in dynamic spectrum access networks focuses on the development of the resource allocation and spectrum management schemes under the assumption of the light-tailed behavior. Contrary to this conventional assumption, the emerging mobile Internet and multimedia applications necessarily lead to the heavy-tailed behavior in network traffic. For example, it has been experimentally validated that multimedia traffic, such as variable-bit-rate (VBR) video sequences, not only exhibits heavy-tailed marginal distribution, but also possesses strong time correlation, e.g., long range dependence and self similarity [4]. It has been shown that such heavy-tailed behavior not only has a significant impact on the spectrum sensing performance [23] but also greatly degrades the delay performance of SUs [21]. Despite its importance, the network performance and control of dynamic spectrum access networks in the heavy-tailed/long range dependent environment is still an under-explored area. In this paper, we aim to study the network stability of dynamic spectrum access networks in the presence of mixed heavy-tailed and light-tailed traffic and provide valuable insights for designing effective spectrum allocation schemes accordingly.

The most commonly adopted stability criterion is strong stability, which requires that there exists a resource allocation scheme, such as a scheduling algorithm, under which each user have finite time-average expected queue length [14]. Although strong stability is a desirable property for ensuring explicit QoS requirements, it may not be achievable in a heavy-tailed

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environment. On one hand, it is known that the heavy tail arrival traffic inherently lead to heavy-tailed transmission delay under any channel conditions, which can further lead to unbounded average queueing delay [21]. On the other hand, recent research shows that by competing wireless channel with the users having heavy-tailed traffic, the users with light-tailed traffic can experience significantly reduced QoS performance, such as unbounded average queueing delay [11]. Moreover, even if strong stability exists in the presence of heavy-tailed traffic, it does not necessarily imply the boundedness of the higher order moments, such as delay variance (jitter), which is one of the key metrics for the QoS oriented applications such as VoIP, on-line gaming and video conferencing [15].

The above observations indicate that the conventional strong stability criterion is difficult to characterize the QoS performance of next-generation wireless networks, e.g., LTE and DSA networks, which will be dominated by a variety of Internet, multimedia, and cloud applications, which can generate both heavy-tailed and light-tailed traffic. Therefore, the design of effective network control solutions for DSA networks necessitates a new stability criterion, which can effectively characterize the QoS performance in the presence of heavy tails. Toward this, in this paper, we introduce a new stability criterion, namely *moment stability*, which requires that all the network users with light-tailed traffic arrivals always have bounded queueing delay with finite mean and variance. Compared to strong stability, moment stability not only requires the finiteness of lower order moments, such as mean, but also demands the boundedness of higher order moments, such as variance, provided that such moments exist. What is more important, moment stability prevents heavy-tailed traffic (e.g., video conferencing and on-line gaming traffic) from significantly degrading the queueing performance of light-tailed traffic (e.g., email deliveries, audio/voice traffic, and temperature and humidity readings). Based on the definition of moment stability, the *network stability region* is defined as the closure of the set of all arrival rate vectors for which the queues of all network users can be stabilized by a feasible scheduling policy. Moreover, a scheduling policy is *throughput optimal* if it stabilizes the system for any arrival rates in the stability region.

To maximize the utilization of spectrum vacancies of primary networks, throughput optimal scheduling algorithms are desirable for DSA networks, which determine the optimal transmission times and channels for SUs so that the largest set of traffic rates of SUs can be supported, while maintaining the desired network stability. As the subclasses of the work-conserving scheduling policies, the maximum-weight scheduling and many of its variants [18] are known to be throughput optimal by making scheduling decision based on the queue lengths of network users. Although the throughput optimality of maximum-weight scheduling policies have been demonstrated in a variety of networking scenarios under the conventional assumptions of light-tailed traffic, their feasibility in DSA networks with heavy-tailed traffic is still unexplored.

In this paper, we study the fundamental impact of heavy-tailed environment on network stability under the celebrated maximum weight scheduling policies. Towards this, we first show the difficulty of achieving strong stability in the presence

of heavy-tailed traffic under the classic maximum weight scheduling policy. Then, we derive the necessary and sufficient conditions for the existence of a feasible scheduling policy to achieve moment stability. The network stability region yielded from these conditions is shown to be directly related to the statistics of SU traffics, PU activities, the number of SUs contending for the spectrum, and the total number of PU channels available to SUs. Moreover, we show that the maximum-weight- α scheduling algorithm, which makes the scheduling decision based on the queue lengths raised to the α -th power, is throughput optimal with respect to moment stability. Its throughput optimality holds independent of the marginal distribution of SU's arrival traffic (i.e., heavy-tailed or light-tailed distributed) and the time correlation of SU's arrival traffic (i.e., long range dependent or short range dependent). The contributions of this paper are summarized as follows.

- We define a new stability criterion, namely the moment stability, to characterize the QoS performance of DSA networks in the presence of heavy-tailed traffic.
- We derive the stability/capacity region of DSA networks and reveal the underlying relationship between the stability region and the networks settings regarding primary networks and DSA networks.
- We show that the classic maximum weight scheduling fails to achieve moment stability in the presence of heavy-tailed traffic even though SUs can exploit the transmission opportunities of multiple PU channels.
- We prove that under maximum-weight- α scheduling algorithm, DSA network can achieve the maximum throughput, while maintaining moment stability, and we also demonstrate such throughput optimality is independent of the traffic statistics of SUs, including the marginal distribution and time correlation structure of the traffic flows.

Note that this paper is based on our preliminary research in [20]. However, different from [20] which defines the network stability based on asymptotic queue length, this paper introduces a new and more generic stability criterion based on time average expected queue length [14] (see Definition 6), which necessitates a new stability analysis. What is more important, our previous work is based on the simple assumption of independent and identically distributed (i.i.d.) traffic arrivals from time slot to time slot for SUs, which can simplify the analysis, while failing to capture the inherent correlation across different time slots in network traffic, e.g., Internet traffic and video streams. In this paper, we address this problem by studying the impact of traffic temporal correlation on the stability performance. In addition, a comprehensive literature review and new simulation results are also given in this paper.

The rest of this paper is organized as follows. Section II summarizes the related work. Section III introduces system model and preliminaries. Section IV formally introduces moment stability and presents the critical conditions on the existence of moment stability. Section VI gives the simulation results to verify the derived theoretical results. Finally, Section VII concludes this paper.

II. RELATED WORK

The network stability depends on the finiteness of the queue length distribution. In [9] and [22], the queuing delay of SUs in a multi-channel cognitive network was investigated. Specifically, using large deviation approximation, [9] aimed to analyze the stationary queue distribution of SUs under light-tailed traffic assumption. Similarly, [22] studied the moments of the SUs' queue length under light-tailed environment. Different from [9] and [22], which consider the queuing performance under the light-tailed SU traffic, we aim to investigate the behavior of the SU's queue length in the presence of heavy-tailed traffic and study the fundamental impact of heavy-tailed environment on the network stability of dynamic spectrum access networks. To the best of our knowledge, little work on the analysis of the stability performance has been done for dynamic spectrum access networks.

In this paper, we investigate the stability performance of DSA networks under the celebrated maximum weight scheduling algorithms, which were first proposed by Tassiulas and Ephremides [18]. The maximum weight scheduling algorithms have drawn lots of attentions because of their throughput optimality, i.e., they can stabilize a queuing system for every supportable set of traffic arrival rates. What is more important and attractive, such throughput optimality is simply achieved by comparing the queue lengths of network users. So far, the maximum weight scheduling algorithms have been applied in a variety of networking scenarios, e.g., Internet backbone networks [16], satellite networks [13], mobile ad-hoc networks [5], and cellular networks [2]. Moreover, it has been shown that the maximum weight scheduling algorithms can also achieve high throughput and low average delay in cognitive radio or DSA networks [19], [25]. However, the performance of maximum weight scheduling algorithms in the literature are generally studied under the light-tailed traffic conditions, which is largely departing from the increased proliferation of video streaming, cloud computing, and Internet file transfer over mobile devices, which inevitably cause the rise of heavy tailed traffic over wireless networks [4], [8], [24]. Therefore, it is necessary to revisit the performance of the maximum weight scheduling algorithms under heavy-tailed traffic, which can provide valuable insights into the design of efficient network control protocols for next-generation wireless networks, e.g., DSA networks.

It is worth to note that [6] and [12] are among the first research efforts to study the performance of maximum weight- α scheduling under heavy tailed traffic. Specifically, the work in [6] considers a simple queueing system of two users competing a single server and shows that the maximum weight- α scheduling is effective to mitigate the impact of the queue with heavy-tailed traffic on the other queue with light-tailed traffic. In [12], the single-hop switched network is considered, where each traffic flow in the network is only served by one switch and then exits the network. Under this network setting, the similar results of [6] still hold.

Different from [6], [12] which only consider the network with single channel, we study maximum weight- α scheduling under DSA networks, where an arbitrary number of SUs can access a large number of PU channels with dynamically changing

channel availability. This dynamic and complex network setting can induce strong dependence on the queue length across multiple SUs competing multiple channels. which greatly complicates the queueing analysis. What is more important, the existing work in [6], [12] assumes the i.i.d. traffic arrivals from time slot to time slot. Such assumptions are largely departing from the experimental results that verify that multimedia and Internet traffic is not i.i.d. across time slots, but exhibits a strong dependence over a long range of time scales, such as long range dependence [10]. To close such gap, in this paper, we also study the impact of the strong temporal correlation in network traffic on the stability performance of maximum weight- α scheduling. In addition, the impact of time-correlated traffic on the routing performance is studied in [7], under the assumption that the network users have light-tailed arrival traffic and share a single wireless channel without being impacted by primary networks.

III. SYSTEM MODEL AND PRELIMINARIES

A. Preliminaries

In this paper we use the following notations. For any two real functions $a(t)$ and $b(t)$, let $a(t) \sim b(t)$ denote $\lim_{t \rightarrow \infty} a(t)/b(t) = 1$. For any two non-negative random variables (r.v.s) X and Y , we say that $X \leq_{a.s.} Y$ if $X \leq Y$ almost surely, and $X \leq_{s.t.} Y$ if X is stochastically dominated by Y , i.e., $P(X > t) \leq P(Y > t)$ for all $t \geq 0$. We say $X \stackrel{d}{=} Y$ if X and Y are equal in distribution. Also, let $F(x) = P(X \leq x)$ denote the cumulative distribution function (cdf) of a non-negative r.v. X . Let $\bar{F}(x) = P(X > x)$ denote its tail distribution function.

Definition 1: A r.v. X is heavy-tailed (HT) if for all $\theta > 0$

$$\lim_{x \rightarrow \infty} e^{\theta x} \bar{F}(x) = \infty, \quad (1)$$

or, equivalently, if for all $z > 0$

$$E[e^{zX}] = \infty. \quad (2)$$

Definition 2: A r.v. X is light-tailed (LT) if it is not heavy-tailed or, equivalently, if there exists $z > 0$ such that

$$E[e^{zX}] < \infty. \quad (3)$$

Generally speaking, a r.v. is HT if its tail distribution decreases slower than exponentially. On the contrary, a r.v. is LT if its tail distribution decreases exponentially or faster. Some typical HT distributions include Pareto and log-normal, while the typical LT distributions include exponential and Gamma.

Based on the existence of the moments, we define the tail coefficient of a non-negative random variable.

Definition 3: The tail coefficient $\kappa(X)$ of a nonnegative random variable X is defined by

$$\kappa(X) = \sup \{k \geq 0 : E[X^k] < \infty\} \quad (4)$$

The tail coefficient defines the threshold order above which a random variable has infinite moments. Some HT distributions, such as Pareto, have finite tail coefficient or equivalently have

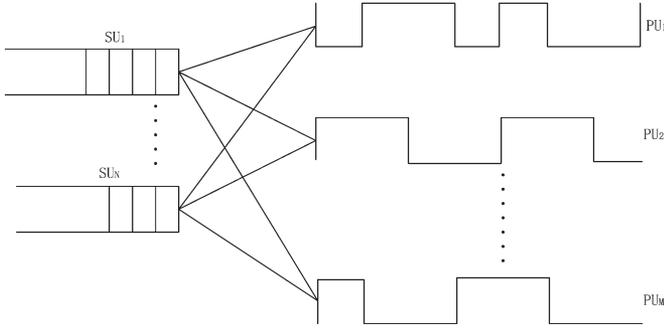


Fig. 1. System model with N SUs and M time-varying PU channels.

infinite moments of certain orders, while some HT distribution, such as log-normal, may have infinite tail coefficient or equivalently have finite moments of all orders. In this work, we focus on heavy tail distributed random variables with finite tail index because they can effectively characterize lots of network attributes such as the frame length of variable bit rate (VBR) traffic, the session duration of licensed users in WLANs, and files sizes on Internet servers. An important class of HT distributions which have finite tail coefficient is the regularly varying distributions

Definition 4: A r.v. X is called regularly varying with tail index $\beta > 0$, denoted by $X \in \mathcal{RV}(\beta)$, if

$$\bar{F}(x) \sim x^{-\beta} \mathcal{L}(x), \quad (5)$$

where $L(x)$ is a slowly varying function.

Regularly varying distributions are a generalization of Pareto/Zipf/power-law distributions. The tail index β indicates how heavy the tail distribution is, where smaller values of β imply heavier tail. Moreover, for a r.v. $X \in \mathcal{RV}(\beta)$, the tail coefficient $\kappa(X)$ of X is equal to the tail index β , which defines the maximum order of bounded moments X can have. Specifically, if $0 < \beta < 1$, X has infinite mean and variance. If $1 < \beta < 2$, X has finite mean and infinite variance.

B. System Model

We consider a dynamic spectrum access network consisting of N SUs and M PU channels, as shown in Fig. 1. The time is slotted and during each time slot, only one packet is transmitted. Let $\mathbf{S} = (S_1(t), S_2(t), \dots, S_M(t))$ denote the states of PU channels. $S_i(t) \in \{0, 1\}$, $\forall i \in 1, \dots, M$ with $S_i(t) = 0$ if channel i is busy and $S_i(t) = 1$ if channel i is idle. The processes $(S_1(t), S_2(t), \dots, S_M(t))$ are independent with each other with the channel busy probability p_i , i.e., $P(S_i(t) = 1) = p_i$. At each time slot t , each SU $i \in \{1, 2, \dots, N\}$ receives $A_i(t)$ packets. The arrival process $A_i(t)$, $i = 1, \dots, N$ is independent from each other and independent of the PU channel states. At each time slot, a scheduling/control policy allocates the detected idle channels to the SUs with knowledge only of the current queue lengths and instantaneous channel states. Since our primary objective is to study the impact of heavy-tailed traffic on network stability, we consider the scenario where the sensing errors are negligible. The above network model presents the downlink or uplink scheduling problem for

the centralized networks. The practical networks represented by this model include cellular, WiFi and mesh networks with coexisting licensed and unlicensed users. In this work, the marginal distribution of the arrival process $A(t)$ can follow either heavy-tailed or light-tailed distribution, i.e., $A(t) \in LT$ or HT . Moreover, $A(t)$ is not necessarily i.i.d. from slot to slot. Let $\rho(k) = E[A(t)A(t-k)]$ denote the autocorrelation function of $A(t)$. The arrival process, e.g., Markov modulated process, is short range dependent if its autocorrelation function decays exponentially, i.e., $\rho(k) \sim b^k$ as $k \rightarrow \infty$ and $0 < b < 1$. On the contrary, the arrival process, e.g., fractal brownian motion, is long range dependent if its autocorrelation function decays hyperbolically, i.e., $\rho(k) \sim k^{b-1}$ as $k \rightarrow \infty$ and $0 < b < 1$ is the fractal exponent. The quantity $H = (b+1)/2$ is referred to as the Hurst parameter, which characterizes the speed of decay of the autocorrelation function. Since $0 < b < 1$, the long range dependent process has slowly decaying autocorrelation function with $0.5 < H < 1$, which gives rise to the extreme burstiness in the network traffic. In particular, it is shown that the video traffic can have heavy-tailed marginal distribution, i.e., $A(t) \in HT$, along with long range dependent time correlation, i.e., $\rho(k) \sim k^{b-1}$ as $k \rightarrow \infty$ and $0 < b < 1$ [14].

C. Queueing Dynamics

Let $Q_i(t)$ denote the number of packets in the queue q_i of SU i by the end of time slot t . Define $h_{ij}(t)$ as the number of packets which can be released from queue q_i if channel j is allocated to queue i at time slot t . Based on the system model defined in the previous section, $h_{ij} \in \{0, 1\}$, $\forall i, j$. Then, the queueing dynamics of the SU i can be represented by

$$Q_i(t+1) = Q_i(t) - \sum_{j=1}^M h_{ij}(t) S_j(t) + A_i(t) \quad (6)$$

subject to

$$h_{ij}(t) \in \{0, 1\}, \quad \forall i, j \quad (7)$$

$$0 \leq \sum_{j=1}^M h_{ij}(t) \leq 1 \quad \forall i \quad (8)$$

$$0 \leq \sum_{i=1}^N h_{ij}(t) \leq 1 \quad \forall j \quad (9)$$

where the second constraint implies that each SU can only be allocated with one channel, while the third constraint means that each channel can only be assigned to one SU. By defining $H_i(t)$ as the number of packets which depart from queue q_i at time slot t under a certain control policy, the queueing dynamics in (6) can be rewritten by

$$Q_i(t+1) = Q_i(t) - H_i(t) + A_i(t). \quad (10)$$

Note that based on the above model, $H_i(t) \in \{0, 1\}$, $\forall i = 1, \dots, N$.

IV. DIFFICULTY OF ACHIEVING STRONG STABILITY WITH HEAVY-TAILED TRAFFIC

In this section, we first introduce the conventional stability criterion, the strong stability. Then, we show that the strong stability is difficult to achieve in dynamic spectrum access networks under heavy-tailed traffic.

Definition 5: A SU i is strongly stable if its time average queue length has bounded mean, i.e.,

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t E[Q_i(\tau)] < \infty. \quad (11)$$

A dynamic spectrum access network is strongly stable if there exists a scheduling algorithm, under which all SUs are strongly stable.

The following lemma shows that strong stability is not achievable for the SUs with heavy-tailed arrivals with tail index less than three.

Lemma 1: Let Q_i denote the steady-state queue length of SU i . If SU i has HT arrivals with tail index β_i , i.e., $A_i(t) \in \mathcal{RV}(\beta_i)$, then under any scheduling algorithms, Q_i necessarily has the tail coefficient $\min_{i \leq M} \beta_i - 1 < \kappa(Q_i) < \beta_i - 1$.

Proof: For SU i with HT arrivals, we prove the upper bound of $\kappa(Q_i)$ by considering the best case for SU i by assuming SU i has higher priority than any other SUs, which means that SU i is allocated to an available idle channel whenever its queue length is not empty. Thus, SU i acts as a $Geo^{X|}/Geo/1$ queue. If $A_i(t) \in \mathcal{RV}(\beta_i)$, it is known that the tail distribution of the steady-state queue length is also regularly varying and is one order heavier than that of the arrival process, which implies $\kappa(Q_i) < \beta_i - 1$. To prove the lower bound of $\kappa(Q_i)$, we consider a composite queue q_c which has the arrival process $A_c(t) = \sum_{i=1}^N A_i(t)$ and experiences the same PU channel as the original queuing system. Since $A_i(t) \in \mathcal{RV}(\beta_i)$, it implies by regular variation that the arrival process $A_v(t) \in \mathcal{RV}(\beta_c)$, where $\beta_c = \min_{1 \leq i \leq N} \beta_i$. Let Q_c denote the steady state queue length of q_c . we have that $\kappa(Q_v) = \beta_c - 1$. Since the queue length Q_i is stochastically dominated by the composite queue length Q_c . This implies that $\kappa(Q_i) > \beta_c - 1 = \min_{1 \leq i \leq N} \beta_i - 1$. ■

This lemma indicates that for a SU i with HT arrival traffic, its queueing delay is at least one order heavier than its arrival traffic under any scheduling algorithms. Therefore, if the tail index β_i of the arrival traffic is less than three, then the tail coefficient $\kappa(Q_i)$ of the queue length Q_i is less than two. This indicates unbounded variance for the queueing delay because by Definition 3, the tail coefficient of a random variable defines its maximum order of finite moments.

Next, we show that strong stability is also difficult to achieve for SUs with light-tailed arrivals because the conventional scheduling policies, which are effective under the light-tailed traffic, may have difficulty in achieving strong stability in the presence of mixed heavy-tailed and light-tailed arrivals. More specifically, recent work in [6], [12] has shown that by sharing a single wireless channel, users with heavy-tailed traffic arrivals can lead to unbounded average queueing delay for the users with light-tailed traffic arrivals. However, since in a

DSA network, there always exist multiple PU channels that can be shared among SUs, there rises a new and critical question regarding whether the parallelism (e.g., multiple channel access opportunities) will solve the unbounded queue length problem induced by classic maximum weight scheduling. In the following Lemma, we first show that using multiple channels cannot solve the problem. Then, we reveal the sufficient conditions under which the unbounded queue length problem still rise even through there are transmission opportunities over multiple PU channels.

Specifically, we consider a dynamic spectrum access network, which consists of three SU queues and two PU channels. Assume that queue 1 and queue 2 have HT traffic with tail coefficient c_1 and c_2 , i.e., $A_1(t) \in \mathcal{RV}(c_1)$ and $A_2(t) \in \mathcal{RV}(c_2)$, and queue 3 has light-tailed arrivals, i.e., $A_3(t) \in LT$. We show in the following Lemma 2 that under certain conditions, the maximum weight scheduling leads to the unbounded expected steady-state queue length for queue 3 even if it has light-tailed arrivals.

At each time slot t , the maximum weight scheduling algorithm solves the following optimization problem

$$\text{Find : } h_{ij}, \quad \forall i = \{1, 2, 3\} \quad j = \{1, 2\} \quad (12)$$

$$\text{Maximize : } \sum_{i,j} h_{ij}(t) Q_i(t) S_j(t) \quad (13)$$

$$\text{subject to : } (7), (8), \text{ and } (9). \quad (14)$$

Lemma 2: Assume queues q_1 , q_2 and q_3 have arrivals $A_1(t) \in \mathcal{RV}(c_1)$, $A_2(t) \in \mathcal{RV}(c_2)$, and $A_3(t) \in LT$, respectively. If $c_1 + c_2 < 3$, under the maximum weight scheduling policy, the expected queue length of queue q_3 is unbounded, i.e., $E[Q_3(t)] = \infty$.

Proof: Assume the queueing system is in the steady state. Suppose that a message m of size $A_3(t)$ arrives to q_3 at time slot t . Define $U_3(t)$ as the queueing delay of the message m , which is the difference between the time when the Head-of-Line (HoL) packet of the message is served and the time when the message arrives to the queue. We evaluate the queueing delay $U_3(t)$ in the following two scenarios: $\mathcal{E} = \{Q_3(t) < Q_1(t) \wedge Q_3(t) < Q_2(t)\}$ and its complementary set \mathcal{E}^c .

Consider the case \mathcal{E} . Under the maximum weight scheduling policy, both q_1 and q_2 keep being served until q_3 has a queue length longer than at least one of the two queues. In this case, the smallest queueing delay occurs in the following scenario. As soon as the HoL packet of the message m arrives to q_3 , both q_1 and q_2 do not receive any messages and the two channels keep idle so that q_1 and q_2 are served at each time slot, until one of q_1 and q_2 reaches the same queue length as q_3 . At this moment, q_3 begins to be served and the following condition holds

$$Q_3(t) + \sum_{i=1}^{\Delta} A_3(i) \geq \min(Q_1(t), Q_2(t)) - \Delta \quad (15)$$

where Δ is the number of time slots q_3 spends catching up with either q_1 or q_2 , whichever has the shorter queue length. In this case, the queueing delay $U_3(t)$ is lower bounded by

$$U_3(t) \geq Q_3(t) + \Delta \quad (16)$$

which implies

$$E[U_3(t)|\mathcal{E}] = E[Q_3(t)|\mathcal{E}] + E[\Delta|\mathcal{E}]. \quad (17)$$

By (15), independence of Δ and $A_i(t)$, and iterated expectations, $E[Q_3(t)|\mathcal{E}]$ is lower bounded by

$$E[\Delta|\mathcal{E}] \geq \frac{1}{\lambda_3 + 1} (E[\min(Q_1(t), Q_2(t))|\mathcal{E}] - E[Q_3(t)|\mathcal{E}]) \quad (18)$$

which, combining with (17), yields

$$\begin{aligned} E[U_3(t)|\mathcal{E}] &\geq \frac{1}{\lambda_3 + 1} (E[\min(Q_1(t), Q_2(t))|\mathcal{E}] \\ &\quad - \frac{\lambda_3}{\lambda_3 + 1} E[Q_3(t)|\mathcal{E}]) \\ &\geq \frac{1}{\lambda_3 + 1} E[\min(Q_1(t), Q_2(t))|\mathcal{E}]. \end{aligned} \quad (19)$$

This implies

$$E[U_3(t)1_{\mathcal{E}}] \geq \frac{1}{\lambda_3 + 1} E[\min(Q_1(t), Q_2(t))1_{\mathcal{E}}] \quad (20)$$

where $1_{\mathcal{E}}$ is the indicator function which equals 1 if \mathcal{E} occurs and 0 otherwise.

We now consider the case \mathcal{E}^c , where $Q_3(t)$ is larger than at least one of $Q_1(t)$ and $Q_2(t)$, i.e., $Q_3(t) > \min(Q_1(t), Q_2(t))$. By the fact $U_3(t) > Q_3(t)$, it follows

$$E[U_3(t)|\mathcal{E}^c] \geq E[\min(Q_1(t), Q_2(t))|\mathcal{E}^c], \quad (21)$$

which implies

$$E[U_3(t)1_{\mathcal{E}^c}] \geq \frac{1}{\lambda_3 + 1} E[\min(Q_1(t), Q_2(t))1_{\mathcal{E}^c}]. \quad (22)$$

This, in conjunction with (20), implies

$$E[U_3(t)] \geq \frac{1}{\lambda_3 + 1} E[\min(Q_1(t), Q_2(t))]. \quad (23)$$

We next evaluate $E[\min(Q_1(t), Q_2(t))]$ by constructing two fictitious queues, \tilde{q}_1 and \tilde{q}_2 , which have the same packet arrivals as q_1 and q_2 , respectively. Both \tilde{q}_1 and \tilde{q}_2 have a service rate 1, i.e., each queue is connected to a PU channel, which is always idle. Suppose both the original and fictitious queueing systems are in the steady state. It is easy to check, under any scheduling policies, the queue length of the fictitious queue is less than or equal to that of the original queue, for every sample path. This means the steady-state queue length of the fictitious queue is stochastically dominated by that of the original queue. Define $\tilde{Q}_1(t)$ and $\tilde{Q}_2(t)$ as the steady-state queue lengths at \tilde{q}_1 and \tilde{q}_2 , respectively. We have

$$P(Q_i(t) > \tau) \geq P(\tilde{Q}_i(t) > \tau), \quad i = 1, 2. \quad (24)$$

Since \tilde{q}_1 and \tilde{q}_2 are $GI/GI/1$ queues with regularly varying message size, it is known that the tail distribution of the queue length is one order heavier than the tail distribution of the

message size. Therefore, by the assumption that $A_1(t) \in \mathcal{RV}(c_1)$ and $A_2(t) \in \mathcal{RV}(c_2)$, we have

$$\lim_{\tau \rightarrow \infty} \frac{\log [P(\tilde{Q}_i(t) > \tau)]}{\log \tau} = -c_i + 1, \quad i = 1, 2. \quad (25)$$

Moreover, by the independence of $\tilde{Q}_1(t)$ and $\tilde{Q}_2(t)$, we have

$$\begin{aligned} P(\min(Q_1(t), Q_2(t)) > \tau) &\geq P(\tilde{Q}_1(t) > \tau \wedge \tilde{Q}_2(t) > \tau) \\ &= P(\tilde{Q}_1(t) > \tau) P(\tilde{Q}_2(t) > \tau). \end{aligned}$$

This, combining with (25) and the assumption $c_1 + c_2 < 3$, implies

$$\lim_{\tau \rightarrow \infty} \frac{\log [\min(Q_1(t), Q_2(t)) > \tau]}{\log \tau} \geq -(c_1 + c_2 - 2) > -1 \quad (26)$$

which, by applying the Theorem 2 in [3], implies $E[\min(Q_1(t), Q_2(t))] = \infty$. Combining with (23), we have $E[U_3(t)] = \infty$. This, by Little's law, means that the expected number of messages in q_3 is unbounded and consequently the steady-state queue length of q_3 has infinite mean. This completes the proof. ■

The above Lemma implies that in the presence of heavy-tailed arrivals, the maximum weight scheduling cannot guarantee strong stability since the queueing delay for the SU queues with light-tailed arrivals is necessarily unbounded. This conclusion is independent of the exact distribution of arrival processes, the incoming traffic intensity (i.e., $E[A_i]$) of the SU queues, and the PU channel status (i.e., the channel idle probability p_i). We conjecture that under more general case where more SUs and PU channels are present, the similar conclusions also hold as long as the SU queues with light-tailed arrivals have to compete the available spectrum with the SU queues with heavy-tailed arrivals.

V. MOMENT STABILITY OF DYNAMIC SPECTRUM ACCESS NETWORKS

In the previous section, we showed that it is difficult to achieve the strong stability in the heavy-tailed environment. In this section, we formally define the moment stability, which aims to prevent the heavy-tailed traffic, e.g., video conferencing and on-line gaming traffic, from significantly degrading the queueing performance of light-tailed ones, e.g., email deliveries, audio/voice traffic, and temperature/humidity readings. Then, we prove the necessary and sufficient conditions under which there exists a feasible scheduling algorithm to achieve moment stability.

Definition 6: A dynamic spectrum access network is moment stable, if there exists a scheduling algorithm, under which the time average queue length of any SU with light-tailed arrival has bounded mean and variance, i.e., if $A_i(t) \in \mathbf{LT} \forall i \leq N$, then

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t E[Q_i(\tau)] < \infty, \quad \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t E^2[Q_i(\tau)] < \infty.$$

A. Necessary Condition of Moment Stability

Theorem 1: If there exists a scheduling policy that achieves moment stability of the system, then

$$\sum_{i \in Q} \lambda_i \leq |Q| - \sum_{k=1}^{|Q|} P(K < k), \quad \forall Q \subset \{1, \dots, N\} \quad (27)$$

where $\lambda_i = E[A_i(t)]$, $|Q|$ denotes the cardinality of set Q , and K is the number of idle channels among total M channels at each time slot t , which follows poisson binomial distribution, denoted by $K \sim PB(\mathbf{p}, M)$, $\mathbf{p} = (p_1, \dots, p_M)$, i.e.,

$$P(K < k) = \sum_{l=0}^k \sum_{A \in F_l} \left(\prod_{j \in A} p_j \prod_{j \in A^c} (1 - p_j) \right), \quad (28)$$

where A^c is the complementary set of A , and F_l is the collection of all subsets of l integers that are selectable from set $\{1, \dots, M\}$.

Intuitively speaking, the right hand in (27) is the maximum time-average throughput the dynamic spectrum access network can achieve, under the constraints that at each time slot, each SU can only access one PU channel, while each PU channel can only serve one SU. It can be shown that the inequality in (27) is also the necessary condition for the existence of the strong stability provided that such stability is achievable, which, by Lemma 1, requires that either all SUs have LT arrivals or the minimum tail coefficient of all HT arrivals is larger than three.

Proof: Suppose the system is moment stable under certain resource allocation policy, which, by Definition 6, implies that the system is steady-state stable. This means for each queue, the incoming rate is equal to the service rate, i.e., $E[A_i(t)] = E[H_i(t)]$. Thus, for any subset $Q \subset \{1, \dots, N\}$, we have

$$\sum_{i \in Q} E[A_i(t)] = \sum_{i \in Q} E[H_i(t)] \quad (29)$$

which, by defining $K(t)$ as the number of idle channels at time slot t , can be rewritten as

$$\sum_{i \in Q} E[A_i(t)] = E \left[E \left[\sum_{i \in Q} H_i(t) | K(t), Q_i(t-1), i \in Q \right] \right]. \quad (30)$$

The event $B = \{K(t), Q_i(t-1), i \in Q\}$ can be partitioned into three disjoint sets

$$\begin{aligned} B_1 &= \{K(t) = 0\} \\ B_2 &= \{K(t) = 0\}^c \wedge \{Q_i(t-1) = 0, i \in Q\} \\ B_3 &= \{K(t) = 0\}^c \wedge \{Q_i(t-1) = 0, i \in Q\}^c. \end{aligned} \quad (31)$$

It is easy to verify that

$$E \left[\sum_{i \in Q} H_i(t) | B_i \right] = 0, \quad i = 1, 2. \quad (32)$$

As to event B_3 , we can further divide it into two disjoint sets

$$\begin{aligned} B_3^{(1)} &= \{K(t) < |Q|\} \wedge \{Q_i(t-1) = 0, i \in Q\}^c \\ B_3^{(2)} &= \{K(t) \geq |Q|\} \wedge \{Q_i(t-1) = 0, i \in Q\}^c \end{aligned} \quad (33)$$

which, in conjunction with (32) and (30), implies

$$\sum_{i \in Q} E[A_i(t)] = E \left[\sum_{i \in Q} H_i(t) 1_{B_3^{(1)}} \right] + E \left[\sum_{i \in Q} H_i(t) 1_{B_3^{(2)}} \right]. \quad (34)$$

Define $k_j = \{K(t) = j\} \wedge \{Q_i(t-1) = 0, i \in Q\}^c$ as the event that there are j idle channels and at least one of the queues is not empty. For the first term on the right side of (34), we have

$$\begin{aligned} E \left[\sum_{i \in Q} H_i(t) 1_{B_3^{(1)}} \right] &= \sum_{j=1}^{|Q|-1} \sum_{i \in Q} E[H_i(t) | k_j] P(k_j) \\ &\leq \sum_{j=1}^{|Q|-1} j P(\{K(t) = j\} \\ &\quad \wedge \{Q_i(t-1) = 0, i \in Q\}^c) \\ &\leq \sum_{j=1}^{|Q|-1} j P(K(t) = j). \end{aligned} \quad (35)$$

The second inequality is due to the fact that $\sum_{i \in Q} H_i(t) \leq K(t)$ if $K(t) \leq |Q| - 1$. For the second term on the right side of (34), we have

$$\begin{aligned} E \left[\sum_{i \in Q} H_i(t) 1_{B_3^{(2)}} \right] &= \sum_{j=|Q|}^M \sum_{i \in Q} E[H_i(t) | k_j] P(k_j) \\ &\leq |Q| \sum_{j=|Q|}^M P(\{K(t) = j\} \\ &\quad \wedge \{Q_i(t-1) = 0, i \in Q\}^c) \\ &\leq |Q| P(K(t) \geq |Q|). \end{aligned} \quad (36)$$

The second inequality holds because the number of channels allocated can not exceed the maximum number of queues, which implies $\sum_{i \in Q} H_i(t) \leq |Q|$ if $K(t) \geq |Q|$. Combining (34), (35), and (36), we have

$$\begin{aligned} \sum_{j \in Q} \lambda_j &\leq \sum_{j=1}^{|Q|-1} j P(K(t) = j) + |Q| P(K(t) \geq |Q|) \\ &= \sum_{j=1}^{|Q|-1} \left(\sum_{i=j}^{|Q|-1} P(K(t) = i) + |Q| P(K(t) \geq |Q|) \right) \\ &= |Q| - \sum_{j=1}^{|Q|} P(K(t) < j). \end{aligned} \quad (37)$$

Since $K(t)$ follows poisson binomial distribution, this completes the proof. \blacksquare

B. Sufficient Condition of Moment Stability

We next prove the sufficient condition of moment stability by first introducing the maximum-weight- α scheduling, which associates each queue with a different parameter α and makes the scheduling decision based on the queue lengths raised to the α -th power, which works as follows. For N queues $\{q_i\}_{1 \leq i \leq N}$, each queue q_i is assigned with a positive parameter α_i , which is the threshold order below which q_i has finite moments

under any scheduling algorithms. Specifically, as indicated in Lemma 1, $\alpha_i = \kappa(A_i(t)) - 1$ if queue i has HT arrivals $A_i(t)$ with tail index $\kappa(A_i(t))$. If queue i has LT arrivals, α_i can be any positive value larger than two. At each time slot t , the scheduling algorithm solves the following optimization problem

$$\text{Find : } h_{ij}, \quad \forall i \leq N \quad j \leq M \quad (38)$$

$$\text{Maximize : } \sum_{i,j} h_{ij}(t) Q_i(t)^{\alpha_i} S_j(t) \quad (39)$$

$$\text{subject to : } (7), (8), \text{ and } (9). \quad (40)$$

Note that if all parameters $\{\alpha_i\}_{1 \leq i \leq N}$ are equivalent, maximum-weight- α scheduling becomes the conventional maximum-weight scheduling.

Theorem 2: The dynamic spectrum access network is moment stable under the scheduling policy defined in (38), if

$$\sum_{i \in Q} \lambda_i < |Q| - \sum_{k=1}^{|Q|} P(K < k) \quad \forall Q \subset \{1, \dots, N\} \quad (41)$$

where $K \sim PB(\mathbf{p}, M)$, $\mathbf{p} = (p_1, \dots, p_M)$ and each queue i has the α_i -th moment of its time average queue length upper bounded by

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \sum_{i=1}^N E[Q_i(t)^{\alpha_i}] \leq \left(-\frac{2N}{d} \right) \left(\sum_{i=1}^N W_i \left(-\frac{d}{2N} \right) + V(\tau_\Delta) \right) \quad (42)$$

where $d = \max_{Q \subset \{1, \dots, N\}} \{ \sum_{i \in Q} \lambda_i + \Delta - \sum_{k=1}^{|Q|} P(K > k) \}$ and $W_i(\cdot)$ follows (47) if $1 \leq \alpha_i \leq \kappa(A_i(t))$ and (49) if $0 < \alpha_i < 1$ and $V(\tau_\Delta)$ is a constant directly related the time correlation of $A_i(t)$, which defined in (56), (57), and (58). Δ is an arbitrary positive value with $\Delta < c_{max}$. c_{max} is the maximum of c such that $\{\lambda\}_{i \leq N} + c\mathbf{1} \in \Lambda$, where $\mathbf{1}$ is a all-one vector with size of N and Λ is the closure of the set of all arrival rate vectors defined in (41). τ_Δ is the minimum τ such that $|E[A_i(t)|A_i(t-\tau), \dots, A_i(1)] - \lambda_i| < \Delta, \forall i \leq M$.

The Theorem above indicates that under the proposed scheduling algorithm, each SU queue can achieve the queueing delay with desirable bounded moment as long as its order is less than the maximum achievable one. The major advantage of such algorithm is that it can prevent the queues with heavy-tailed arrivals from impacting the queues with light-tailed arrivals. For example, assume there exist three queues, with arrival processes of $A_1(t) \in \mathcal{RV}(1.5)$, $A_2(t) \in \mathcal{RV}(1.5)$, and $A_3(t) \in LT$, using conventional maximum weight scheduling algorithm, it is shown by Lemma 2 that all three queues have unbounded queueing delay. In contrary, under the proposed algorithm, let queue 1 and 2 be assigned with $\alpha_1 = 0.5$ and $\alpha_2 = 0.5$, respectively. By setting proper value for α_3 , we can ensure that queue 3 has bounded moments of any orders even though queue 1 and 2 still have unbounded delay (since their maximum achievable order of unbounded moment is 0.5). Particularly, if letting $2 > \alpha_3 > 1$, queue 3 is guaranteed to have bounded mean delay. If letting $\alpha_3 > 2$, queue 3 will have bounded delay variance (jitter).

The above Theorem indicates the maximum weight- α scheduling can achieve moment stability independent of the marginal distribution (heavy-tailed or light-tailed) and the time correlation structure (long range dependent or short range dependent) of SU arrival traffic, although the time correlation can impact the upper bound of the time-average moments of the SUs through the constant $V(\tau_\Delta)$, where τ_Δ necessarily exists because of the decaying property of the autocorrelation of both long range dependent and short range dependent traffic.

Proof: Let $\mathcal{Q}(t) = (Q_1(t), \dots, Q_N(t))$ denote a vector process of queue lengths of N SUs. We define the Lyapunov function:

$$L(\mathcal{Q}(t)) = \sum_{i=1}^N L(Q_i(t)) \quad (43)$$

$$\text{where } L(Q_i(t)) = \frac{Q_i(t)^{\alpha_i+1}}{\alpha_i+1}. \quad (44)$$

We next evaluate each term $L(Q_i(t))$ under two cases: $1 \leq \alpha_i \leq \kappa(A_i(t)) - 1$ and $0 < \alpha_i < 1$. For the first case, using queueing dynamics in (10) and Taylor expansions with the Lagrange form of the remainder [11], we have

$$\begin{aligned} L(Q_i(t+1)) &= \frac{1}{\alpha_i+1} (Q_i(t) + A_i(t) - H_i(t))^{\alpha_i+1} \\ &= \frac{Q_i(t)^{\alpha_i+1}}{\alpha_i+1} + \Delta_i(t) Q_i(t)^{\alpha_i} + \alpha_i \frac{\Delta_i(t)^2}{2} \delta^{\alpha_i-1} \end{aligned} \quad (45)$$

where $\Delta_i(t) = A_i(t) - H_i(t)$ and $\delta \in [Q_i(t) - 1, Q_i(t) + A_i(t)]$. Therefore, by the fact that $\Delta_i(t)^2 \leq A_i(t)^2 + 1$ and $(Q_i(t) + A_i(t))^{\alpha_i-1} < 2^{\alpha_i-1} (Q_i(t)^{\alpha_i-1} + A_i(t)^{\alpha_i-1})$, for any positive constant θ , we have

$$\begin{aligned} E[L_i(Q_i(t+1)) - L_i(Q(t)) | \mathcal{Q}(t)] &= Q_i(t)^{\alpha_i} E[\Delta_i(t) | \mathcal{Q}(t)] + \frac{\alpha_i}{2} E[\Delta_i(t)^2 \delta^{\alpha_i-1} | \mathcal{Q}(t)] \\ &\leq E[(A_i(t) - H_i(t)) Q_i(t)^{\alpha_i} | \mathcal{Q}(t)] \\ &\quad + 2^{\alpha_i-2} \alpha_i E[A_i(t)^2 + 1] Q_i(t)^{\alpha_i-1} \\ &\quad + 2^{\alpha_i-2} \alpha_i E[A_i(t)^{\alpha_i+1} + A_i(t)^{\alpha_i-1}] \\ &\leq E[(A_i(t) - H_i(t) + \theta) Q_i(t)^{\alpha_i} | \mathcal{Q}(t)] + W_i(\theta) \end{aligned} \quad (46)$$

where

$$\begin{aligned} W_i(\theta) &= (\theta^{-1} 2^{\alpha_i-2} \alpha_i E[A_i(t)^2 + 1])^{\alpha_i-1} \\ &\quad + 2^{\alpha_i-2} \alpha_i E[A_i(t)^{\alpha_i+1} + A_i(t)^{\alpha_i-1}]. \end{aligned} \quad (47)$$

The last inequality in (46) holds because $1 < \alpha_i < \kappa(A_i(t)) - 1$, which implies that $E[A_i(t)^2]$, $E[A_i(t)^{\alpha_i+1}]$, and $E[A_i(t)^{\alpha_i-1}]$ are finite.

For the second case $0 < \alpha_i < 1$, by the similar arguments, we obtain

$$\begin{aligned} E[L_i(Q_i(t+1)) - L_i(Q(t)) | \mathcal{Q}(t)] &\leq E[(A_i(t) - H_i(t) + \theta) Q_i(t)^{\alpha_i} | \mathcal{Q}(t)] + W_i(\theta) \end{aligned} \quad (48)$$

where

$$W_i(\theta) = \theta + 1 + E[A_i(t)^{\alpha_i+1}]. \quad (49)$$

By (43), (46), and (48), the Lyapunov drift is upper bounded by

$$\begin{aligned} & E[L(\mathcal{Q}(t+1)) - L(\mathcal{Q}(t))] \\ & \leq \sum_{i=1}^N E[A_i(t)Q_i(t)^{\alpha_i}] + \sum_{i=1}^N (\theta Q_i(t)^{\alpha_i} + W_i(\theta)) \\ & \quad - E\left[\sum_{i=1}^N H_i(t)Q_i(t)^{\alpha_i}\right] \\ & = T_I + T_{II} - T_{III}. \end{aligned} \quad (50)$$

We now evaluate the expectation of term I, i.e., T_I , of (50) under three cases: $2 < \alpha_i$, $1 < \alpha_i < 2$ and $0 < \alpha_i < 1$. First, by the queueing dynamics, we obtain

$$Q_i(t) < Q_i(t-\tau) + \sum_{k=1}^{\tau} A_i(t-\tau). \quad (51)$$

This, combining with Taylor expansions with the Lagrange form of the remainder and the fact $Q_i(t-\tau) < \sum_{k=1}^{\tau} A_i(t-\tau)$, implies that if $2 < \alpha_i$, then

$$Q_i(t)^{\alpha_i} < Q_i(t-\tau)^{\alpha_i} + (\alpha_i + 2^{\alpha_i} \alpha_i^2) \left(\sum_{k=1}^{\tau} A_i(t-\tau) \right)^{\alpha_i}. \quad (52)$$

If $1 < \alpha_i < 2$, then

$$Q_i(t)^{\alpha_i} < Q_i(t-\tau)^{\alpha_i} + (\alpha_i + \alpha_i^2) \left(\sum_{k=1}^{\tau} A_i(t-\tau) \right)^2. \quad (53)$$

If $0 < \alpha_i < 1$ then

$$Q_i(t)^{\alpha_i} < Q_i(t-\tau)^{\alpha_i} + \sum_{k=1}^{\tau} A_i(t-\tau). \quad (54)$$

Then, the expectation of term I of (50) follows

$$E[A_i(t)Q_i(t)] < E[A_i(t)Q_i(t-\tau)^{\alpha_i}] + V(\tau) \quad (55)$$

where $V(\tau)$ is some constant, which is related to the time correlation in $A_i(t)$. More specifically, we have if $2 < \alpha_i$, then

$$\begin{aligned} V(\tau) & = V(E[(A_i(t-\tau)A(t))^{\alpha_i}]) \\ & = (\alpha_i + 2^{\alpha_i-1} \alpha_i^2) \sum_{k=1}^{\tau} E[(A_i(t-\tau)A(t))^{\alpha_i}]. \end{aligned} \quad (56)$$

If $1 < \alpha_i < 2$, then

$$\begin{aligned} V(\tau) & = V\left(E\left[(A_i(t-\tau)A(t))^2\right]\right) \\ & = (\alpha_i + \alpha_i^2) \sum_{k=1}^{\tau} E\left[(A_i(t-\tau)A(t))^2\right]. \end{aligned} \quad (57)$$

If $0 < \alpha_i < 1$ then

$$V(\tau) = V(E[(A_i(t-\tau)A(t))]) = \sum_{k=1}^{\tau} \rho_i(\tau). \quad (58)$$

As described in the theorem, for an arbitrary value $\Delta < c_{max}$, we can find τ_{Δ} , which is the minimum τ such that $|E[A_i(t)]A_i(t-\tau), \dots, A_i(1)] - \lambda_i| < \Delta, \forall i \leq M$. For $\tau > \tau_{\Delta}$, it follows by (55) that

$$E[A_i(t)Q_i(t)] < E[(\lambda_i + \Delta)Q_i(t-\tau)^{\alpha_i}] + V(\tau). \quad (59)$$

To simplify (59), we first define the following notations. At each time slot t , we arrange the queues in a decreasing order of their queue lengths raised to the α_i th power, i.e., $Q_{q_1}(t)^{\alpha_1}, \dots, Q_{q_N}(t)^{\alpha_N}$ with $Q_{q_i}(t)^{\alpha_i} \geq Q_{q_{i+1}}(t)^{\alpha_{i+1}}$, where ties are broken randomly. Then, we have

$$\begin{aligned} & \sum_{i=1}^N Q_i(t-\tau)^{\alpha_i} (\lambda_i + \Delta) = \sum_{j=1}^N Q_{q_j}(t-\tau)^{\alpha_{q_j}} (\lambda_{q_j} + \Delta) \\ & = \sum_{j=1}^{N-1} \left(Q_{q_j}(t-\tau)^{\alpha_{q_j}} - Q_{q_{j+1}}(t-\tau)^{\alpha_{q_{j+1}}} \right) \sum_{n=1}^j (\lambda_{q_n} + \Delta) \\ & \quad + Q_{q_N}(t-\tau)^{\alpha_{q_N}} \sum_{n=1}^N (\lambda_{q_n} + \Delta). \end{aligned} \quad (60)$$

We now evaluate the expectation of term II, i.e., T_{II} , of (50). Specifically, we have

$$T_{II} = E\left[E\left[\sum_{i=1}^N H_i(t)Q_i(t)^{\alpha_i} \mid \mathcal{Q}(t)\right]\right] \quad (61)$$

where

$$\begin{aligned} & E\left[\sum_{i=1}^N H_i(t)Q_i(t)^{\alpha_i} \mid \mathcal{Q}(t)\right] \\ & = \sum_{j=1}^M E\left[\sum_{i=1}^N H_i(t)Q_i(t)^{\alpha_i} \mid \mathcal{Q}(t), K(t) = j\right] P(K(t) = j) \\ & = \sum_{j=1}^M P(K(t) = j) \sum_{i=1}^j Q_{q_i}(t)^{\alpha_{q_i}} \\ & = \sum_{j=1}^N P(K(t) = j) \sum_{i=1}^j Q_{q_i}(t)^{\alpha_{q_i}} \\ & \quad + \sum_{j=N+1}^M P(K(t) = j) \sum_{i=1}^N Q_{q_i}(t)^{\alpha_{q_i}} \\ & = \sum_{j=1}^N Q_{q_j}(t)^{\alpha_{q_j}} \sum_{i=1}^N P(K(t) = i) \\ & \quad + \sum_{j=1}^N Q_{q_j}(t)^{\alpha_{q_j}} P(K(t) > N) \\ & = \sum_{j=1}^N Q_{q_j}(t)^{\alpha_{q_j}} P(K(t) \geq j). \end{aligned} \quad (62)$$

By some computations, we can rewrite (62) as follows

$$\begin{aligned} & \sum_{j=1}^N Q_{q_j}(t)^{\alpha_{q_j}} P(K(t) \geq j) \\ & = \sum_{j=1}^{N-1} (Q_{q_j}(t)^{\alpha_{q_j}} - Q_{q_{j+1}}(t)^{\alpha_{q_{j+1}}}) \sum_{n=1}^j P(K(t) \geq n) \\ & \quad + Q_{q_N}(t)^{\alpha_{q_N}} \sum_{n=1}^N P(K(t) \geq n). \end{aligned} \quad (63)$$

Combining (63), (60), and (50), we obtain

$$\begin{aligned}
& \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t E [L(\mathcal{Q}(\tau+1)) - L(\mathcal{Q}(\tau))] \\
& \leq \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \left\{ \sum_{j=1}^{N-1} \left((Q_{q_j}(t)^{\alpha_{q_j}} - Q_{q_{j+1}}(t)^{\alpha_{q_{j+1}}}) \right. \right. \\
& \quad \left. \left. \times \sum_{n=1}^j (\lambda_{q_n} + \Delta - P(K(t) \geq n)) \right) \right. \\
& \quad \left. + Q_{q_N}(t)^{\alpha_{q_N}} \sum_{n=1}^N (\lambda_{q_n} + \Delta - P(K(t) \geq n)) \right. \\
& \quad \left. + \sum_{i=1}^N \theta Q_i(t)^{\alpha_i} + \sum_{i=1}^N W_i(\theta) + V(\tau_{\Delta}) \right\}. \quad (64)
\end{aligned}$$

By defining

$$d = \max_{Q \subset \{1, \dots, N\}} \left\{ \sum_{i \in Q} \lambda_i + \Delta - \sum_{k=1}^{|Q|} P(K > k) \right\} \quad (65)$$

which is a negative constant by the conditions given in the theorem, we can rewrite (64) as

$$\begin{aligned}
& \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t E [L(\mathcal{Q}(t+1)) - L(\mathcal{Q}(t))] \\
& \leq \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \left\{ \left(\frac{d}{N} + \theta \right) \sum_{i=1}^N Q_i(t)^{\alpha_i} \right. \\
& \quad \left. + \sum_{i=1}^N W_i(\theta) + V(\tau_{\Delta}) \right\}.
\end{aligned}$$

The last inequality holds because q_1 has the largest α -th power queue length. Letting $\theta = -d/(2N)$ and using telescoping sums, we have

$$\begin{aligned}
& \limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=1}^t \sum_{i=1}^N E [Q_i(t)^{\alpha_i}] \\
& \leq \left(-\frac{2N}{d} \right) \left(\sum_{i=1}^N W_i(-\frac{d}{2N}) + V(\tau_{\Delta}) \right) \quad (66)
\end{aligned}$$

where $W_i(\cdot)$ is defined in (47) and (49), respectively and $V(\tau_{\Delta})$ is defined in (56), (57) and (58). ■

Theorem 3: The sufficient and necessary conditions for moment stability is

$$\sum_{i \in Q} \lambda_i < |Q| - \sum_{k=1}^{|Q|} (P(K < k)) \quad \forall Q \subset \{1, \dots, N\} \quad (67)$$

where $K \sim PB(\mathbf{p}, M)$, $\mathbf{p} = (p_1, \dots, p_M)$ and the scheduling policy defined in (38) is throughput optimal, which stabilizes any set of arrival rates within the stability region.

As indicated by (67), the stability region of a dynamic spectrum access network is characterized by the statistics of SU traffic (i.e., λ_i), the PU activities (i.e., p_i), the number of SUs contending the spectrum (i.e., $|Q|$), and the total number of PU channels available to SUs (i.e., M). This region holds for any feasible work conserving policies, which utilize all idle slots of PU channels for transmissions unless SUs have empty queues. Since the work conserving policies are feasible when sensing errors are negligible, (67) actually provides the outer bound of the network stability region under any sensing performance. It can be proven that the above stability region also holds for the strong stability if such stability exists. In this case, the network stability regions under the moment stability and the strong stability overlap with each other. However, the moment stability is stronger than the strong stability. Specifically, if the minimum tail coefficient of all arrivals is larger than 2, both strong stability and moment stability exist, while the latter case not only guarantees the finiteness of the mean but also ensures the finiteness of the higher order moments, such as variance. This is an important property for the QoS oriented applications such as on-line gaming and video conferencing.

C. Impact of Channel Diversity

So far, all the results in the previous sections consider the scenario where every SU transmits using the same data rate as long as the PU channel is free. In this subsection, we analyze the impact of channel diversity on the stability performance under heavy tailed environment. More specifically, let p_{ij} denote the packet delivery ratio of SU_i operating over the channel j . Let $p = (p_{ij})_{i \leq N, j \in M}$ denote packet delivery ratio vector. Due to the channel diversity, p_{ij} can be different for different user i over different channel j . Let $S = (S_i(t))_{i \leq M}$ denote the network channel state. Let \mathcal{S} denote the set consisting of all possible network channel states. We define the maximal schedule as a vector $H = (h_{ij})_{i \leq N, j \leq M}$, which satisfies the constraints defined in (8) and (9). We define \mathcal{H} as the set consisting of all possible maximal schedules. Let $R_H^S = (R_H^S(i))_{i \leq N}$ denote a n -dimensional vector where

$$R_H^S(i) = \sum_{j=1}^M h_{ij} p_{ij} S_j(t), \quad (68)$$

if the maximal schedule H is selected under the channel state S . Otherwise, $R_H^S(i) = 0$.

Based on the definitions above, the network stability region Λ under the channel diversity case can be given as follows

$$\Lambda = \{0 < \boldsymbol{\lambda} < \boldsymbol{\mu}, \boldsymbol{\mu} \in \text{Conv}(R_H^S)\} \quad (69)$$

where $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_N)$ and $\text{Conv}(R_H^S)$ is the convex hull of set $R_{\mathcal{H}}^S = \{R_H^S\}_{\{S \in \mathcal{S}, H \in \mathcal{H}\}}$, i.e.,

$$\text{Conv}(R_{\mathcal{H}}^S) = \left\{ \sum_{n=1}^{|\mathcal{H}| \times |\mathcal{S}|} w_n R_H^S P(S) \right. \\
\left. |w_n > 0, \sum_{n=1}^{|\mathcal{H}| \times |\mathcal{S}|} w_n = 1, R_H^S \in R_{\mathcal{H}}^S \right\}$$

where $P(S)$ is probability that channel state is S .

Lemma 3: The sufficient and necessary conditions for moment stability is $\lambda \in \Lambda$.

The proof follows the similar procedures as proving Theorem 3 and thus is omitted here for brevity. It is worth to note that the throughput-optimal scheduling algorithm, which achieves the moment stability under the channel diversity case, aims to solve the following optimization problem at each time slot t

$$\text{Find : } h_{ij}, \forall i \leq N \quad j \leq M \quad (70)$$

$$\text{Maximize : } \sum_{i,j} h_{ij}(t) Q_i(t)^{\alpha_i} p_{ij} S_j(t) \quad (71)$$

$$\text{subject to : } (7), (8), \text{ and } (9). \quad (72)$$

VI. SIMULATION RESULTS

In this section, we use simulations to illustrate our theoretical results. More specifically, we choose Pareto and exponential distributions to represent HT and LT distributions, respectively. We say a random variable $X \in \mathcal{PAR}(\alpha, x_m)$ if it follows Pareto distribution with parameters α and x_m , i.e., $P(X > x) = (x_m/x)^\alpha$. We say a random variable $X \in \mathcal{EXP}(\lambda)$ if it follows exponential distribution with parameter λ , i.e., $P(X > x) = e^{-\lambda x}$.

We consider a scenario where three SUs are sharing two PU channels. We assume SU_1 and SU_2 are transmitting heavy tailed traffic with traffic arrival processes $A_1(t) \in \mathcal{PAR}(1.5, 1)$ and $A_2(t) \in \mathcal{PAR}(1.2, 1)$ and SU_3 is sending light tailed traffic with $A_3(t) \in \mathcal{EXP}(1/3)$. Consequently, we have $\lambda_1 = E[A_1(t)] = 3$, $\lambda_2 = E[A_2(t)] = 6$, and $\lambda_3 = E[A_3(t)] = 3$. We assume the data rate r of each channel is 18. Channel 1 has an idle probability of 0.6, i.e., $P(S_1(t) = 0) = 0.6$ and channel 2 has an idle probability of 0.4, i.e., $P(S_2(t) = 0) = 0.4$, which means the channel 1 and 2 have a throughput $\mu_1 = rP(S_1(t) = 0) = 10.8$ and $\mu_2 = rP(S_2(t) = 0) = 7.2$, respectively. In this case, the network is steady-state stable because $\sum_{i=1}^3 \lambda_i < \mu_1 + \mu_2$. All the following simulation results are plotted on log-log coordinates, by which HT distribution with tail index α can manifest itself as a straight line with the slope equal to $-\alpha$.

We first investigate the difficulty of achieving strong stability under heavy tailed environment. It is shown in Fig. 2 that under the conventional maximum weight scheduling algorithm, the queue lengths of all the SUs follow heavy tailed distribution with a tail index smaller than 1 because their tail distributions decay slower than the reference Pareto distribution with tail index of 1. This means the SU_3 with light tailed traffic arrivals also has unbounded queueing delay. This is consistent with the Theorem 2, which indicates if the tail index of SU_1 and SU_2 satisfy the condition $\alpha_1 + \alpha_2 < 3$, then SU_3 with light tailed traffic necessarily has infinite average queue length. The above observation implies that heavy tailed traffic can significantly degrade network stability even under multi-channel case.

We next show that the moment stability can be achieved by applying the maximum weight- α scheduling algorithm defined in (38). More specifically, we assign the queues of SU_1 , SU_2 , and SU_3 with weight 0.5, 0.2, and 2, respectively. As indicated by Theorem 3, under such settings, the maximum

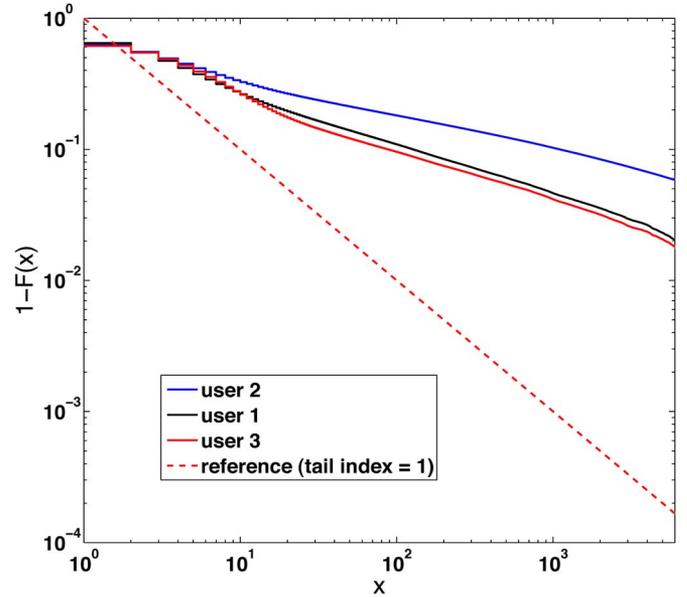


Fig. 2. Queue length distribution of SUs under conventional maximum weight scheduling.

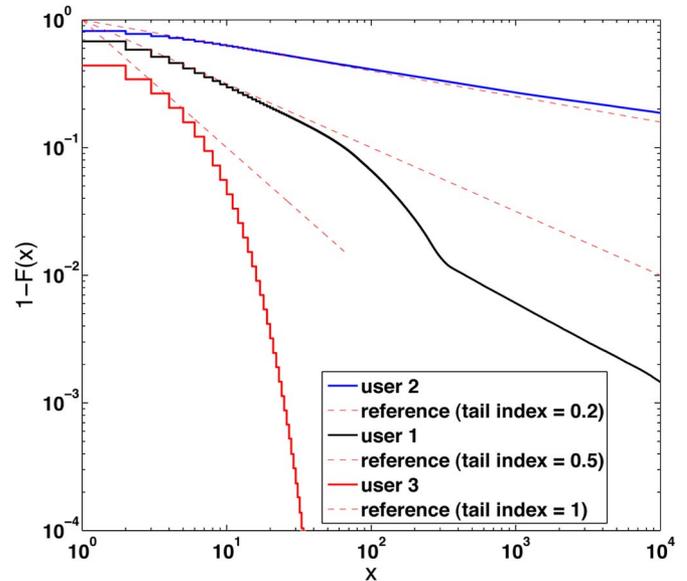


Fig. 3. Queue length distribution of SUs under maximum weight- α scheduling.

weight- α scheduling algorithm can guarantee that SU_3 with light tailed traffic has bounded average queue length, which, as demonstrated in Fig. 2, can not be achieved by applying the conventional maximum weight scheduling algorithm. In particular, it is shown in Fig. 3 that the tail distribution of SU_3 has a slope or decaying rate greater than 1, which implies the queue length of SU_3 is of the finite mean. Moreover, it can be seen that the SU_1 and SU_2 , which send heavy tailed traffic, experience a queueing delay following heavy tailed distribution, which exhibits itself as a straight line parallel to that of the reference Pareto distribution with tail index 0.5 and 0.2, respectively. It is worth to note that according to Lemma 1, 0.5 and 0.2 are also the highest orders of the finite moments SU_1 and SU_2 can have under any scheduling algorithms.

VII. CONCLUSION

In this paper, we introduce a new stability criterion, namely the moment stability, which, compared to the conventional stability criterion, can effectively characterize the QoS performance of dynamic spectrum access networks in the presence of heavy-tailed traffic. Towards this, the necessary and sufficient conditions for the existence of a scheduling algorithm to achieve the moment stability are obtained and the corresponding network stability region is derived. Moreover, we show that the maximum-weight- α scheduling algorithm is throughput optimal with respect to moment stability and thus can effectively prevent heavy-tailed traffic, e.g., video conferencing and on-line gaming traffic, from significantly degrading the queueing performance of light-tailed ones, e.g., email deliveries, audio/voice traffic, and temperature/humidity readings. Moreover, we show that the throughput optimality of the maximum-weight- α scheduling algo holds independent of the marginal distribution of SU's arrival traffic (i.e., heavy-tailed or light-tailed distributed) and the time correlation of SU's arrival traffic (i.e., long range dependent or short range dependent).

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