A Wavelet Approach to Wideband Spectrum Sensing for Cognitive Radios

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Abstract— **In cognitive radio networks, the first cognitive task preceding any form of dynamic spectrum management is the sensing and identification of spectrum holes in wireless environments. This paper develops a wavelet approach to efficient spectrum sensing of wideband channels. The signal spectrum over a wide frequency band is decomposed into elementary building blocks of subbands that are well characterized by local irregularities in frequency. As a powerful mathematical tool for analyzing singularities and edges, the wavelet transform is employed to detect and estimate the local spectral irregular structure, which carries important information on the frequency locations and power spectral densities of the subbands. Along this line, a couple of wideband spectrum sensing techniques are developed based on the local maxima of the wavelet transform modulus and the multi-scale wavelet products. The proposed sensing techniques provide an effective radio sensing architecture to identify and locate spectrum holes in the signal spectrum.**

I. Introduction

Current wireless systems are characterized by wasteful static spectrum allocation, fixed radio functions, and limited network coordination between mobile devices, resulting in a surprisingly large portion of the radio spectrum goes unused. The emerging paradigm of Dynamic Spectrum Access shows promise of alleviating today's spectrum scarcity problem by ushering in new forms of spectrum agile networks [1]. Key to this new paradigm are cognitive radios (CRs) that are aware of and can sense the environments, learn from the environments, and perform functions to best serve their users without causing harmful interference to other authorized users [2]. The cognitive process starts with the passive sensing of RF stimuli [3]. As such, the first cognitive task preceding any form of dynamic spectrum management is to develop wireless spectral detection and estimation techniques for sensing and identification of spectrum holes.

Depending on the regimes of spectrum utilization, the front-end architecture of CRs can be quite different [4]. In early stage of CR network deployment, the spectrum utilization is expected to be low (around 5%) and there is little spectrum scarcity. In this case, the radio front-end starts with a tunable narrowband bandpass filter (BPF) to search one narrow frequency band at a time. Focusing on each narrow band, existing spectrum sensing techniques are largely categorized into energy detection [5] and feature detection [6]. When the spectrum utilization is medium (below 20%) resulting in medium spectrum scarcity, the radio front-tend should adopt a wideband architecture to search over multiple frequency bands at a time. Multiple narrowband BPFs can be employed to form a filterbank for wideband sensing [4], but this architecture requires an increased number of components and the filter range of each BPF is preset. In future networks where spectrum utilization is high (above 20%), the significant spectrum scarcity would call for different spectrum sharing mechanisms such as ultra-wideband cognitive radios [7], which in turn entail different sensing tasks for spectrum overlay.

In this paper, we focus on the wideband spectrum sensing task, without resorting to multiple narrowband BPFs. Our goal is to identify the frequency locations of non-overlapping spectrum bands and categorize these bands into *black, gray* or *white* spaces, corresponding to the power spectral density (PSD) levels being high, medium or low [3]. In a peer-to-peer network adopting the equal-sharing spectrum allocation paradigm, white spaces are treated as spectrum holes that can be picked by the CR for opportunistic use. Evidently, the cognitive network of interest concerns spectrum identification more than the detailed spectral shape over the entire wideband. Thus, many traditional spectral estimation techniques become irrelevant or unnecessarily complicated [8]. Recognizing the distinct nature of CR sensing, we model the entire wideband under scrutiny as a train of consecutive frequency subbands, where the power spectral characteristic is smooth within each subband but exhibits a discontinuous change between adjacent subbands. Such changes are in fact irregularities in PSD, which carry key information on the locations and intensities of spectrum holes. An attractive mathematical tool for analyzing singularities and irregular structures is the wavelet transform, which can characterize the local regularity of signals [9]. Thus, it is well motivated to investigate the wavelet transform approach to wideband spectrum sensing for CRs.

There has been considerable research on wavelet analysis for time series and images [9]-[13]. Singularity detection and processing with wavelets have been applied to filtering and denoising [9], [13], compression [10], and applications in image processing and elsewhere. Targeting the CR sensing task, this paper derives wavelet-based techniques for detecting irregular edges in the signal PSD as opposed to irregularities in time series. A couple of dynamic sensing solutions are formulated based on the maxima of waveform transform modulus [9] and the peaks of multiscale products [13], which result in detection and estimation of the locations of spectral irregularities. We also estimate the average PSD level within each identified subband, which carries critical information on spectrum holes available for opportunistic sharing.

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II. Problem Formulation for Spectrum Sensing

Suppose that a total of B Hz in the frequency range $[f_0, f_N]$ is available for a wideband wireless network. Being cognitive, this network supports heterogeneous wireless devices that may adopt different wireless technologies for transmissions over different bands in the frequency range. A CR at a particular place and time needs to sense the wireless environment in order to identify spectrum holes for opportunistic use[1]. Suppose that the radio signal received by the CR occupies N spectrum bands, whose frequency locations and PSD levels are to be detected and identified. These spectrum bands lie within $[f_0, f_N]$ consecutively, with their frequency boundaries located at $f_0 < f_1 < \cdots f_N$. The n-th band is thus defined by $B_n : \{f \in B_n : f_{n-1} \le f < f_n\}$, $n = 1, 2, \ldots, N$. The PSD structure of a wideband signal is illustrated in Fig. 1. The following basic assumptions are adopted.

- *a1*) The frequency boundaries f_0 and $f_N = f_0 + B$ are known to the CR. Even though the actual received signal may occupy a larger band, this CR regards $[f_0, f_N]$ as the wide band of interest and seeks white spaces only within this spectrum range.
- *a2*) The number of bands N and the locations $f_1, \ldots, f_{N-1}$ are unknown to the CR. They remain unchanged within a time burst, but may vary from burst to burst in the presence of slow fading.
- *a3*) The PSD within each band B_n is smooth and almost flat, but exhibits discontinuities from its neighboring bands B_{n-1} and B_{n+1}. As such, irregularities in PSD appear at and only at the edges of the N bands[2].
- *a4*) The ambient noise is additive and white, with zero mean and two-sided PSD $S_w(f) = N_0/2, \forall f$.

In the absence of noise, the normalized (unknown) power spectral shape within each band B_n is denoted by $S_n(f)$, which satisfies the following conditions:

$$S_n(f) = 0, \quad \forall f \notin B_n;$$
$$\int_{f_{n-1}}^{f_n} S_n(f) df = f_n - f_{n-1}.$$

According to *a3*), we may approximate $S_n(f)$ as:

$$S_n(f) = \begin{cases} 1, & \forall f \in B_n. \\ 0, & \forall f \notin B_n. \end{cases} \tag{1}$$

With *a3*) and *a4*), the PSD of the observed signal $r(t)$ at the CR front-end can be written as

$$S_r(f) = \sum_{n=1}^{N} \alpha_n^2 S_n(f) + S_w(f), \qquad f \in [f_0, f_N] \tag{2}$$

where α_n^2 indicates the signal power density within the n-th band. The corresponding time-domain signal is

$$r(t) = \sum_{n=1}^{N} \alpha_n p_n(t) + w(t) \tag{3}$$

where $S_n(f)$ is the signal spectrum of $p_n(t)$ and $w(t)$ denotes the additive noise with PSD $S_w(f)$. For example, the signal component occupying B_n can be a pulse train in the form $p_n(t) = \sum_{k=-\infty}^{\infty} b_k h(t - kT_s) e^{j2\pi f_{c,n} t}$, where $\{b_k\}$ are digitally modulated symbols, $h(t)$ is a pulse shaper of bandwidth $(f_n - f_{n-1})$, and $f_{c,n} = (f_{n-1} + f_n)/2$ is the center frequency of this band. The spectral shape $S_n(f)$ is thus proportional to $|\mathcal{F}\{h(t)\}|^2$, with $\mathcal{F}\{\cdot\}$ denoting Fourier Transform.

The wideband spectrum sensing problem of our interest is formulated as follows:

For a CR that receives $r(t)$ with PSD $S_r(f)$ as in (2), how to estimate the following parameters characterizing the wideband spectral environment: N, $\{f_n\}_{n=1}^{N-1}$ and $\{\alpha_n^2\}_{n=1}^{N}$?

We seek answers to this problem without resorting to multiple narrowband BPFs. The use of N BPFs not only causes increased number of receiver components, but also faces challenges in tuning the local oscillator of each BPF in the absence of knowledge on N as well as the intended passband range $[f_{n-1}, f_n]$, $n = 1, \ldots, N$ [4].

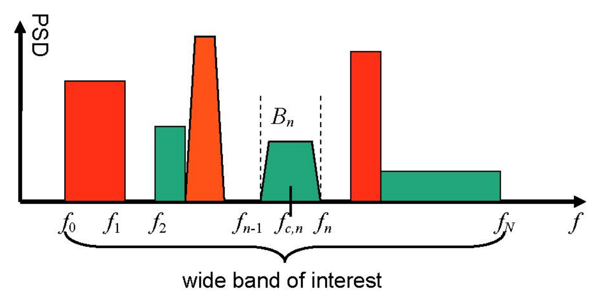

Fig. 1. N frequency bands with piecewise smooth PSD.

III. Wavelet Approach to Spectrum Sensing

Based on *a3*) and with reference to Fig. 1, wideband spectrum sensing can be viewed as an edge detection problem in an image depicted by the PSD $S_r(f)$ in frequency. Edges in this image correspond to the locations of frequency discontinuities $\{f_i\}_{i=1}^{N-1}$, which are to be identified. This section shows that the wavelet transform can effectively characterize the edges exhibited in the local singular structure of the PSD.

In adopting the wavelet approach to spectrum sensing, we note at the outset that the wavelet transform in existing applications is applied *in lieu of* Fourier transform (FT) to characterize a time series such as $r(t)$, or a spatial pixel graph such as in imaging [9]. In our problem, the domain of interest is frequency f, which is in fact the duality of time t *after* FT. As such, the noise component dealt in our problem has different characteristics from that in conventional wavelet analysis.

[1]Dynamic spectrum sharing not only concerns the identification of spectrum holes, but also the detection of primary license-holders when a primary-secondary network (as oppose to a peer-to-peer network) is of interest. The latter task is a binary-hypothesis signal detection problem, while this paper focuses on the former sensing task relevant to both network paradigms.

[2]Spectral spikes may arise in communication signals (e.g., due to signal cyclostationarity), but are not treated as PSD discontinuities. The treatment on this issue will be discussed in Section III.E.

A. Wavelet Transform of Signal PSD

Let $\phi(f)$ be a wavelet smoothing function with a compact support, m vanishing moments and m times continuously differentiable. The strictly positive integer m is selected depending on Lipschitz exponent, which is a measure for the local regularity of the signal of interest [9]. Widely-used examples for $\phi(f)$ include the Gaussian function and the perfect reconstruction filter bank (PRFB) [12]. The dilation of $\phi(f)$ by a scale factor s is given by

$$\phi_s(f) = \frac{1}{s}\phi\left(\frac{f}{s}\right). \tag{4}$$

For dyadic scales, s takes values from powers of 2, i.e., $s = 2^j$, $j = 1, 2, \ldots, J$. Letting $*$ denote convolution, the continuous wavelet transform (CWT) of $S_r(f)$ is given by

$$\mathcal{W}_s S_r(f) = S_r * \phi_s(f). \tag{5}$$

We note that the CWT in (5) is carried out in the frequency domain, while the function of interest $S_r(f)$ itself relates to the received time-domain function $r(t)$ via the FT. A direct way to compute $\mathcal{W}_s S_r(f)$ is to first perform the FT on the autocorrelation function $R_r(\tau) := E\{r(t)r(t+\tau)\}$ such that $S_r(f) = \mathcal{F}\{R_r(\tau)\}$, followed by the convolution operation in (5).

Equivalently, $\mathcal{W}_s S_r(f)$ can be computed from $r(t)$ in an alternative way. Let $\Phi_s(\tau) := \mathcal{F}\{\phi_s(-f)\} = \mathcal{F}^{-1}\{\phi_s(f)\} = \Phi(s\tau)$ represent the inverse FT of the wavelet function. The inverse FT of $\mathcal{W}_s S_r(f)$ is given by $\mathcal{W}_s S_r(\tau) := \mathcal{F}^{-1}\{\mathcal{W}_s S_r(f)\}$, which is related to $\Phi_s(\tau)$ and $R_r(\tau)$ via

$$\mathcal{W}_s S_r(\tau) = R_r(\tau) \cdot \Phi(s\tau). \tag{6}$$

Therefore, an alternative to (5) is given by

$$\mathcal{W}_s S_r(f) = \mathcal{F}\{\mathcal{W}_s S_r(\tau)\} = \mathcal{F}\{R_r(\tau) \cdot \Phi(s\tau)\}. \tag{7}$$

Once the wavelet $\phi_s(f)$ and its FT pair $\Phi(s\tau)$ are determined, the computation of the CWT $\mathcal{W}_s S_r(f)$ involves either convolution and FT (on $R_r(\tau)$) operations as in (5), or product and FT (on the product) operations as in (7).

B. Spectrum Sensing via Wavelet Modulus Maxima

For the PSD $S_r(f)$ of interest, edges and irregularities at the scale s are defined as local sharp variation points of $S_r(f)$ smoothed by $\phi_s(f)$. As we know, the edges of a function are often signified in the shapes of its derivatives. With the CWT, the first-order and second-order derivatives of $S_r(f)$ smoothed by the scaled wavelet $\phi_s(f)$ can be expressed respectively by

$$\begin{aligned}\mathcal{W}'_s S_r(f) &= s\frac{d}{df}(S_r * \phi_s)(f) \\ &= S_r * (s\frac{d\phi_s}{df})(f) = -s\mathcal{F}\{\tau R_r(\tau)\Phi_s(s\tau)\};\end{aligned} \tag{8}$$

$$\begin{aligned}\mathcal{W}''_s S_r(f) &= s^2\frac{d^2}{df^2}(S_r * \phi_s)(f) \\ &= S_r * (s^2\frac{d^2\phi_s}{df^2})(f) = s^2\mathcal{F}\{\tau^2 R_r(\tau)\Phi_s(s\tau)\}.\end{aligned} \tag{9}$$

Similar to (5) and (7), the computation of $\mathcal{W}'_s S_r(f)$ and $\mathcal{W}''_s S_r(f)$ each has two equivalent expressions.

It is shown in [9] that the local extrema of the first derivative and the zero-crossings of the second derivative characterize the signal irregularities. In particular, the local maxima of the wavelet modulus are sharp variation points, which tend to be more accurate than local minima points (corresponding to slow variation points) for spectrum sensing purposes. The identification of $\{f_n\}_{n=1}^N$ can thus be realized based on the following proposition.

Proposition 1. *Boundaries $\{f_n\}$ of consecutive frequency bands $\{B_n\}$ with piecewise smooth PSD can be acquired from $r(t)$ by picking the local maxima of the wavelet modulus $\mathcal{W}'_s S_r(f)$ in (8) with respect to f as*

$$\hat{f}_n = \text{maxima}_f \ \{|\mathcal{W}'_s S_r(f)|\}, \qquad f \in (f_0, f_N) \tag{10}$$

or from the zero-crossing points of $\mathcal{W}''_s S_r(f)$ in (9) as

$$\hat{f}_n = \text{zeros}_f \ \{\mathcal{W}''_s S_r(f)\} \qquad s.t. \ \ \mathcal{W}''_s S_r(\hat{f}_n) = 0. \tag{11}$$

Detecting the zero-crossings of $\mathcal{W}''_s S_r(f)$ or the local extrema of $\mathcal{W}'_s S_r(f)$ are similar procedures. When searching for $\hat{f}_n$ via either procedure, the scale factor s can be set to the dyadic scales $s = 2^j$, $j = 1, \ldots, J$. Only those modulus maxima or zero crossings that propagate to coarser (i.e., larger) scales are retained, while others are removed as noise [9].

C. Spectrum Sensing via Multiscale Wavelet Products

In Proposition 1, the desired local maxima of wavelet modulus are tracked by their propagation to multiple coarser scales, with the goal of denoising. Such an idea of exploiting the multiscale correlation can be carried out in a direct (albeit nonlinear) way, giving rise to multiscale analysis techniques. In [13], edge detection and estimation is analyzed based on forming multiscale point-wise products of smoothed gradient estimators. This approach is intended to enhance multiscale peaks due to edges, while suppressing noise. Adopting this technique to our spectrum sensing problem and restricting to dyadic scales, we construct the multiscale product of J CWT gradients as

$$\mathcal{U}_J S_r(f) = \prod_{j=1}^{J} \mathcal{W}'_{s=2^j} S_r(f) \tag{12}$$

where the derivative of the smoothed PSD $S_r(f)$ is given by (8).

Based on Proposition 1, it is evident that the frequency edges $\{f_n\}$ of interest (which are local maxima of $|\mathcal{W}'_s S_r(f)|$ for all $s \leq 2^J$) show up as the local maxima of $|\mathcal{U}_J S_r(f)|$. On the other hand, noise-induced spurious local maxima of $|\mathcal{W}'_s S_r(f)|$ are random at each scale and tend not to propagate through all J scales; hence, they do not show up as the local maxima of the product $\mathcal{U}_J S_r(f)$.

Summing up, we reach the following proposition as an alternative means of spectrum sensing.

Proposition 2. *Boundaries $\{f_n\}$ of consecutive frequency bands $\{B_n\}$ with piecewise smooth PSD can be acquired from $r(t)$ by picking the local maxima of the multiscale product $\mathcal{U}_J S_r(f)$ in (12) with respect to f as*

$$\hat{f}_n = \text{maxima}_f \ \{|\mathcal{U}_J S_r(f)|\}, \qquad f \in (f_0, f_N). \qquad (13)$$

D. Spectral Density Estimation

After $\{f_n\}_{n=1}^{N-1}$ have been detected and estimated via Proposition 1 or 2, the remaining task of spectrum sensing is to estimate the PSD levels $\{\alpha_n\}_{n=1}^{N}$. To this end, we compute the average PSD within the band B_n, $n = 1, \ldots, N$, in the form

$$\beta_n = \frac{1}{f_n - f_{n-1}} \int_{f_{n-1}}^{f_n} S_r(f) df. \qquad (14)$$

Based on the assumptions *a3*) *a4*) and the approximation (1) on the PSD shape, it is evident that β_n is related to the unknown α_n^2 by $\beta_n \approx \alpha_n^2 + N_0/2$. The noise PSD $N_0/2$ can be measured offline, or deduced from an empty band, say the n'-th one, that satisfies $\alpha_{n'}^2 = 0$ and $\beta_{n'} = N_0/2$ for $f \in B_{n'}$. Such an empty band almost always exists, since the spectrum utilization in current wireless systems is rather low (below 20%). Apparently, $\beta_{n'} = N_0/2$ is the smallest possible value for all $\{\beta_n\}$. Summing up, we present a simple estimator for α_n^2 as follows.

Proposition 3. *For each frequency band B_n with piecewise-flat PSD as in (1), its spectral density α_n^2 can be estimated from $S_r(f)$ as*

$$\hat{\alpha}_n^2 = \beta_n - \min_{n'} \beta_{n'}, \qquad n = 1, \ldots, N \qquad (15)$$

where $\{f_n\}$, used for computing $\{\beta_n\}$ in (14), can be replaced by their estimates obtained from the wavelet approach.

The spectral density estimator in Proposition 3 is quite simple, while more elaborated methods are possible to estimate both $\{\alpha_n^2\}$ and $\{S_n(f)\}$, even when the signal PSD is not piecewise flat. Such solutions can take advantage of the attractive properties of the wavelet transform in providing 'complete' reconstructions of functions with local structures [9]. Details are omitted here for space limit.

Albeit simple, the estimator in (15) is adequate for solving the sensing problem of our interest. The primary goal of our sensing problem is to identify the frequency locations of bands $\{B_n\}$ and categorize them into *black, gray,* or *white* spaces [3], corresponding to PSD levels α_n^2 being high, medium or low. Therefore, coarse estimation of α_n^2 suffices for frequency space categorization.

E. Noise Characteristic in the Wavelet Approach

In our wavelet approach, local maxima of the wavelet transform modulus may arise not only due to the frequency edges $\{f_n\}$ of interest, but also due to additional sources: isolated impulses, spikes, very-narrow-band interference (vNBI), and additive white noise. It is of interest to investigate the degrading effect of these sources on spectrum sharing. The following remarks are in due.

- Isolated impulses/spikes and vNBIs appear as narrow peaks in an otherwise white or gray space. For wideband receivers with built-in capability to handle vNBI, it is preferred not to identify these peaks during spectrum sensing, such that the entire white/gray space is treated as being opportunistically available for sharing. In this case, results based on multiscale products are preferred for the inherent ability to suppress isolated impulses, depending on the amount of smoothing utilized [13]. On the other hand, for narrowband receivers that rely on channelized spectrum allocation, the information on vNBIs is useful to acquire during sensing.
- Regarding the ambience noise, it is interesting to observe that the noise effect in our wavelet approach to the spectrum sensing problem is not as harmful as in conventional wavelet applications. In the latter case, the wavelet transform is imposed on a time series or an image whose noise component is random (e.g., the Gaussian noise $w(t)$ in (3)), causing a large number of spurious local extrema at finer scales. In contrast, in our problem the CWT is applied to the PSD $S_r(f)$ in (2), whose additive noise component $S_w(f)$ is white/flat. Thus, there is few spurious edges incurred by $S_w(f)$.

IV. Simulations

We consider a wide band of interest in the range of $[50, 250]$ MHz. Fig. 2(a) illustrates the PSD $S_r(f)$ observed by a CR. The noise floor in the PSD is quite large at $S_w(f) = 200$. During the observed burst of transmissions in the network, there are a total of $N = 6$ bands $\{B_n\}$, with frequency boundaries at $\{f_n\}_{n=0}^{6} = [50, 120, 170, 200, 220, 224, 250]$ MHz. Among these bands (marked on Fig. 2(a)), B_1, B_3 and B_5 have relatively high signal PSD at levels 24, 30, and 36, respectively, while B_2 has low signal PSD at a level of 3, all with reference to $S_w(f) = 200$. The rest two bands, B_4 and B_6 are not occupied and are thus spectrum holes.

In all tests, we use the Gaussian wavelet along with four dyadic scales $s = 2^j$, $j = 1, 2, 3, 4$. Fig. 2(b) depicts the wavelet modulus computed from (8), while Fig. 2(c) plots the multiscale products of wavelets expressed in (12). Edges in the PSD $S_r(f)$ are clearly captured by the wavelet transform in all curves. As the scale factor s^j increases, the wavelet transform becomes smoother within each frequency band, retaining the lower-variation contour of the noisy PSD. In particular, the multiscale product method in 2(c) is very effective in suppressing the spurious local extrema caused by noise, resulting in better detection and estimation performance.

The simple spectral density estimation scheme in Proposition 3 is used to estimate the noise and signal PSD levels. The estimated values are

$$\{\hat{\alpha}_n^2\} = [24.3566,\ 4.1266,\ 29.4695,\ 0,\ 38.3268,\ 0.7608]$$

corresponding to the true signal PSD values $[24, 3, 30, 0, 36, 0]$ respectively, and

$$\hat{S}_w(f) = 199.4313$$

corresponding to the true noise PSD value 200. Such estimation accuracy is adequate in classifying the corresponding frequency band into the coarse categories of white, gray and black spaces. The sensing capacity of the wavelet approach is evident even when the signal to noise ratio is quite low.

V. Summary

This paper formulates the cognitive spectrum identification task as an spectral edge detection problem and exploits the wavelet approach for spectrum sensing of wideband channels. Solutions based on the local maxima of both gradient wavelet modulus and multiscale wavelet products are derived and tested. The proposed schemes are able to scan over a wide bandwidth to simultaneously identify all piecewise smooth subbands, without prior knowledge on the number of subbands within the frequency range of interest. The wavelet approach offers evident advantages over the conventional use of multiple narrowband BPFs, in terms of both implementation costs and flexibility in adapting to dynamic PSD structures.

Since the wavelet approach targets wideband spectrum sensing, it may require high sampling rates in order to characterize the entire wide bandwidth. Nevertheless, the requirements on sampling rates can be reduced when the sensing task primarily concerns a relaxed spectral estimation problem of identification of band types and spectrum holes, or when guard bands are inserted during CR transmissions such that it suffices to obtain rough location estimates of spectrum holes.

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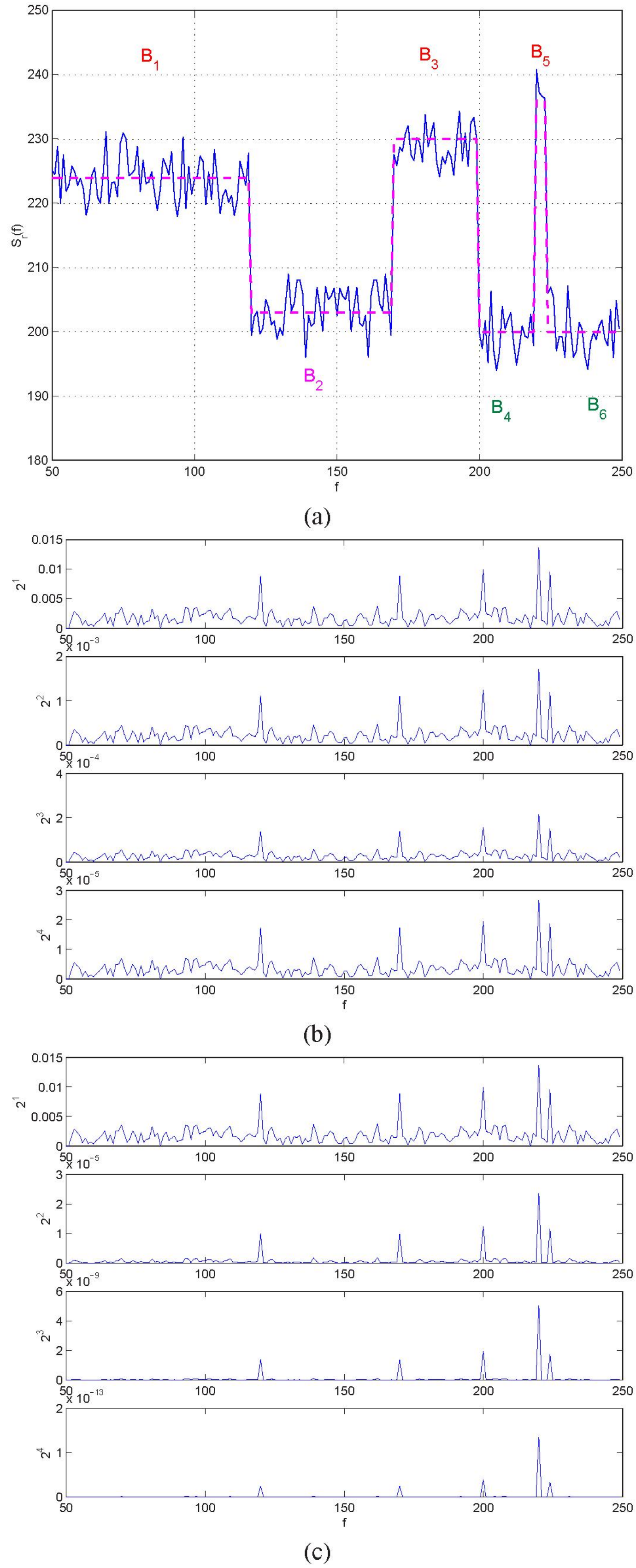

Fig. 2. (a) Original signal PSD; (b) wavelet transform modulus at scales $2^j, j = 1:4$; (c) multiscale wavelet products.