Network Stability of Cognitive Radio Networks in the Presence of Heavy Tailed Traffic

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***Abstract*—The heavy tailed nature in dynamic spectrum networks challenges the applicability of conventional network stability criterions. To encounter this, a new stability criterion, namely *moment stability*, is introduced, which requires that the queue length of each secondary user has finite moments for every achievable order. Then, the necessary and sufficient conditions for the existence of a resource allocation policy to achieve moment stability are derived. Moreover, the network stability region yielded from these conditions is shown to be directly related to the statistics of secondary user traffics, primary user activities, the number of secondary users contending the spectrum, and the total number of primary user channels available to secondary users. In addition, a throughput-optimal policy, which stabilizes the network for any arrival rates in the stability region, is also introduced. In the end, the impact of the heavy tailed primary user traffic on the network stability is investigated, which shows that the tail heaviness of the primary user traffics determines what types of the network stability are achievable for a dynamic spectrum networks.**

I. Introduction

Dynamic spectrum access (DSA) is an emerging technique that allows the secondary users (SUs) to share the spectrum in an opportunistic manner [1]. Using such scheme, the SUs can access the unoccupied spectrum during idle periods of the primary users (PUs), and stop transmissions when the PU channels become busy. The achievable Quality of Service (QoS) performance of secondary users is significantly affected by the dynamically changing PU traffic and the resource allocation policies used by the SUs.

Heavy tailed traffic has been widely observed in a variety of computer and communication networks such as Ethernet, WLAN, mobile ad-hoc networks, and cellular networks. Such heavy tailed network traffic can be either caused by the inherent heavy tailed distribution in the traffic source such as the file size on the Internet servers, the web access pattern, the scene length of VBR (variable bit rate) video and MPEG streams [6], or caused by the network protocols themselves such as retransmissions and random access schemes [7]. Different from the conventional light tailed traffic (i.e., Markovian or Poisson traffic), heavy tailed traffic exhibits high burstiness or dependence over a long range of time scale. Such highly bursty nature can induce significant performance degradations including the considerably reduced network throughput, queue stability, and system scalability.

Currently, the majority of research in dynamic spectrum access networks focuses on the development of the resource allocation and spectrum management schemes under the assumption of the light tailed behavior of primary and secondary users. Contrary to this conventional assumption, significant empirical evidence establishes that both PU and SU traffics can actually exhibit the heavy tailed nature. As for the primary users, it is shown that the call holding time of mobile users in 3G cellular networks and the session duration of licensed users in WLANs show heavy tailed statistics [12][3]. On the other hand, the emerging applications such as battlefield surveillance, video conferencing, and on-line gaming require the secondary users to support multimedia traffic, which is inherently bursty and heavy tailed distributed. Particularly, it has been shown that such heavy tailed behavior not only has a significant impact on the spectrum sensing performance [12] but also greatly degrades the delay performance of secondary users [11] [10]. Despite its importance, the network performance and control of dynamic spectrum access networks in the realistic heavy tailed environment is still an under-explored area. In this paper, we aim to study the network stability of dynamic spectrum access networks in the presence of heavy tails and provide valuable insights for designing effective spectrum allocation schemes accordingly.

The three most common types of stability in the literature include rate stability, steady-state stability, and strong stability [5][8]. Particularly, rate stability regulates the relationship between the time average arrival rates and service rates, while steady-state stability demands the existence of steady-state distributions. Different from the first two definitions, strong stability has more strict criterion by requiring each queue to have finite time-average expected queue length, which is a desirable property for most of the queueing systems with explicit QoS requirements.

However, thanks to the heavy tailed nature of PU and SU traffics, the conventional stability criterion may need to be revisited for facilitating the design of QoS oriented resource allocation schemes over highly dynamic environment. This can be seen from two perspectives. (1) Although the strong stability has been proven to be achievable in many complex systems, such stability performance is difficult to obtain in a heavy tailed environment. Specifically, it is known that the queuing systems with heavy tail arrival traffic or heavy tailed service time inherently lead to heavy tailed queueing delay, which may have unbounded expectation and variance. What is more important, recent research [4] found that when two queues share the same server under maximum weight scheduling policy, the queue with light tailed traffic can experience infinite mean delay if the other queue has heavy tailed traffic with infinite variance. (2) Moreover, even if strong stability exists in the presence of heavy tailed traffic, it does not necessarily imply the boundedness of the higher order moments, such as delay variance (jitter), which is one of the key metrics for the QoS oriented applications such as VoIP, on-line gaming and video conferencing.

The above observations motivate us to investigate the network stability from a new perspective. Specifically, we introduce *moment stability*, which, in the context of cognitive radio networks, requires that the steady-state queue length of each secondary user i has finite moments up to its maximum achievable order.(see Section III for more details). Accordingly, the *network stability region* is defined as the closure of the set of all arrival rate vectors for which the queues of all secondary users can be stabilized by a feasible scheduling policy. Moreover, a scheduling policy is *throughput optimal* if it stabilizes the system for any arrival rates in the stability region.

In this paper, we consider a cognitive radio network in which multiple SUs opportunistically exploit the spectrum holes of multiple PU channels. Each PU channel is modeled by an ON/OFF process, in which channel stays in ON state if it is busy and OFF, otherwise. Each SU is associated with an input queue and a random number of packets arrives to the queue at each time slot. Each SU is equipped with one transceiver, which can switch to any PU channel, but no two SUs can share the same channel at the same time. For the detailed description of this model, see Section II.

We first show the difficulty of achieving moment stability in the presence of heavy tailed traffic by proving the infeasibility of maximum weight scheduling policy, which is known to be throughput optimal with respect to strong stability. Then, we derive the necessary and sufficient conditions for the existence of a feasible scheduling policy to achieve moment stability. The network stability region yielded from these conditions is shown to be directly related to the statistics of SU traffics, PU activities, the number of SUs contending the spectrum, and the total number of PU channels available to SUs. Moreover, a throughput-optimal policy, which stabilizes any set of arrival rate vectors in the stability region, is also introduced. In the end, the impact of the heavy tailed PU traffic on the network stability is investigated, which shows that the network stability is determined by the tail heaviness of the PU traffic.

The rest of this paper is organized as follows. Section II introduces system model and preliminaries. Section III formally introduces moment stability, presents the network stability region, and investigates the impact of heavy tail PU channel on the stability performance. Finally, Section IV concludes this paper.

II. System Model and Preliminaries

A. System Model

We consider a cognitive radio network consisting of N SUs and M PU channel, as shown in Fig.1. Time is slotted and during each time slot, only one packet is transmitted. Let $\mathbf{S} = (S_1(t), S_2(t), ..., S_M(t)$ denote the states of PU channels. $S_i(t) \in \{0,1\}, \forall i \in 1, ..., M$ with $S_i(t) = 0$ if channel i is busy and $S_i(t) = 1$ if channel i is idle. The processes $(S_1(t), S_2(t), ..., S_M(t)$ are independent with each other and $S_i(t)$ are i.i.d. from slot to slot, distributed according to Bernoulli process with expected mean p_i, i.e., $P(S_i(t) = 1) = p_i$. At each time slot t, each secondary user $i \in \{1, 2, ..., N\}$ receives $A_i(t)$ packets, which is i.i.d. from slot to slot, and the arrival process $A_i(t), i = 1, ..., N$ is independent from each other and independent of the PU channel states. At each time slot, a scheduling/control policy allocates the detected idle channels to the secondary users with knowledge only of the current queue lengths and instantaneous channel states. Since our primary objective is to study the impact of heavy tailed traffic on network stability, we consider the scenario where the sensing errors are negligible. The above network model presents the downlink or uplink scheduling problem for the centralized networks. The practical networks represented by this model include cellular, WiFi and mesh networks with coexisting licensed and unlicensed users.

B. Queueing Dynamics

Let $Q_i(t)$ denote the number of packets in the queue q_i of secondary user i by the end of time slot i. Define $h_{ij}(t)$ as the number of packets which can be released from queue i if channel j is allocated to queue i at time slot t. Based on the above model, $h_{ij} \in \{0,1\}, \forall i, j$. Then, the queueing dynamics of the secondary user i can be represented by

$$Q_i(t+1) = Q_i(t) - \sum_{j=1}^{M} h_{ij}(t) S_j(t) + A_i(t) \qquad (1)$$

subject to

$$h_{ij}(t) \in \{0,1\}, \quad \forall i, j \qquad (2)$$

$$0 \le \sum_{j=1}^{M} h_{ij}(t) \le 1 \quad \forall i \qquad (3)$$

$$0 \le \sum_{i=1}^{N} h_{ij}(t) \le 1 \quad \forall j \qquad (4)$$

where the second constraint implies that each secondary user can only be allocated with one channel, while the third constraint means that each channel can only be assigned to one secondary user. By defining $H_i(t)$ as the number of packets which depart from queue i at time slot t under a certain control policy, the queueing dynamics in (1) can be rewritten by

$$Q_i(t+1) = Q_i(t) - H_i(t) + A_i(t) \qquad (5)$$

Note that based on the above model, $H_i(t) \in \{0,1\}, \forall i = 1, ..., N$.

C. Preliminaries

In this paper we use the following notations. For any two real functions $a(t)$ and $b(t)$, we let $a(t) \sim b(t)$ denote $\lim_{t\to\infty} a(t)/b(t) = 1$. For any two non-negative r.v.s X and Y, we say that $X \le_{a.s.} Y$ if $X \le Y$ almost surely, and $X \le_{s.t.} Y$ if X is stochastically dominated by Y, i.e., $P(X > t) \le P(Y > t)$ for all $t \ge 0$. We say $X \overset{d}{=} Y$ if X

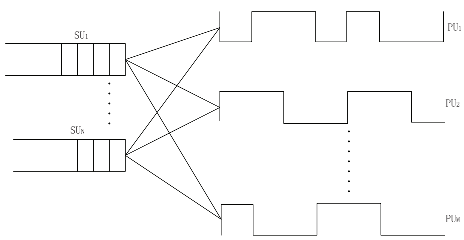

Fig. 1. System model.

and Y are equal in distribution. Also, let $F(x) = P(X \leq x)$ denote the cumulative distribution function (cdf) of a non-negative r.v. X. Let $\overline{F}(x) = P(X > x)$ denote its tail distribution function.

Definition 1 *A r.v. X is heavy tailed (HT) if for all $\theta > 0$*

$$\lim_{x \to \infty} e^{\theta x} \overline{F}(x) = \infty, \tag{6}$$

or, equivalently, if for all $z > 0$

$$E[e^{zX}] = \infty. \tag{7}$$

Definition 2 *A r.v. X is light tailed (LT) if it is not heavy tailed or, equivalently, if there exists $z > 0$ such that*

$$E[e^{zX}] < \infty. \tag{8}$$

Remark 1 Generally speaking, a r.v. is HT if its tail distribution decreases slower than exponentially. On the contrary, a r.v. is LT if its tail distribution decreases exponentially or faster. Some typical HT distributions include Pareto and log-normal, while the typical LT distributions include exponential and Gamma.

Based on the existence of the moments, we define the tail coefficient of a non-negative random variable.

Definition 3 *The tail coefficient κ_X of a nonnegative random variable X is defined by*

$$\kappa_X = \sup\{k \geq 0 : E[X^k] < \infty\} \tag{9}$$

Remark 2 In other words, the tail coefficient defines the threshold order above which a random variable has infinite moments. Some HT distributions, such as Pareto, have finite tail coefficient or equivalently have infinite moments of certain orders, while some HT distribution, such as log-normal, may have infinite tail coefficient or equivalently have finite moments of all orders. In this research, we focus on the random variables which are heavy tail distributed with finite tail coefficient.

An important subclass of HT distributions which have finite tail coefficient is the regularly varying distributions

Definition 4 *A r.v. X is called regularly varying with tail index $c > 0$, denoted by $X \in \mathcal{RV}(c)$, if*

$$\overline{F}(x) \sim x^{-c} \mathcal{L}(x), \tag{10}$$

where $L(x)$ is a slowly varying function.

Remark 3 Regularly varying distributions are a generalization of Pareto/Ziph/power-law distributions. The tail index c indicates how heavy the tail distribution is, where smaller values of c imply heavier tail. Moreover, for a r.v. $X \in \mathcal{RV}(c)$, the tail coefficient κ_X of X is equal to the tail index c, which defines the maximum order of bounded moments X can have. Specifically, if $0 < \alpha < 1$, X has infinite mean and variance. If $1 < \alpha < 2$, X has finite mean and infinite variance.

III. Network Stability of Dynamic Spectrum Access Networks

In this section, the definition of moment stability is introduced, and the necessary and sufficient conditions for the existence of moment stability are derived. Then, the impact of heavy tailed PU channel on network stability is studied.

A. Stability Definitions

We consider three types of stability for the queueing system of the secondary users: steady-state stability, strongly stability, and moment stability.

Definition 5 *The cognitive radio network is steady-state stable if the queue length process of each secondary user is positive recurrent.*

Definition 6 *The cognitive radio network is strongly stable if the steady-state queue length of each secondary user has finite mean.*

Definition 7 *Let Q_i denote the steady-state queue length of queue i. Define β_i as the maximum achievable order of finite moments of Q_i under any scheduling policies.*

Under the queueing dynamics defined in (5), the maximum achievable order β_i can be given by the following Lemma.

Lemma 1 *If queue i has HT arrivals with tail coefficient c_i, i.e., $A_i(t) \in \mathcal{RV}(c_i)$, then $\beta_i = c_i - 1$. If queue i has LT arrivals, i.e., $A_i(t) \in LT$, then $\beta_i = \infty$.*

Proof: Consider the best case for queue i by assuming queue i has higher priority than any other queues, which means queue i is allocated to an available idle channel whenever its queue length is not empty. Thus, queue i acts as a $Geo^{[X]}/Geo/1$ queue. If $A_i(t) \in \mathcal{RV}(c_i)$, it is known that the tail distribution of the steady-state queue length is also regularly varying and is one order heavier than that of the arrival process, which implies $\beta_i = c_i - 1$. If $A_i(t) \in LT$, it is know that the i-th moment of the steady-state queue length scales with $i+1$-th moment of the service time, which is also LT and always has finite moments for any order. This means that the queue length of queue i has finite moments of all orders, thus implying $\beta_i = \infty$. ■

Definition 8 *A cognitive radio network is moment stable if the steady-state queue length of each secondary user i has finite moments up to the order β_i.*

Remark 4 Among the stability definitions above, steady-state stability is the weakest one, which only requires the existence of the stationary distribution for the queue length processes. Compared with strong stability, moment stability not only requires the finiteness of lower order moments, such as mean, but also demands the boundedness of higher order moments, such as variance, provided that such moments exist. What is more important, moment stability guarantees that the probability of occurrence of large queueing delay is minimized in the sense that the maximum achievable order of finite moment β_i characterizes the fastest decaying rate of the tail distribution of queue length, i.e., $\beta_i = -\log P(Q_i > t)/\log t$, as $t \to \infty$ and thus ensuring that each queue has the maximum order of finite moments β_i implies the minimization of the probability of the SU having large queueing delay.

Although moment stability is a desirable property to promise QoS guaranteed applications, the conventional scheduling policies, which are effective under the light tailed traffic, may have difficulty in achieving moment stability in the presence of heavy tails. In the following, we show that the celebrated maximum weight scheduling, which is known to be capable of achieving strong stability in the presence of light tail traffic, can induce unbounded queueing delay for SUs with mixed heavy tailed and light tailed arrivals. To see this, we consider a dynamic spectrum network, which consists of three secondary user queues and two primary user channels. Assume that queue 1 and queue 2 have HT traffic with tail coefficient c_1 and c_2, i.e., $A_1(t) \in \mathcal{RV}(c_1)$ and $A_2(t) \in \mathcal{RV}(c_2)$, and queue 3 has light tailed arrivals, i.e., $A_3(t) \in LT$. We show in the following Lemma 2 that under certain conditions, maximum weight scheduling leads to the unbounded expected steady-state queue length for queue 3 even if it has light tailed arrivals.

At each time slot t, the maximum weight scheduling algorithm chooses the channel allocation which satisfies

$$\max \sum_{i,j} h_{ij}(t)Q_i(t)S_j(t), \quad \forall i = \{1,2,3\} \quad j = \{1,2\} \tag{11}$$

subject to (2), (3), and (4).

Lemma 2 *Assume queue q_1, q_2 and q_3 have arrivals $A_1(t) \in \mathcal{RV}(c_1)$, $A_2(t) \in \mathcal{RV}(c_2)$, and $A_3(t) \in LT$, respectively. If $c_1 + c_2 < 3$, under the maximum weight scheduling policy, the expected queue length of queue q_3 is unbounded, i.e., $E[Q_3(t)] = \infty$.*

Remark 5 The above Lemma implies that in the presence of heavy tailed arrivals, the maximum weight scheduling can achieve neither moment stability nor strong stability since the queueing delay for the SU queues with light tailed arrivals is necessarily unbounded. This conclusion is universal to a certain extent, which is independent of the exact distribution of arrival processes, the incoming traffic intensity (i.e., $E[A_i]$) of the SU queues, and the PU channel status (i.e., the channel idle probability p_i). We conjecture that under more general case where more SUs and PU channels are present, the similar conclusions also hold as long as the the SU queues with light tailed arrivals have to compete the available spectrum with the SU queues with heavy tailed arrivals.

Proof: Assume the queueing system is in the steady state. Suppose that a message m of size $A_3(t)$ arrives to q_3 at time slot t. Define $U_3(t)$ as the queueing delay of the message m, which is the difference between the time when the Head-of-Line (HoL) packet of the message is served and the time when the message arrives to the queue. We evaluate the queueing delay $U_3(t)$ in the following two scenarios: $\mathcal{E} = \{Q_3(t) < Q_1(t) \wedge Q_3(t) < Q_2(t)\}$ and its complementary set $\mathcal{E}^c$.

Consider the case $\mathcal{E}$. Under the maximum weight scheduling policy, both q_1 and q_2 keep being served until q_3 has a queue length longer than at least one of the two queues. In this case, the smallest queueing delay occurs in the following scenario. As soon as the HoL packet of the message m arrives to q_3, both q_1 and q_2 do not receive any messages and the two channels keep idle so that q_1 and q_2 are served at each time slot, until one of q_1 and q_2 reaches the same queue length as q_3. At this moment, q_3 begins to be served and the following condition holds

$$Q_3(t) + \sum_{i=1}^{\Delta} A_3(i) \geq \min(Q_1(t), Q_2(t)) - \Delta \tag{12}$$

where Δ is the number of time slots q_3 spends catching up with either q_1 or q_2, whichever has the shorter queue length. In this case, the queueing delay $U_3(t)$ is lower bounded by

$$U_3(t) \geq Q_3(t) + \Delta \tag{13}$$

which implies

$$E[U_3(t)|\mathcal{E}] = E[Q_3(t)|\mathcal{E}] + E[\Delta|\mathcal{E}]. \tag{14}$$

By (12), independence of Δ and $A_i(t)$, and iterated expectations, $E[Q_3(t)|\mathcal{E}]$ is lower bounded by

$$E[\Delta|\mathcal{E}] \geq \frac{1}{\lambda_3+1}(E[\min(Q_1(t), Q_2(t))|\mathcal{E}] - E[Q_3(t)|\mathcal{E}]) \tag{15}$$

which, combining with (14), yields

$$\begin{aligned} E[U_3(t)|\mathcal{E}] &\geq \frac{1}{\lambda_3+1}(E[\min(Q_1(t), Q_2(t))|\mathcal{E}] \\ &\quad -\frac{\lambda_3}{\lambda_3+1}E[Q_3(t)|\mathcal{E}] \\ &\geq \frac{1}{\lambda_3+1}E[\min(Q_1(t), Q_2(t))|\mathcal{E}] \end{aligned} \tag{16}$$

This implies

$$E[U_3(t)1_{\mathcal{E}}] \geq \frac{1}{\lambda_3+1}E[\min(Q_1(t), Q_2(t))1_{\mathcal{E}}] \tag{17}$$

where $1_{\mathcal{E}}$ is the indicator function which equals 1 if $\mathcal{E}$ occurs and 0 otherwise.

We now consider the case $\mathcal{E}^c$, where $Q_3(t)$ is larger than at least one of $Q_1(t)$ and $Q_2(t)$, i.e., $Q_3(t) > \min(Q_1(t), Q_2(t))$. By the fact $U_3(t) > Q_3(t)$, it follows

$$E[U_3(t)|\mathcal{E}^c] \geq E[\min(Q_1(t), Q_2(t))|\mathcal{E}^c], \tag{18}$$

which implies

$$E[U_3(t)1_{\mathcal{E}^c}] \geq \frac{1}{\lambda_3+1}E[\min(Q_1(t), Q_2(t))1_{\mathcal{E}^c}]. \tag{19}$$

This, in conjunction with (17), implies

$$E[U_3(t)] \geq \frac{1}{\lambda_3+1}E[\min(Q_1(t), Q_2(t))]. \tag{20}$$

We next evaluate $E[\min(Q_1(t), Q_2(t))]$ by constructing two fictitious queues, $\widetilde{q_1}$ and $\widetilde{q_2}$, which have the same packet arrivals as q_1 and q_2, respectively. Both $\widetilde{q_1}$ and $\widetilde{q_2}$ have a service rate 1, i.e., each queue is connected to a PU channel, which is always idle. Suppose both the original and fictitious queueing systems are in the steady state. It is easy to check, under any scheduling policies, the queue length of the fictitious queue is less than or equal to that of the original queue, for every sample path. This means the steady-state queue length of the fictitious queue is stochastically dominated by that of the original queue. Define $\widetilde{Q}_1(t)$ and $\widetilde{Q}_2(t)$ as the steady-state queue lengthes at $\widetilde{q_1}$ and $\widetilde{q_2}$, respectively. We have

$$P(Q_i(t) > \tau) \geq P(\widetilde{Q}_i(t) > \tau), \quad i = 1, 2 \tag{21}$$

Since $\widetilde{q_1}$ and $\widetilde{q_2}$ are $GI/GI/1$ queues with regularly varying message size, it is known that the tail distribution of the queue length is one order heavier than the tail distribution of the message size. Therefore, by the assumption that $A_1(t) \in \mathcal{RV}(c_1)$ and $A_2(t) \in \mathcal{RV}(c_2)$, we have

$$\lim_{\tau\to\infty} \frac{\log[P(\widetilde{Q}_i(t) > \tau)]}{\log \tau} = -c_i + 1, \quad i = 1, 2 \tag{22}$$

Moreover, by the independence of $\widetilde{Q}_1(t)$ and $\widetilde{Q}_2(t)$, we have

$$\begin{aligned} P(min(Q_1(t), Q_2(t)) > \tau) &\geq P(\widetilde{Q}_1(t) > t \wedge \widetilde{Q}_2(t) > t) \\ &= P(\widetilde{Q}_1(t) > t)P(\widetilde{Q}_2(t) > t) \end{aligned}$$

This, combining with (22) and the assumption $c_1 + c_2 < 3$, implies

$$\lim_{\tau\to\infty} \frac{\log[min(Q_1(t), Q_2(t)) > \tau]}{\log \tau} \geq -(c_1 + c_2 - 2) > -1 \tag{23}$$

which, by applying the Theorem 2 in [2], implies $E[\min(Q_1(t), Q_2(t))] = \infty$. Combining with (20), we have $E[U_3(t)] = \infty$. This, by Little's law, means that the expected number of messages in q_3 is unbounded and consequently the steady-state queue length of q_3 has infinite mean. This completes the proof. ■

B. Necessary Condition of Moment Stability

In the previous section, we show that the conventional scheduling algorithms designed for the light tailed environment could be ineffective in achieving moment stability in the presence of heavy tailed traffic. In this and next sections, we derive the necessary and sufficient conditions under which there exists a feasible scheduling policy to achieve moment stability. In addition, the throughput-optimal scheduling algorithm is introduced, which ensures that each SU queue can have the maximum achievable bounded moments whenever arrival rates lie in the stability region.

Theorem 1 *If there exists a scheduling policy that achieves moment stability of the system, then*

$$\sum_{i\in Q} \lambda_i \leq |Q| - \sum_{k=1}^{|Q|} P(K < k), \quad \forall Q \subset \{1, ..., N\} \tag{24}$$

where $\lambda_i = E[A_i(t)]$, $|Q|$ denotes the cardinality of set Q, and K is the number of idle channels among total M channels at each time slot t, which follows poisson binomial distribution, denoted by $K \sim PB(\boldsymbol{p}, M)$, $\boldsymbol{p} = (p_1, ..., p_M)$, i.e.,

$$P(K < k) = \sum_{l=0}^{k} \sum_{A\in F_l} \left(\prod_{j\in A} p_j \prod_{j\in A^c} (1 - p_j) \right), \tag{25}$$

where A^c is the complementary set of A, and F_l is the collection of all subsets of l integers that are selectable from set $\{1, ..., M\}$.

Remark 6 Intuitively speaking, the right hand in (24) is the maximum time-average throughput the cognitive radio network can achieve, under the constraints that at each time slot, each secondary user can only access one PU channel, while each PU channel can only serve one secondary user.

Remark 7 It can be shown that the inequality in (24) is also the necessary condition for the existence of the strong stability provided that such stability is achievable, which, by Lemma 1, requires that either all secondary users have LT arrivals or the minimum tail coefficient of all HT arrivals is larger than 2.

Proof: Suppose the system is moment stable under certain resource allocation policy, which, by Definition 8, implies that the system is steady-state stable. This means for each queue, the incoming rate is equal to the service rate, i.e., $E[A_i(t)] = E[H_i(t)]$. Thus, for any subset $Q \subset \{1, .., N\}$, we have

$$\sum_{i\in Q} E[A_i(t)] = \sum_{i\in Q} E[H_i(t)] \tag{26}$$

which, by defining $K(t)$ as the number of idle channels at time slot t, can be rewritten as

$$\sum_{i\in Q} E[A_i(t)] = E\left[E\left[\sum_{i\in Q} H_i(t)|K(t), Q_i(t-1), i \in Q\right]\right] \tag{27}$$

The event $B = \{K(t), Q_i(t-1), i \in Q\}$ can be partitioned into three disjoint sets

$$\begin{aligned} B_1 &= \{K(t) = 0\} \\ B_2 &= \{K(t) = 0\}^c \wedge \{Q_i(t-1) = 0, i \in Q\} \\ B_3 &= \{K(t) = 0\}^c \wedge \{Q_i(t-1) = 0, i \in Q\}^c \end{aligned} \tag{28}$$

It is easy to verify that

$$E\left[\sum_{i\in Q} H_i(t)|B_i\right] = 0, \quad i = 1, 2 \tag{29}$$

As to event B_3, we can further divide it into two disjoint sets

$$\begin{aligned} B_3^{(1)} &= \{K(t) < |Q|\} \wedge \{Q_i(t-1) = 0, i \in Q\}^c \\ B_3^{(2)} &= \{K(t) \geq |Q|\} \wedge \{Q_i(t-1) = 0, i \in Q\}^c \end{aligned} \tag{30}$$

which, in conjunction with (29) and (27), implies

$$\sum_{i\in Q} E[A_i(t)] = E\left[\sum_{i\in Q} H_i(t) 1_{B_3^{(1)}}\right] + E\left[\sum_{i\in Q} H_i(t) 1_{B_3^{(2)}}\right] \tag{31}$$

Define $k_j = \{K(t) = j\} \wedge \{Q_i(t-1) = 0, i \in Q\}^c$ as the event that there are j idle channels and at least one of the queues is not empty. For the first term on the right side of (31), we have

$$
\begin{aligned}
E\left[\sum_{i\in Q} H_i(t) 1_{B_3^{(1)}}\right] &= \sum_{j=1}^{|Q|-1} \sum_{i\in Q} E[H_i(t)|k_j]P(k_j) \\
&\le \sum_{j=1}^{|Q|-1} jP\left(\{K(t)=j\}\right. \\
&\qquad \left.\wedge \{Q_i(t-1)=0, i\in Q\}^c\right) \\
&\le \sum_{j=1}^{|Q|-1} jP(K(t)=j) \qquad (32)
\end{aligned}
$$

The second inequality is due to the fact that $\sum_{i\in Q} H_i(t) \le K(t)$ if $K(t) \le |Q|-1$. For the second term on the right side of (31), we have

$$
\begin{aligned}
E\left[\sum_{i\in Q} H_i(t) 1_{B_3^{(1)}}\right] &= \sum_{j=|Q|}^{M} \sum_{i\in Q} E[H_i(t)|k_j]P(k_j) \\
&\le |Q| \sum_{j=|Q|}^{M} P\left(\{K(t)=j\}\right. \\
&\qquad \left.\wedge \{Q_i(t-1)=0, i\in Q\}^c\right) \\
&\le |Q|P(K(t) \ge |Q|) \qquad (33)
\end{aligned}
$$

The second inequality holds because the number of channels allocated can not exceed the maximum number of queues, which implies $\sum_{i\in Q} H_i(t) \le |Q|$ if $K(t) \ge |Q|$. Combining (31), (32), and (33), we have

$$
\begin{aligned}
\sum_{j\in Q} \lambda_j &\le \sum_{j=1}^{|Q|-1} jP(K(t)=j) + |Q|P(K(t)\ge|Q|) \\
&= \sum_{j=1}^{|Q|-1} \left(\sum_{i=j}^{|Q|-1} P(K(t)=i) + |Q|P(K(t)\ge|Q|\right) \\
&= |Q| - \sum_{j=1}^{|Q|} P(K(t)<j) \qquad (34)
\end{aligned}
$$

Since $K(t)$ follows poisson binomial distribution, this completes the proof. ∎

C. Sufficient Condition of Moment Stability

We next prove the sufficient condition of moment stability by first introducing the maximum-weight-α scheduling, which associates each queue with a different parameter α and makes the scheduling decision based on the queue lengths raised to the α-th power. It is shown in [4] under a simple queueing system with two queues and one server, maximum-weight-α scheduling is effective to mitigate the impact of the queue with heavy-tailed traffics on the other queue with light-tailed traffics. In the following, we design a maximum-weight-α based spectrum allocation policy, which is proven to be able to achieve moment stability in the much more complex and time-varying cognitive radio network.

The proposed spectrum allocation policy works as follows. For N queues $\{q_i\}_{1\le i\le N}$, each queue q_i is assigned with a positive parameter $\alpha_i < \beta_i$, where β_i, as defined in Definition 7, is the threshold order below which q_i has finite moments under any scheduling algorithms. Specifically, as indicated in Lemma 1, $\beta_i = \kappa_{A_i(t)} - 1$ if queue i has HT arrivals $A_i(t)$ with tail index $\kappa_{A_i(t)}$. If queue i has LT arrivals, β_i can be any positive value. During each time slot t, the scheduling algorithm chooses the channel allocation which satisfies the condition

$$\max \sum_{i,j} h_{ij}(t) Q_i(t)^{\alpha_i} S_j(t) \qquad (35)$$

subject to (2), (3), and (4). Note that if all parameters $\{\alpha_i\}_{1\le i\le N}$ are equivalent, maximum-weight-α scheduling becomes the conventional maximum-weight scheduling.

Theorem 2 *The cognitive radio network is moment stable under the scheduling policy defined in* (35)*, if*

$$\sum_{i\in Q} \lambda_i < |Q| - \sum_{k=1}^{|Q|} P(K<k) \quad \forall Q \subset \{1,...,N\} \qquad (36)$$

where $K \sim PB(\boldsymbol{p}, M)$, $\boldsymbol{p} = (p_1, ..., p_M)$ and each queue i has the α_i-th moment of its steady-state queue length upper bounded by

$$E[Q_i(t)^{\alpha_i}] \le \left(-\frac{2N}{d}\right) \sum_{i=1}^{N} W_i\left(-\frac{d}{2N}\right) \qquad (37)$$

where $d = \max_{Q\subset\{1,...,N\}} \left\{\sum_{i\in Q} \lambda_i - \sum_{k=1}^{|Q|} P(K>k)\right\}$ and $W_i()$ follows (42) *if $1 \le \alpha_i \le \beta_i$ and* (44) *if $0 < \alpha_i < 1$.*

Remark 8 The Theorem above indicates that under the proposed scheduling algorithm, each SU queue can achieve the queueing delay with desirable bounded moment as long as its orders is less than the maximum achievable one. The major advantage of such algorithm is that it can prevent the queues with heavy tailed arrivals from impacting the queues with light tailed arrivals. For example, assume there exist three queues, with arrival processes of $A_1(t) \in \mathcal{RV}(1.5)$, $A_2(t) \in \mathcal{RV}(1.5)$, and $A_3(t) \in LT$, using conventional maximum weight scheduling algorithm, it is shown by Lemma 2 that all three queues have unbounded queueing delay. In contrary, under the proposed algorithm, let queue 1 and 2 be assigned with $c_1 = 0.5$ and $c_2 = 0.5$, respectively. By setting proper value for c_3, we can ensure that queue 3 has bounded moments of any orders even though queue 1 and 2 still have unbounded delay (since their maximum achievable order of unbounded moment is 0.5). Particularly, if letting $2 > c_3 > 1$, queue 3 is guaranteed to have bounded mean delay. If letting $c_3 > 2$, queue 3 will have bounded delay variance (jitter).

Proof: Let $\mathcal{Q}(t) = (Q_1(t), ..., Q_N(t))$ denote a vector process of queue lengths of N secondary users. We define the Lyapunov function:

$$L(\mathcal{Q}(t)) = \sum_{i=1}^{N} L(Q_i(t)) \qquad (38)$$

where

$$L(Q_i(t)) = \frac{Q_i(t)^{\alpha_i+1}}{\alpha_i+1} \quad (39)$$

We next evaluate each term $L(Q_i(t))$ under two cases:$1 \leq \alpha_i \leq \kappa(A_i(t)) - 1$ and $0 < \alpha_i < 1$. For the first case, using queueing dynamics in (5) and Taylor's expansions with the Lagrange form of the remainder [4], we have

$$\begin{aligned} L(Q_i(t+1)) &= \frac{1}{\alpha_i+1}(Q_i(t)+A_i(t)-H_i(t))^{\alpha_i+1} \\ &= \frac{Q_i(t)^{\alpha_i+1}}{\alpha_i+1} + \Delta_i(t)Q_i(t)^{\alpha_i} + \alpha_i\frac{\Delta_i(t)^2}{2}\delta^{\alpha_i-1} \end{aligned} \quad (40)$$

where $\Delta_i(t) = A_i(t) - H_i(t)$ and $\delta = [Q_i(t)-1, Q_i(t)+A_i(t)]$. Therefore, by the fact that $\Delta_i(t)^2 \leq A_i(t)^2+1$ and $(Q_i(t)+A_i(t))^{\alpha_i-1} < 2^{\alpha_i-1}(Q_i(t)^{\alpha_i-1}+A_i(t)^{\alpha_i-1})$, for any positive constant θ, we have

$$\begin{aligned} &E[L_i(Q_i(t+1)) - L_i(Q(t))|\mathcal{Q}(t)] \\ &= Q_i(t)^{\alpha_i}E[\Delta_i(t)|\mathcal{Q}(t)] + \frac{\alpha_i}{2}E[\Delta_i(t)^2\delta^{\alpha_i-1}|\mathcal{Q}(t)] \\ &\leq E[(A_i(t)-H_i(t))Q_i(t)^{\alpha_i}|\mathcal{Q}(t)] \\ &\quad +2^{\alpha_i-2}\alpha_iE[A_i(t)^2+1]Q_i(t)^{\alpha_i-1} \\ &\quad +2^{\alpha_i-2}\alpha_iE[A_i(t)^{\alpha_i+1}+A_i(t)^{\alpha_i-1}] \\ &\leq E[(A_i(t)-H_i(t)+\theta)Q_i(t)^{\alpha_i}|\mathcal{Q}(t)] + W_i(\theta) \end{aligned} \quad (41)$$

where

$$\begin{aligned} W_i(\theta) \;=\; & \left(\theta^{-1}2^{\alpha_i-2}\alpha_iE[A_i(t)^2+1]\right)^{\alpha_i-1} \\ & +2^{\alpha_i-2}\alpha_iE[A_i(t)^{\alpha_i+1}+A_i(t)^{\alpha_i-1}] \end{aligned} \quad (42)$$

The last inequality in (41) holds because $1 < \alpha_i < \kappa(A_i(t))-1$, which implies that $E[A_i(t)^2]$, $E[A_i(t)^{\alpha_i+1}]$, and $E[A_i(t)^{\alpha_i-1}]$ are finite.

For the second case $0 < \alpha_i < 1$, by the similar arguments, we obtain

$$\begin{aligned} &E[L_i(Q_i(t+1)) - L_i(Q(t))|\mathcal{Q}(t)] \\ &\leq E[(A_i(t)-H_i(t)+\theta)Q_i(t)^{\alpha_i}|\mathcal{Q}(t)] + W_i(\theta) \end{aligned} \quad (43)$$

where

$$W_i(\theta) = \theta + 1 + E[A_i(t)^{\alpha_i+1}]. \quad (44)$$

By (38), (41), and (43), the conditional Lyapunov drift is upper bounded by

$$\begin{aligned} & E[L(\mathcal{Q}(t+1)) - L(\mathcal{Q}(t))|\mathcal{Q}(t)] \\ \leq\; & \sum_{i=1}^{N}((\lambda_i+\theta)Q_i(t)^{\alpha_i} + W_i(\theta)) \\ & -E\left[\sum_{i=1}^{N}H_i(t)Q_i(t)^{\alpha_i}|\mathcal{Q}(t)\right] \end{aligned} \quad (45)$$

We now evaluate the expectation on the right side of (45). We first define the following notations. At each time slot t, we arrange the queues in a decreasing order of their queue lengthes raised to the α_i th power, i.e., $Q_{q_1}(t)^{\alpha_1}, ..., Q_{q_N}(t)^{\alpha_N}$ with $Q_{q_i}(t)^{\alpha_i} \geq Q_{q_{i+1}}(t)^{\alpha_{i+1}}$, where ties are broken randomly. Then, we have

$$\begin{aligned} & E\left[\sum_{i=1}^{N}H_i(t)Q_i(t)^{\alpha_i}|\mathcal{Q}(t)\right] \\ &= \sum_{j=1}^{M}E\left[\sum_{i=1}^{N}H_i(t)Q_i(t)^{\alpha_i}|\mathcal{Q}(t), K(t)=j\right]P(K(t)=j) \\ &= \sum_{j=1}^{M}P(K(t)=j)\sum_{i=1}^{j}Q_{q_i}(t)^{\alpha_{q_i}} \\ &= \sum_{j=1}^{N}P(K(t)=j)\sum_{i=1}^{j}Q_{q_i}(t)^{\alpha_{q_i}} \\ &\qquad + \sum_{j=N+1}^{M}P(K(t)=j)\sum_{i=1}^{N}Q_{q_i}(t)^{\alpha_{q_i}} \\ &= \sum_{j=1}^{N}Q_{q_j}(t)^{\alpha_{q_j}}\sum_{i=1}^{N}P(K(t)=i) \\ &\qquad + \sum_{j=1}^{N}Q_{q_j}(t)^{\alpha_{q_j}}P(K(t)>N) \\ &= \sum_{j=1}^{N}Q_{q_j}(t)^{\alpha_{q_j}}P(K(t)\geq j) \end{aligned} \quad (46)$$

By some computations, we can rewrite (46) as follows

$$\begin{aligned} & \sum_{j=1}^{N}Q_{q_j}(t)^{\alpha_{q_j}}P(K(t)\geq j) \\ &= \sum_{j=1}^{N-1}(Q_{q_j}(t)^{\alpha_{q_j}} - Q_{q_{j+1}}(t)^{\alpha_{q_{j+1}}})\sum_{n=1}^{j}P(K(t)\geq n) \\ &\quad +Q_{q_N}(t)^{\alpha_{q_N}}\sum_{n=1}^{N}P(K(t)\geq n) \end{aligned} \quad (47)$$

Similarly, we can obtain

$$\begin{aligned} & \sum_{i=1}^{N}Q_i(t)^{\alpha_i}\lambda_i = \sum_{j=1}^{N}Q_{q_j}(t)^{\alpha_{q_j}}\lambda_{q_j} \\ &= \sum_{j=1}^{N-1}(Q_{q_j}(t)^{\alpha_{q_j}} - Q_{q_{j+1}}(t)^{\alpha_{q_{j+1}}})\sum_{n=1}^{j}\lambda_{q_n} \\ &\quad +Q_{q_N}(t)^{\alpha_{q_N}}\sum_{n=1}^{N}\lambda_{q_n} \end{aligned} \quad (48)$$

Combining (47), (48), and (45), we obtain

$$
\begin{aligned}
&E[L(\mathcal{Q}(t+1))-L(\mathcal{Q}(t))|\mathcal{Q}(t)]\\
&=\sum_{j=1}^{N-1}(Q_{q_j}(t)^{\alpha_{q_j}}-Q_{q_{j+1}}(t)^{\alpha_{q_{j+1}}})\sum_{n=1}^{j}\lambda_{q_n}\\
&\quad+Q_{q_N}(t)^{\alpha_{q_N}}\sum_{n=1}^{N}\lambda_{q_n}\\
&\quad-\sum_{j=1}^{N-1}(Q_{q_j}(t)^{\alpha_{q_j}}-Q_{q_{j+1}}(t)^{\alpha_{q_{j+1}}})\sum_{n=1}^{j}P(K(t)\geq n)\\
&\quad-Q_{q_N}(t)^{\alpha_{q_N}}\sum_{n=1}^{N}P(K(t)\geq n)\\
&\quad+\sum_{i=1}^{N}\theta Q_i(t)^{\alpha_i}+\sum_{i=1}^{N}W_i(\theta)\\
&\leq\sum_{j=1}^{N-1}\Big((Q_{q_j}(t)^{\alpha_{q_j}}-Q_{q_{j+1}}(t)^{\alpha_{q_{j+1}}})\\
&\qquad\sum_{n=1}^{j}(\lambda_{q_n}-P(K(t)\geq n))\Big)\\
&\quad+Q_{q_N}(t)^{\alpha_{q_N}}\sum_{n=1}^{N}(\lambda_{q_n}-P(K(t)\geq n))\\
&\quad+\sum_{i=1}^{N}\theta Q_i(t)^{\alpha_i}+\sum_{i=1}^{N}W_i(\theta)
\end{aligned}\tag{49}
$$

By defining

$$d=\max_{Q\subset\{1,\ldots,N\}}\left\{\sum_{i\in Q}\lambda_i-\sum_{k=1}^{|Q|}P(K>k)\right\}\tag{50}$$

which is a negative constant by (36), we can rewrite (49) as

$$
\begin{aligned}
&E[L(\mathcal{Q}(t+1))-L(\mathcal{Q}(t))|\mathcal{Q}(t)]\\
&\leq dQ_{q_1}(t)^{\alpha_{q_1}}+\sum_{i=1}^{N}\theta Q_i(t)^{\alpha_i}+\sum_{i=1}^{N}W_i(\theta)\\
&\leq(\frac{d}{N}+\theta)\sum_{i=1}^{N}Q_i(t)^{\alpha_i}+\sum_{i=1}^{N}W_i(\theta)
\end{aligned}\tag{51}
$$

The last inequality holds because q_1 has the largest α-th power queue length. Letting $\theta=-d/(2N)$, the Lyapunov drift can be bounded by

$$
\begin{aligned}
&E[L(\mathcal{Q}(t+1))-L(\mathcal{Q}(t))|\mathcal{Q}(t)]\\
&\leq(\frac{d}{2N})\sum_{i=1}^{N}Q_i(t)^{\alpha_i}+\sum_{i=1}^{N}W_i(-\frac{d}{2N})
\end{aligned}\tag{52}
$$

By Foster's criterion for ergodic Markov chain, the queueing length process converges in distribution. Using iterated mean and telescoping sums, we have

$$\sum_{i=1}^{N}E[Q_i(t)^{\alpha_i}]\leq(-\frac{2N}{d})\sum_{i=1}^{N}W_i(-\frac{d}{2N})\tag{53}$$

where $W_i()$ is defined in (42) and (44), respectively. ■

The results in Theorem 1 and 2 can be summarized as follows

Theorem 3 *The sufficient and necessary conditions for moment stability is*

$$\sum_{i\in Q}\lambda_i<|Q|-\sum_{k=1}^{|Q|}(P(K<k))\quad\forall Q\subset\{1,...,N\}\tag{54}$$

where $K\sim PB(\boldsymbol{p},M)$, $\boldsymbol{p}=(p_1,...,p_M)$ and the scheduling policy defined in (35) *is throughput optimal, which stabiles any set of arrival rates within the stability region.*

Remark 9 As indicated by (54), the network stability region of a cognitive radio network is characterized by the statistics of secondary user traffics, primary user activities, the number of secondary users contending the spectrum, and the total number of primary user channels available to secondary users. This region holds for any feasible work conserving policies, which utilize all idle slots of PU channels for transmissions unless secondary users have empty queues. Since work conserving policies are feasible when sensing errors are negligible, (54) actually provides the outer bound of the network stability region under any sensing performance.

Remark 10 It can be proven that the above stability region also holds for strong stability if such stability exists. In this case, the network stability regions under the two criterions overlap with each other. However, moment stability is stronger than strong stability. Specifically, if the minimum tail coefficient of all arrivals is larger than 2, both strong stability and moment stability exist, while the latter case not only guarantees the finiteness of mean but also ensures the finiteness of the higher order moments, such as variance. This is a important property for the QoS oriented applications such as on-line gaming and video conferencing.

D. Network Stability with Heavy Tailed PU channel

In the previous sections, we study the network stability under LT PU channel, in which both idle and busy periods follow LT distribution. In this section, we investigate the impact of HT PU channel on the stability of cognitive radio networks. Particularly, we consider a cognitive network consisting of one HT PU channel and N secondary users. Let $S_H(t)\in\{0,1\}$ denote the states of the PU channel with $S_H(t)=0$ if channel is busy and $S_H(t)=1$ if channel is idle. Let p_{on} denote the probability that PU channel enters busy state and U denote the number of slots the channel stay in the busy state, which follows a HT distribution with tail coefficient $c_U>1$, i.e., $U\in\mathcal{RV}(c_U)$.

Under the above network model, we have the following Lemma regarding the maximum order β_i of finite moments for the queue length of each queue i

Lemma 3 *If queue i has HT arrivals with tail coefficient c_i, i.e., $A_i(t)\in\mathcal{RV}(c_i)$, then $\beta_i=\min(c_i,c_U)-1$. If queue i has LT arrivals, i.e., $A_i(t)\in LT$, then $\beta_i=c_U-1$.*

Remark 11 The proceeding Lemma indicates the HT nature of PU channel can greatly degrade the stability of cognitive radio network. More specifically, if each SU has the traffic arrival with tail coefficient $c_i > c_U$, the tail coefficient c_U of the PU channel's busy period determines the existence of finite mean and variance for the queue lengthes of both HT and LT queues, e.g., if $c_U < 3$, the variance of the queue length of each SU is unbounded and if $c_U < 2$, no finite expectation exists. When $c_U > 3$, whether finite mean and variance exist or not is determined by the tail coefficient c_i of the arrivals of each SU queue.

Proof: The proof is similar to the one for proving Lemma 1 and omitted here for brevity. ■

Similar to the case of LT PU channel, we can prove that the conventional maximum weight scheduling algorithm, which can ensure strong stability with light tailed arrivals, can not achieve moment stability in the presence of heavy tails. Particularly, at each time slot, the maximum weight scheduling algorithm chooses the channel allocation which satisfies

$$\max \sum_{i=1}^{N} Q_i(t) S_H(t) \tag{55}$$

where $c_i < \beta_i$. The queueing performance under the policy above is given as follows.

Lemma 4 *Under maximum weight scheduling algorithm, all queues have heavy tailed distributed queue length, with the same tail coefficient equal to* $\min_{1 \leq i \leq N}(c_i, c_U) - 1$.

Remark 12 The above Lemma indicates that under maximum weight scheduling policy, all queues have the same performance as the worst queue which has the arrivals with the heaviest tail or the smallest tail coefficient. Particularly, if the worst queue has unbound delay and delay variance, so do all queues.

Proof: The proof utilizes the similar arguments as the proof of Theorem 4 in [9] and is omitted for brevity. ■

We now show that the maximum-weight-α scheduling is also throughput-optimal with respect to moment stability under heavy tailed PU channel. Particularly, at each time slot, the scheduling algorithm assigns queue i with parameter $\alpha_i < \beta_i$ and chooses the channel allocation which satisfies

$$\max \sum_{i=1}^{N} Q_i(t)^{\alpha_i} S_H(t) \tag{56}$$

Theorem 4 *The cognitive radio network is moment stable under the scheduling algorithm defined in* (56) *if*

$$\sum_{i=1}^{N} \lambda_i < \frac{1 - p_{on}}{p_{on} E[U] + (1 - p_{on})} \tag{57}$$

Remark 13 Since the right hand of above equation is the maximum throughput of PU channel, the scheduling algorithm defined in (56) is also throughput optimal.

Proof: The proof is similar to the those in the proof of Theorem 2 and the key ideas are given in the following, We first construct an embedded Markovian chain by sampling the system at the time instance when an idle time slot just starts. Let t_n denote the time instance at which the n-th idle time slot begins. The queue length processes at t_n, i.e., $Q_i^e(n) = Q_i(t_n)$, constitute a Markovian chain. The queueing dynamics can be represented by

$$Q_i^e(n+1) = Q_i^e(n) + A_i^e(n) - H_i^e(n) \tag{58}$$

where $H_i^e(n) \in \{0, 1\} \quad \forall i = 1, ..., N$, and $A_i^e(n)$ is total packet arrivals in the duration D from t_{n+1} to t_n. It can be shown by large deviation theorem that $A_i^e(n)$ follows heavy tail distribution with tail coefficient $\min(\kappa_{A_i(t)}, \alpha_U)$. Then, by the Lyapunov drift arguments similar to the those in the proof of Theorem 2, the queueing process in (58) is steady-state stable and the α_i-th moment of the queue length of each queue i is bounded provided that $\sum_{i=1}^{N} \lambda_i < \frac{1-p_{on}}{p_{on} E[U] + (1-p_{on})}$. ■

IV. Conclusions

In this paper, we introduce a new stability criterion, namely moment stability, which is a stronger form stability than the conventional stability criterions and facilitates the design of QoS oriented resource allocation schemes over dynamic spectrum access networks. Towards this, the necessary and sufficient conditions for the existence of a resource allocation scheme to achieve moment stability are obtained and the corresponding network stability region is derived. In addition, the throughput-optimal policy, which achieves the network stability region, is also introduced. In the end, it is shown that the tailness of the heavy tailed PU traffic determines what types of the network stability are achievable in the network.

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