

# Control Channel Establishment in Cognitive Radio Networks using Channel Hopping

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**Abstract**—In decentralized cognitive radio (CR) networks, enabling the radios to establish a control channel (i.e., “rendezvous” to establish a link) is a challenging problem. The use of a dedicated common control channel simplifies the rendezvous process but may not be feasible in many opportunistic spectrum sharing scenarios due to the dynamically changing availability of all the channels, including the control channel. To address this problem, researchers have proposed the use of channel hopping protocols for enabling rendezvous in CR networks. Most, if not all, of the existing channel hopping schemes only provide ad hoc approaches for generating channel hopping sequences and evaluating their properties. In this paper, we present a systematic approach, based on *quorum systems*, for designing and analyzing channel hopping protocols for the purpose of control channel establishment. The proposed approach, called *Quorum-based Channel Hopping (QCH)* system, can be used for implementing rendezvous protocols in CR networks that are robust against link breakage caused by the appearance of incumbent user signals. We describe two synchronous QCH systems under the assumption of global clock synchronization, and two asynchronous channel hopping systems that do not require global clock synchronization. Our analytical and simulation results show that the proposed channel hopping schemes outperform existing schemes under various network conditions.

**Index Terms**—Control channel, cognitive radio, channel hopping, quorum system, medium access control.

## I. INTRODUCTION

IT IS WIDELY believed that cognitive radio (CR) technology is one of the key enabling technologies that can address the spectrum scarcity problem. CR networks need to utilize available spectrum in a dynamic and opportunistic fashion without causing interference to co-located incumbent networks. Because entities of an incumbent network have absolute priority in accessing their spectrum bands, they are often called *incumbent* or *primary* users whereas entities of a CR network are often called *secondary* users. In order to enable dynamic and opportunistic utilization of spectrum, nodes must have the ability to locate each other in a multi-channel environment via a “rendezvous” process. A pair of nodes wishing to communicate with each other use one or more rendezvous to exchange control information that will enable the establishment of a link. When a central entity, such as a base station, is not available, the rendezvous process needs to be carried out in a distributed manner. To address this difficult problem, the vast majority of the existing MAC

protocols for CR networks rely on a dedicated global or group control channel [4], [14], [20], [29]. Assuming a common control channel certainly simplifies the rendezvous process as well as other medium access-related issues. However, relying on a common control channel has a number of important drawbacks. A common control channel may become a bottleneck (a la *control channel saturation problem*) or create a single point of failure. More importantly, the dynamically changing availability of spectrum may make it impossible to maintain a common control channel. In a CR network, because secondary nodes must vacate any channel as soon as incumbent signals appear in the channel, the availability of any channel cannot be guaranteed, thus making it impossible to guarantee the availability of the common control channel.

In the design of multi-channel MAC protocols, the use of *channel hopping (CH)* (a.k.a. parallel rendezvous) techniques have been proposed to avoid the bottleneck of a single control channel [2], [25]. CH protocols are very useful in the context of medium access control in CR networks because they provide an effective method of implementing rendezvous without relying on a common control channel.

To provide reliable performance in a CR network, a CH protocol needs to satisfy two critical requirements. The first requirement is to guarantee the periodic overlap between any pair of CH sequences so that a pair of nodes that wish to establish a link can rendezvous. To minimize channel access delay, the *time-to-rendezvous (TTR) value between two sequences needs to be bounded and small*. The second requirement is to *guarantee that any two CH sequences will rendezvous in more than one channel within a sequence period*. The inability to guarantee rendezvous in more than one channel can be a problem in CR networks because the single rendezvous channel may become unavailable due to the appearance of incumbent signals. The proposed quorum system-based methodology enables the design of CH schemes that satisfy the two requirements—i.e., the CH protocol guarantees rendezvous in multiple channels while ensuring that the TTR has an upper bound.

Most, if not all, of the existing CH schemes [2], [8], [23], [25], [26] only provide ad-hoc approaches for generating CH sequences and evaluating their properties. In this paper, we present a systematic approach, based on *quorum systems*, for designing and analyzing CH protocols for the purpose of control channel establishment. The proposed approach, called *Quorum-based Channel Hopping (QCH)* system, utilizes the intersection property of quorum systems to generate CH sequences that enable rendezvous on multiple channels between any two CH sequences. Our approach guarantees rendezvous on multiple channels *with* and *without* the assumption of global clock synchronization. Under the assumption of global

Manuscript received 28 November 2009; revised 25 May 2010. A preliminary version of portions of this material has been presented at ACM MobiCom 2009 [3]. This work was supported in part by the National Science Foundation under grants CNS-0627436, CNS-0716208, and CNS-0910531.

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Digital Object Identifier 10.1109/JSAC.2011.110403.

clock synchronization, we describe two QCH systems. The first system minimizes the upperbound of the TTR value between any two CH sequences. The second system evenly distributes the rendezvous points over different timeslots during a CH period, thereby alleviating the *rendezvous convergence* problem. We also describe two asynchronous CH systems that do not require global clock synchronization: the first system guarantees rendezvous on two distinct channels within a fixed and small TTR value between any two CH sequences; the second system enables rendezvous on every channel, which provides the maximum robustness to link breakage caused by the appearance of incumbent signals.

The rest of this paper is organized as follows: we provide the network model and describe the problem in Section II. In Section III, we describe the QCH system. We present two synchronous QCH systems in Section IV. In Sections V and VI, we describe two asynchronous CH systems respectively. In Section VII, we discuss the rendezvous performance of CH systems when incumbent signals are detected. In Section VIII, we compare our proposed CH systems and existing CH schemes, and discuss the details of implementing the CH system in MAC protocols. In Section IX, we present the simulation results. We discuss related work in Section X, and conclude the paper in Section XI.

## II. PRELIMINARIES

### A. Network Model

Suppose there are  $N$  licensed channels in a CR network, labeled as  $0, 1, \dots, N - 1$ . To formulate the channel hopping system, we assume that time is divided into multiple CH periods, where each CH period is composed of  $T$  timeslots. For the sake of discussions, we assume that each timeslot is of unit duration so that each CH period is also  $T$ . Moreover, we assume that each node has a single half-duplex radio, and that all nodes in the CR network are synchronized (i.e., global clock synchronization) unless specified otherwise. In Sections V and VI, we will discuss the design of CH systems that do not require global clock synchronization.

A CH sequence determines the order with which a node visits all of the channels. We represent a CH sequence  $\mathbf{u}$  of period  $T$  as a set of *pairs* (2-tuples):

$$\mathbf{u} = \{(0, u_0), (1, u_1), \dots, (i, u_i), \dots, (T - 1, u_{(T-1)})\},$$

where  $u_i \in [0, N - 1]$  represents the *channel index* of sequence  $\mathbf{u}$  in the  $i^{\text{th}}$  timeslot of a CH period ( $i$  is the *slot index*).

Given two CH sequences  $\mathbf{u}$  and  $\mathbf{v}$ , if a pair  $(i, j) \in \mathbf{u} \cap \mathbf{v}$ ,  $(i, j)$  is called an *overlap* between  $\mathbf{u}$  and  $\mathbf{v}$ . In this case, the  $i^{\text{th}}$  timeslot is called a *rendezvous slot* and channel  $j$  is called a *rendezvous channel* between  $\mathbf{u}$  and  $\mathbf{v}$ . If a pair of nodes select  $\mathbf{u}$  and  $\mathbf{v}$  respectively as their CH sequences, then the resulting rendezvous channel can be used as a *pair-wise control channel*—i.e., the two nodes can exchange control information in channel  $j$  at the  $i^{\text{th}}$  timeslot of every CH period.

Let  $I_j(\mathbf{u}, \mathbf{v})$  denote a function that indicates whether channel  $j$  is a rendezvous channel between two sequences  $\mathbf{u}$  and  $\mathbf{v}$ , i.e.,

$$I_j(\mathbf{u}, \mathbf{v}) = \begin{cases} 1, & \text{if } \exists i \in [0, T - 1], \text{ s.t. } (i, j) \in \mathbf{u} \cap \mathbf{v}, \\ 0, & \text{otherwise.} \end{cases}$$

Now let us use  $C(\mathbf{u}, \mathbf{v})$  to denote the *number of rendezvous channels* between two sequences  $\mathbf{u}$  and  $\mathbf{v}$ , i.e.,

$$C(\mathbf{u}, \mathbf{v}) = \sum_{j=0}^{N-1} I_j(\mathbf{u}, \mathbf{v}).$$

### B. Problem Statement

In the opportunistic spectrum sharing paradigm, the value of  $C(\mathbf{u}, \mathbf{v})$  directly impacts the *robustness* of the pair-wise control channel established using sequences  $\mathbf{u}$  and  $\mathbf{v}$ . Recall that secondary users share spectrum opportunistically with incumbent users who have priority access rights. In such a scenario, secondary users are required to vacate the currently occupied channels when incumbent signals are detected in them. This requirement poses a difficult challenge in the design of MAC protocols for CR networks—in particular, in terms of how to establish control channels in such a way that enables the reliable exchange of control information despite the unpredictable appearance of incumbent signals. The robustness of the control channels established using sequences  $\mathbf{u}$  and  $\mathbf{v}$  is proportional to the value of  $C(\mathbf{u}, \mathbf{v})$ , since this value determines the number of distinct channels in which the rendezvous occur within a sequence period. If the rendezvous are spread out over a greater number of distinct channels, then the probability of link breakage caused by the inability to exchange control packets (due to the appearance of incumbent signals) decreases. Thus, we have the following *channel hopping system design problem*.

*Problem 1:* Given  $T$ , the CH system design problem is to devise a set of CH sequences of period  $T$ , denoted as  $H$ , which satisfies the following two properties:

- 1)  $\forall \mathbf{u} \in H, |\mathbf{u}| = T$ ;
- 2)  $D(H) \geq 1$ , where  $D(H) = \min_{\mathbf{u}, \mathbf{v} \in H} \{C(\mathbf{u}, \mathbf{v})\}$ .

The set  $H$  is called a *CH system* of period  $T$ , and  $D(H)$  is the *degree of overlapping* of the CH system  $H$ .

## III. THE QUORUM-BASED CHANNEL HOPPING SYSTEM

### A. The Quorum System

First, we provide a brief introduction to the quorum system to facilitate the understanding of QCH.

*Definition 1:* Given a finite universal set  $U = \{0, \dots, n - 1\}$  of  $n$  elements, a quorum system  $S$  under  $U$  is a collection of non-empty subsets of  $U$ , which satisfies the intersection property:  $p \cap q \neq \emptyset, \forall p, q \in S$ . Each  $p \in S$  (which is a subset of  $U$ ) is called a *quorum*.

It is readily apparent that a CH system  $H$  defined in Problem 1 is a quorum system under the universal set  $U = \{(i, j) | i \in [0, T - 1], j \in [0, N - 1]\}$ , since it satisfies the intersection property: any two sequences in  $H$  have at least one overlap. Each CH sequence in  $H$  is a quorum.

1) *Cyclic Quorum Systems:* The *cyclic quorum system*, first introduced in [18], can be constructed using cyclic difference sets. Here, we provide some definitions related to cyclic quorum systems since they are utilized to design channel hopping schemes in Sections IV and V.

*Definition 2:* A set  $D = \{a_1, a_2, \dots, a_\kappa\} \subset \mathbf{Z}_n$  is called a relaxed cyclic  $(n, \kappa)$ -difference set if for every  $d \not\equiv 0 \pmod{n}$  there exists at least one ordered pair  $(a_i, a_j)$ , where

$a_i, a_j \in D$ , such that  $a_i - a_j \equiv d \pmod{n}$ . Here,  $\mathbf{Z}_n$  denotes the set of nonnegative integers less than  $n$ .

**Definition 3:** A group of sets  $B_i = \{a_1 + i, a_2 + i, \dots, a_\kappa + i\} \pmod{n}$ ,  $i \in \{0, 1, \dots, n-1\}$  is a cyclic quorum system if and only if  $D = \{a_1, a_2, \dots, a_\kappa\}$  is a relaxed cyclic  $(n, \kappa)$ -difference set.

For example,  $D = \{0, 1, 3\}$  is a relaxed cyclic  $(7, 3)$ -difference set under  $\mathbf{Z}_7$  since each  $d \in \{1, \dots, 6\}$  is congruent to the difference of two elements in  $D$ . Given  $D$ ,  $S = \{B_0, B_1, \dots, B_6\}$  is a cyclic quorum system under  $\mathbf{Z}_7$ , where  $B_i = \{0 + i, 1 + i, 3 + i\} \pmod{7}$ ,  $i = \{0, 1, \dots, 6\}$ . It was proven in [15] that any quorum  $q$  in a cyclic quorum system under  $U = \{0, \dots, n-1\}$  must satisfy  $|q| \geq \sqrt{n}$ , where  $|q|$  denotes the cardinality of  $q$ .

Given any  $n$ , a difference set as small as  $\kappa \approx \sqrt{n}$  can be found when  $\kappa^2 - \kappa + 1 = n$  and  $\kappa - 1$  is a prime power. Such a difference set is called the *Singer* difference set [5], which is the *minimal* difference set whose size  $\kappa$  approximates the lower bound  $\sqrt{n}$ . Hence, cyclic quorum systems defined by the Singer difference sets are *minimal* cyclic quorum systems in the sense that their quorum sizes are close to the theoretical lower bound. For example, the set  $\{1, 2, 4\}$  under  $\mathbf{Z}_7$  is a Singer difference set when  $\kappa = 3$ .

Any set  $D$  that contains  $\lceil \frac{n+1}{2} \rceil$  elements of  $\mathbf{Z}_n$  is a relaxed cyclic  $(n, \lceil \frac{n+1}{2} \rceil)$ -difference set and a cyclic quorum system  $S = \{B_0, B_1, \dots, B_{n-1}\}$  can be constructed based on  $D$  according to Definition 3. Since  $D$  contains more than half of the elements in  $\mathbf{Z}_n$ , we refer to such a cyclic quorum system,  $S$ , as a *majority* cyclic quorum system. For example,  $S = \{\{0, 1, 2\}, \{1, 2, 3\}, \{2, 3, 0\}, \{3, 0, 1\}\}$  is a majority cyclic quorum system under  $\mathbf{Z}_4$ .

2) *Load of Quorum Systems:* Here, we provide some definitions regarding the load of quorum systems. In the context of quorum systems, a *strategy* is a rule giving each quorum an access frequency so that the frequencies sum up to one. In other words, a strategy gives the frequency of picking each quorum. A strategy induces a *load* on each element, which represents the fraction of the time the element is used. Specifically, an element's load is the summation of the frequencies of all quorums that the element belongs to. Below, we provide more precise definitions.

**Definition 4:** Let a quorum system  $S = \{q_0, q_1, \dots, q_{\kappa-1}\}$  be given over a universal set  $U$ . Then  $W \in [0, 1]^\kappa$  is a strategy for  $S$  if it is a probability distribution over the quorums  $q_j \in S$ , i.e.,  $\sum_{j=0}^{\kappa-1} W_j = 1$ .

The *system load*,  $\mathcal{L}(S)$ , is the minimal load on the busiest element, minimizing over the strategies.

**Definition 5:** Let a strategy  $W$  be given for a quorum system  $S = \{q_0, q_1, \dots, q_{\kappa-1}\}$  over a universal set  $U$ . For an element  $i \in U$ , the load induced by  $W$  on  $i$  is  $l_W(i) = \sum_{q_j \in S: i \in q_j} W_j$ . The load induced by a strategy  $W$  on a quorum system  $S$  is  $\mathcal{L}_W(S) = \max_{i \in U} l_W(i)$ . The *system load* on a quorum system  $S$  is  $\mathcal{L}(S) = \min_W \{\mathcal{L}_W(S)\}$ , where the minimum is taken as the system load over all strategies  $W$ .

## B. The Quorum-based Channel Hopping System

In this subsection, we introduce an algorithm that uses a quorum system to construct a CH system. We refer to

this algorithm as Algorithm 1. Without loss of generality, suppose we want to construct a CH system  $H$  where every pair of CH sequences rendezvous in  $m$  different channels, viz  $D(H) = m$ . We randomly select  $m$  channels from  $\{0, \dots, N-1\}$  to construct a *set of rendezvous channels*, such as  $R = \{h_0, h_1, \dots, h_{m-1}\}$ . In our construction algorithm, every CH sequence is composed of  $m$  frames and each frame is composed of  $k$  slots ( $k$  is called the frame length). Hence, the period of each CH sequence is  $T = m \cdot k$ . We use the following example to explain the construction algorithm.

Suppose the set of rendezvous channels is  $R = \{0, 1, 2\}$ , each CH sequence is composed of  $m = 3$  frames, and each frame has  $k = 3$  slots.

- 1) First construct a universal set,  $U = \mathbf{Z}_k = \{0, 1, 2\}$ ;
- 2) Construct a quorum system  $S$  under  $U$ ,  $S = \{\{0, 1\}, \{0, 2\}, \{1, 2\}\}$ ;<sup>1</sup>
- 3) Using the quorum  $q_0 = \{0, 1\} \in S$ , we construct a CH sequence  $\mathbf{u}$  by (timeslot, channel) assignments. We make  $k$  channel assignments for timeslots in the  $(j+1)^{th}$  frame of  $\mathbf{u}$  according to the following equation:

$$u_i = \begin{cases} h_j, & \text{if } (i \bmod k) \in q_0, \\ h, & \text{if } (i \bmod k) \notin q_0. \end{cases}$$

where  $j \in [0, m-1]$  and  $h$  is a randomly selected channel from  $\{0, \dots, N-1\}$ . In the example shown in Figure 1, the (timeslot, channel) assignments for the 1<sup>st</sup> frame of sequence  $\mathbf{u}$  is obtained using the quorum  $q_0 = \{0, 1\} \in S$  and the 1<sup>st</sup> channel in  $R$ ,  $h_0 = 0$ —i.e.,  $u_0 = 0, u_1 = 0, u_2 = h$ . The resulting CH sequence is  $\mathbf{u} = \{(0, 0), (1, 0), (2, h), (3, 1), (4, 1), (5, h), (6, 2), (7, 2), (8, h)\}$ ;

- 4) Repeat Step (3) for each of the other quorums in  $S$  (i.e.,  $q_1 = \{0, 2\}$  and  $q_2 = \{1, 2\}$ ) to construct two other sequences,  $\mathbf{v}$  and  $\mathbf{w}$ . The three CH sequences— $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$ —are the elements of the set  $Q$ , which contains  $|S| = 3$  CH sequences.

The sequences in  $Q$  are illustrated in Figure 1. Algorithm 1 constructs each sequence in  $Q$  by making  $k$  (timeslot, channel) assignments for each of the  $m$  rendezvous channels. One quorum in  $S$  is needed to generate each CH sequence in  $Q$ . Thus,  $|Q| = |S|$ .

Note that  $\forall \mathbf{u}, \mathbf{v} \in Q$ , there are two corresponding quorums  $p, q \in S$  used for constructing  $\mathbf{u}$  and  $\mathbf{v}$ , respectively. Because of the intersection property of  $S$ ,  $\mathbf{u}$  and  $\mathbf{v}$  overlap in exactly  $m$  distinct channels—viz, the channels  $h_d \in R$ ,  $d \in [0, m-1]$  (see lines 3 and 6 of Algorithm 1). Also note that all of the sequences in  $Q$  have the same period, viz,  $T = m \cdot k$  slots. Therefore,  $Q$  is a CH system that satisfies the properties in Problem 1. We refer to the CH system constructed using Algorithm 1 as a *quorum-based channel hopping (QCH)* system.

## C. Metrics for Evaluating CH Systems

We introduce two metrics—*maximum time-to-rendezvous* (MTTR) and *load*—that are used to evaluate CH systems. Note

<sup>1</sup>The desired properties of the CH system determines the particular quorum system that is constructed. In Section IV, we discuss a number of quorum systems that can be used to construct QCH systems with specific properties.

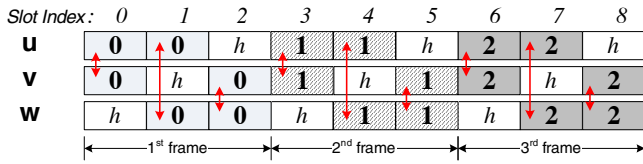


Fig. 1. An illustration of QCH system  $Q$  with  $m = 3$  and  $k = 3$ . Any two sequences overlap on three channels. We use the quorum system  $S = \{\{0, 1\}, \{0, 2\}, \{1, 2\}\}$  over  $U = \{0, 1, 2\}$  to construct  $Q$ . The numbers inside the slots denote channel index values: bold-font numbers denote channel indexes from  $R$  and  $h$  denotes channel indexes randomly chosen from  $\{0, \dots, N - 1\}$ .

that these metrics can be used to evaluate all CH systems, not just quorum-based CH systems.

1) *Maximum Time-to-Rendezvous*: The first metric we introduce is the *maximum time-to-rendezvous* (MTTR) for a CH system, which is defined as the maximum time for any pair of sequences in a CH system to rendezvous. Let  $M(H)$  denote the MTTR of a CH system  $H$ . In a QCH system  $Q$  constructed using Algorithm 1, the MTTR value is equal to the frame length  $k$ . In the QCH system  $Q$  given in Figure 1, the MTTR is  $M(Q) = 3$ . It is obvious that the MTTR value impacts the medium access delay of MAC protocols that utilize channel hopping since the exchange of control information is not possible without rendezvous. Networks that require stringent delay requirements will require a CH system with a small MTTR value. For example, in a mobile CR network, neighboring nodes have to exchange time-sensitive control information frequently—information such as spectrum sensing related control information, node location, link connectivity, etc. In Section IV-A, we describe a methodology for designing a QCH system that is optimal in terms of minimizing the MTTR.

2) *Load of CH Systems*: In CH MAC protocols, spreading out the rendezvous in time and frequency (i.e., channels) is important in order to take full advantage of the frequency diversity of multi-channel medium access. If a large proportion of neighboring nodes rendezvous on the same channel, then channel congestion can occur and lead to a control channel bottleneck problem—we use the term *rendezvous convergence* to refer to such a problem. Some CH MAC protocols (e.g., SSCH [2]) implement “customized” mechanisms to prevent rendezvous convergence. Ideally, a CH protocol should spread out the rendezvous over all channels evenly.

One advantage of using the proposed approach to devise CH schemes is that it can formally characterize the rendezvous convergence problem using the measure of *load* which is used in the study of quorum systems. In quorum systems, a *strategy* is a probabilistic rule that gives the frequency of accessing each quorum so that the frequencies sum up to one. Since a CH system is in essence a quorum system, we can use the definition of load given above to create an analogous definition for the load of a CH system. Let  $W_0$  denote the following strategy: each node randomly selects a sequence from a CH system with equal probability. Given a CH system,  $H$ , the load of  $H$  induced by  $W_0$ ,  $\mathcal{L}_{W_0}(H)$ , is the load of the busiest element induced by  $W_0$ ; the busiest element, in this context, refers to the (timeslot, channel)-pair included in the largest number of CH sequences. We define the *load of a CH system*

### Algorithm 1 QCH System Construction Algorithm

**Input:** the total number of channels  $N$ , the set of rendezvous channels  $R = \{h_0, h_1, \dots, h_{m-1}\}$ , the universal set  $U = \mathbf{Z}_k$ , and a quorum system  $S$  under  $U$ .

**Output:** the QCH system  $Q$ .

```

1:  $Q = \emptyset$ .
2: for  $j = 0$  to  $(|S| - 1)$  do
3:   for  $d = 0$  to  $(m - 1)$  do
4:     for  $i = 0$  to  $(k - 1)$  do
5:       if  $i \in q_j$  then
6:          $u_{(i+d \cdot k)} = h_d$ .
7:       end if
8:       if  $i \notin q_j$  then
9:          $u_{(i+d \cdot k)} = h$ , randomly chosen from
            $\{0, \dots, N - 1\}$ .
10:      end if
11:    end for
12:  end for
13:   $Q = Q \cup \mathbf{u}$ .
14: end for

```

as the load of the CH system induced by the particular strategy  $W_0$ . In the QCH system  $Q$  shown in Figure 1, the load is  $\mathcal{L}_{W_0}(Q) = 2/3$ . In Section IV-B, we will discuss a QCH design whose load value approximates the minimal load of QCH systems.

## IV. SYNCHRONOUS QCH SYSTEMS

In this section, we describe two QCH systems that require global clock synchronization.

### A. Minimizing the MTTR in QCH Systems

Minimizing the MTTR of a QCH system is equivalent to minimizing its frame length  $k$ . To design an optimal QCH system that minimizes the MTTR, we first need to solve the following problem:

*Problem 2:* Given a QCH system  $Q$ ,

$$\begin{aligned} & \text{minimize } k, \\ & \text{subject to } \mathcal{L}_{W_0}(Q) < 1. \end{aligned}$$

The constraint  $\mathcal{L}_{W_0}(Q) < 1$  equates to  $\bigcap_{\mathbf{u} \in Q} \mathbf{u} = \emptyset$ , which is needed to avoid the scenario in which the load of the QCH system is equal to one (i.e., there is at least one (timeslot, channel)-pair that is included in all of the sequences in  $Q$ ).

**The lower bound for  $k$ .** To solve Problem 2, we find the lower bound for  $k$  when the load of the QCH system is less than one in the following theorem.

*Theorem 1:* Given a QCH system  $Q$ , a necessary condition for  $\mathcal{L}_{W_0}(Q) < 1$  is  $k \geq 3$ .

*Proof:* We prove this theorem by contradiction. Let  $k \leq 2$  and suppose we have a QCH system  $Q$ , where  $m = D(Q)$ ,  $T = k \cdot m$  and  $\mathcal{L}_{W_0}(Q) < 1$ .

If  $k = 1$ , then  $T = m$ . Since  $C(\mathbf{u}, \mathbf{v}) \geq m \geq 1, \forall \mathbf{u}, \mathbf{v} \in Q$ , all sequences in  $Q$  must be identical. In this case, the load of  $Q$  is 1, which contradicts the constraint  $\mathcal{L}_{W_0}(Q) < 1$ .

If  $k = 2$ , the universal set is  $U = \{0, 1\}$  according to Algorithm 1. Any quorum system  $S$  over  $U = \{0, 1\}$  has a

system load of one, i.e.,  $\mathcal{L}(S) = 1$ . Since the QCH system  $Q$  is constructed using  $S$ ,  $\mathcal{L}_{W_0}(Q) \geq \mathcal{L}(S) = 1$ , and this contradicts the constraint  $\mathcal{L}_{W_0}(Q) < 1$ .

From Figure 1, we can see that there exists a QCH system in which  $k = 3$  and its load is less than one. From the above arguments, it is clear that  $k \geq 3$  is a necessary condition for  $\mathcal{L}_{W_0}(Q) < 1$ . ■

**Construction of the M-QCH system.** The QCH system that achieves the lower bound for  $k$  (i.e.,  $k = 3$ ) is an optimal QCH design in the sense that it minimizes the MTTR while keeping the load less than one. We refer to such a system as an *M-QCH* system, and it can be constructed using Algorithm 1 with a majority cyclic quorum system over a universal set  $U = \mathbf{Z}_3$ . The example QCH system shown in Figure 1 is an M-QCH system.

**Discussions.** An M-QCH system can support  $m$  rendezvous channels ( $m \in [1, N]$ ) and it has the lowest MTTR value (i.e., 3) among all QCH systems. Hence, M-QCH systems are advantageous in establishing control channels with minimal medium access delay. The load of an M-QCH system is  $2/3$ , which is the same as the load of the quorum system  $S = \{\{0, 1\}, \{0, 2\}, \{1, 2\}\}$  over  $U = \{0, 1, 2\}$  under  $W_0$ .

### B. Minimizing the Load

In this subsection, we study the QCH system,  $Q$ , that has the minimal load under the constraint  $M(Q) \leq \tau$ , for a given value of  $\tau$ . To devise such a QCH system, we need to solve the following problem:

**Problem 3:** Given a QCH system  $Q$ ,

$$\begin{aligned} & \text{minimize } \mathcal{L}_{W_0}(Q), \\ & \text{subject to } M(Q) \leq \tau, \end{aligned}$$

where  $\tau$  is the maximum allowed MTTR of  $Q$ .

**The lower bound for load.** We solve Problem 3 by finding the lower bound for  $\mathcal{L}_{W_0}(Q)$  under the constraint  $M(Q) \leq \tau$ .

**Theorem 2:** Given a QCH system  $Q$  where  $M(Q) \leq \tau$ , the minimum load induced by  $W_0$  on  $Q$  is  $\frac{1}{\sqrt{\tau}}$ , i.e.,  $\mathcal{L}_{W_0}(Q) \geq \frac{1}{\sqrt{\tau}}$ .

*Proof:* In a QCH system,  $Q$ , in which  $M(Q) \leq \tau$ , the inequality  $k \leq \tau$  ( $k$  is the frame length) holds since  $M(Q) = k$ . According to Algorithm 1, such a QCH system  $Q$  is constructed using a quorum system  $S$  over  $U = \mathbf{Z}_k$ . Thus, we have  $\mathcal{L}_{W_0}(Q) \geq \mathcal{L}(S)$ . The Propositions 4.1 and 4.2 in [21] state that the following relation is true:  $\mathcal{L}(S) \geq \max\left\{\frac{1}{\varphi(S)}, \frac{\varphi(S)}{k}\right\}$ , where  $\varphi(S)$  is the size of the smallest quorum in the quorum system  $S$ . Using the inequality of arithmetic and geometric means, it can be shown that  $\mathcal{L}(S) \geq \frac{1}{\sqrt{k}}$ . Since  $k \leq \tau$ , we have  $\mathcal{L}_{W_0}(Q) \geq \mathcal{L}(S) \geq \frac{1}{\sqrt{k}} \geq \frac{1}{\sqrt{\tau}}$ . ■

**Construction of the L-QCH system.** As shown in [15], [18], the *minimal cyclic quorum* system is near-optimal in terms of the load. By using a minimal cyclic quorum system over  $U = \mathbf{Z}_\tau$  as the input to Algorithm 1, we can construct a QCH system whose load value is close to the theoretical minimum, which is  $\frac{1}{\sqrt{\tau}}$ . We refer to such a QCH system as an *L-QCH* System.

**Discussions.** An L-QCH system,  $Q$ , has a degree of overlapping of  $D(Q) \in [1, N]$ , and its MTTR is equal to its frame length  $k$ , that is,  $M(Q) = k$ . According to Theorem 2, a

QCH system with its maximum TTR upperbounded by  $\tau$  has a minimum load value of  $\frac{1}{\sqrt{\tau}}$ —an L-QCH system has a load value that is close to this minimum value. The L-QCH system shown in Figure 2 has a load value of  $\frac{3}{7}$ .

## V. ASYNCHRONOUS QCH SYSTEMS

In this section, we describe an *asynchronous* CH system that does *not* require global clock synchronization. The objective is to devise a CH system,  $H$ , that enables any pair of CH sequences to overlap by at least half of a timeslot for every sequence period (i.e., for every  $T$  consecutive timeslots) even under the assumption that slot boundaries are misaligned by an arbitrary amount.

### A. Rotation Closure Property in CH Systems

We extend the concept of the *rotation closure property* of quorum systems [15] so that it is applicable to CH systems. We will show that a CH system with the rotation closure property is an asynchronous CH system that does not require global clock synchronization.

**Definition 6:** Given a non-negative integer  $i$  and a CH sequence  $\mathbf{u}$  in a CH system  $H$  of period  $T$ , we define

$$\text{rotate}(\mathbf{u}, i) = \{(j, v_j) | v_j = u_{(j+i) \bmod T}, j \in [0, T-1]\}.$$

For example, given  $\mathbf{u} = \{(0, 0), (1, 1), (2, 2)\}$  and  $T = 3$ ,  $\text{rotate}(\mathbf{u}, 2) = \{(0, 2), (1, 0), (2, 1)\}$ .

**Definition 7:** A CH system  $H$  with period  $T$  and a degree of overlapping  $m$  is said to have the rotation closure property if  $\forall \mathbf{u}, \mathbf{v} \in H, \mathbf{u} \neq \mathbf{v}, \forall i \in [0, T-1], C(\text{rotate}(\mathbf{u}, i), \mathbf{v}) \geq m$  holds.

Building on the above definitions, the following theorem states that a CH system with the rotation closure property ensures rendezvous even when the slot boundaries are not aligned.

**Theorem 3:** If a CH system  $H$  with period  $T$  and a degree of overlapping  $m$  satisfies the rotation closure property, any pair of CH sequences in  $H$  must overlap by at least  $m/2$  timeslots for every  $T$  consecutive timeslots even when the timeslot boundaries are misaligned by an arbitrary amount.

*Proof:* Suppose that a CH system  $H$  satisfies the rotation closure property and two nodes,  $A$  and  $B$ , each picks a CH sequence from  $H$  randomly—viz,  $\mathbf{u}$  and  $\mathbf{v}$ , respectively. For the sake of our discussions, suppose the length of a timeslot is 1. We consider two cases.

1) When slot boundaries are aligned: Without loss of generality, let us suppose node  $A$ 's clock is  $i$  slots ahead of node  $B$ 's clock. With respect to node  $B$ 's clock, node  $A$ 's sequence  $\mathbf{u}$  is equivalent to  $\text{rotate}(\mathbf{u}, i)$ . Since  $H$  has the rotation closure property,  $C(\text{rotate}(\mathbf{u}, i), \mathbf{v}) \geq m$ . Hence, the two sequences must have at least  $m$  rendezvous channels between them (i.e., overlap by at least  $m$  timeslots). It is obvious that the same result is obtained when we assume that  $A$ 's clock is  $i$  slots behind  $B$ 's clock.

2) When slot boundaries are misaligned: Suppose node  $A$ 's clock is ahead of node  $B$ 's clock by an arbitrary amount of time, say  $i + \delta$ , where  $i \in \mathbf{Z}_T, 0 < \delta < 1$ .

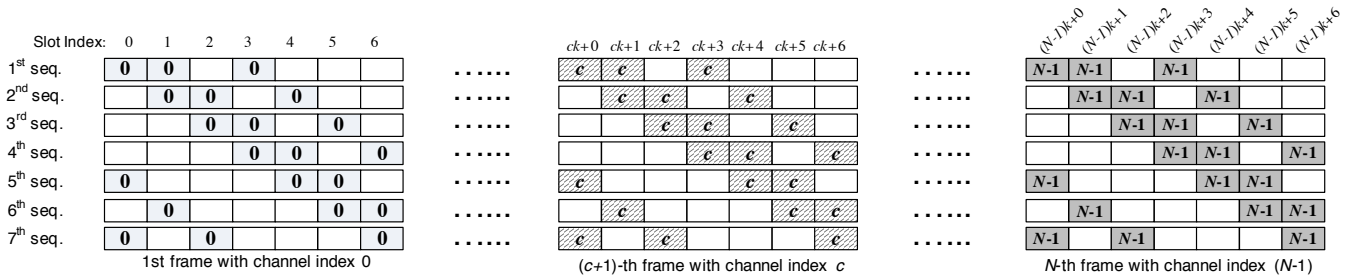


Fig. 2. An L-QCH system,  $Q$ , with  $m = N$  rendezvous channels and  $\tau = 7$ . To construct  $Q$ , we use a minimal cyclic quorum system  $S = \{\{0, 1, 3\}, \{1, 2, 4\}, \{2, 3, 5\}, \{3, 4, 6\}, \{4, 5, 0\}, \{5, 6, 1\}, \{6, 0, 2\}\}$  under  $U = \mathbf{Z}_7$ . Any two sequences in this system overlap on all  $N$  channels. Channel indexes in the “blank” grids are randomly selected from  $\{0, 1, \dots, N - 1\}$ .

- If  $\delta \leq 1/2$ , let us shift left node  $B$ 's sequence by  $\delta$  and designate this sequence as  $\mathbf{v}'$ .<sup>2</sup> It is obvious that the slot boundaries of  $\mathbf{u}$  and  $\mathbf{v}'$  are aligned and the former is ahead of the latter by  $i$  slots in terms of their respective nodes' clocks. Since  $H$  has the rotation closure property,  $C(\text{rotate}(\mathbf{u}, i), \mathbf{v}') \geq m$ , and thus  $\mathbf{u}$  must overlap with  $\mathbf{v}$  by  $m(1 - \delta)$  timeslots. This means that the two sequences overlap with each other by at least  $m/2$  timeslots for every  $T$  consecutive timeslots.
- If  $\delta > 1/2$ , let us shift right node  $B$ 's sequence by  $1 - \delta$  and designate this sequence as  $\mathbf{v}'$ . It is obvious that the slot boundaries of  $\mathbf{u}$  and  $\mathbf{v}'$  are aligned and the former is ahead of the latter by  $(i + 1)$  slots in terms of their respective nodes' clocks. Since  $H$  has the rotation closure property,  $C(\text{rotate}(\mathbf{u}, i + 1), \mathbf{v}') \geq m$ , and thus  $\mathbf{u}$  must overlap with  $\mathbf{v}$  by  $m\delta$  timeslots. This means that the two sequences overlap with each other by at least  $m/2$  timeslots for every  $T$  consecutive timeslots.

### Algorithm 2 A-QCH System Construction Algorithm

**Input:** the total number of channels  $N$ , the set of rendezvous channels  $R = \{h_0, h_1\}$ , the universal set  $U = \mathbf{Z}_k$ , and two cyclic quorum systems  $S$  and  $S'$  over  $U$ .

**Output:** the A-QCH system  $Q$ .

- 1:  $Q = \emptyset$ .
- 2: **for**  $j = 0$  to  $(|S| - 1)$  **do**
- 3:     **for**  $i = 0$  to  $(k - 1)$  **do**
- 4:         **if**  $i \in B_j$  **then**
- 5:              $u_i = h_0$ .
- 6:         **else if**  $i \in B'_j$  **then**
- 7:              $u_i = h_1$ .
- 8:         **else**
- 9:              $u_i = h$  randomly chosen from  $\{0, \dots, N - 1\}$ .
- 10:         **end if**
- 11:     **end for**
- 12:      $Q = Q \cup \mathbf{u}$ .
- 13: **end for**

From Theorem 3, we can conclude that any two nodes that select CH sequences from a system with the rotation closure property can rendezvous with each other during the overlap of their sequences even if they are asynchronous (i.e., slot boundaries are not aligned). If multiple pairs of nodes happen to rendezvous at the same slot on the same channel, they can follow a channel contention procedure (e.g., 802.11 RTS/CTS protocol) to carry out the pair-wise rendezvous.

Henceforth we refer to a CH system that satisfies the rotation closure property as an *asynchronous CH* system. Next, we describe an algorithm, Algorithm 2, for constructing an asynchronous CH system that uses two different types of cyclic quorum systems.

### B. Construction of A-QCH Systems

We refer to the CH system constructed by Algorithm 2 as an *asynchronous quorum-based CH* (A-QCH) system. Using two types of cyclic quorums systems, the constructed A-QCH system guarantees at least two rendezvous channels between any two sequences (i.e.,  $C(\mathbf{u}, \mathbf{v}) \geq 2, \forall \mathbf{u}, \mathbf{v} \in H$ ). This A-QCH system is composed of CH sequences that have only one frame per sequence period (i.e.,  $T = k$ ). In each constructed CH sequence, a subsequence constructed by a

minimal cyclic quorum  $S$  is interleaved with a subsequence constructed by a majority cyclic quorum system  $S'$ . Refer to [18] for methods to construct minimal cyclic quorums and majority cyclic quorums. The description of Algorithm 2 is given below.

- 1) First construct a universal set  $U = \mathbf{Z}_k$ ;
- 2) Find a minimal  $(k, \kappa)$ -difference set  $D = \{a_1, a_2, \dots, a_\kappa\}$  such that  $\kappa < \frac{k}{2}$  and construct a minimal cyclic quorum system based on  $D$ , such as  $S = \{B_i | B_i = \{a_1 + i, a_2 + i, \dots, a_\kappa + i\} \bmod k, i \in [0, k - 1]\}$ ;
- 3) Construct a relaxed cyclic  $(k, \lceil \frac{k+1}{2} \rceil)$ -difference set  $D' = \{a'_1, a'_2, \dots, a'_{\lceil \frac{k+1}{2} \rceil}\}$  such that  $D' \cap D = \emptyset$ . Then, construct a majority cyclic quorum system based on  $D'$ , such as  $S' = \{B'_i | B'_i = \{a'_1 + i, a'_2 + i, \dots, a'_{\lceil \frac{k+1}{2} \rceil} + i\} \bmod k, i \in [0, k - 1]\}$ . Note that  $|S| = |S'| = \bar{k}$ , and  $|D'| = \lceil \frac{k+1}{2} \rceil$ ;
- 4) Use the minimal cyclic quorum system  $S$  for assigning the first rendezvous channel  $h_0$  to appropriate slots in a CH sequence  $\mathbf{u}$  (see lines 4 and 5 in the algorithm);
- 5) Use the majority cyclic quorum system  $S'$  to assign the second rendezvous channel  $h_1$  to appropriate slots in a CH sequence  $\mathbf{u}$  (see lines 6 and 7 in the algorithm).
- 6) The remaining slots in  $\mathbf{u}$  are assigned a channel index

<sup>2</sup>Shifting a node's CH sequence *left/right* by  $\delta$  is equivalent to *advancing/retreating* the node's clock by  $\delta$ .

Slot Index:	0	1	2	3	4	5	6	7	8
	0	0	0	1	0	1	1	1	1
	0	0	1	0	1	1	1	1	0
	0	1	0	1	1	1	1	0	0
	1	0	1	1	1	1	0	0	0
	0	1	1	1	1	0	0	0	1
	1	1	1	1	0	0	0	1	0
	1	1	0	0	0	1	0	1	1
	1	0	0	0	1	0	1	1	1

Fig. 3. An A-QCH system  $Q$  with  $m = 2$  and  $T = k = 9$ . The universal set,  $U$ , is  $\mathbf{Z}_9$ ,  $S$  is constructed using  $D = \{0, 1, 2, 4\}$ , and  $S'$  is constructed using  $D' = \{3, 5, 6, 7, 8\}$ . Note that  $D' \cap D = \emptyset$ . The numbers inside the slots represent the channel index values.

randomly chosen from  $\{0, \dots, N - 1\}$  (see line 9 in the algorithm).

An example A-QCH system is shown in Figure 3. One can readily show that an A-QCH system constructed using Algorithm 2 satisfies the rotation closure property and that the TTR value between any two sequences in an A-QCH system is bounded by the length of its sequence period  $T = k$ .

**Discussions.** Any two CH sequences in an A-QCH system overlap in two channels, i.e., the degree of overlapping of an A-QCH system is two. Given that  $D' \cap D = \emptyset$ ,  $k$  must be no less than  $|D| + |D'|$  so that a CH sequence can accommodate two subsequences constructed using  $S$  and  $S'$ . In [18], Luk and Wong conducted an exhaustive search to find the minimal difference sets under  $\mathbf{Z}_k$  for  $k = 4, \dots, 111$ . The MTTR of an A-QCH system is  $k$ , and the load of an A-QCH system is approximately  $\frac{1}{2}$ , which is also the load value of a majority cyclic quorum system.

In our description of A-QCH systems given above, we used relaxed cyclic difference sets  $D$  and  $D'$  for generating cyclic quorum systems that facilitate the construction of a CH system with the rotation closure property. The specific choices of  $D$  and  $D'$  and the resulting cyclic quorum systems have no significance—i.e., the quorum systems that we have chosen are merely our design choices for constructing an A-QCH system; it is likely that there are other quorum systems that can be used to construct A-QCH systems of similar or different structure.

## VI. ASYNCHRONOUS MAXIMUM OVERLAPPING CH SYSTEM

### A. Limitations of A-QCH

The A-QCH system described in the previous section guarantees rendezvous in only *two* distinct channels (i.e., degree of overlapping is two). If those two channels are unavailable due to the presence of incumbent signals, then node pairs that need to rendezvous in those channels would be unable to exchange control information. To avoid such a problem, an asynchronous CH system that enables rendezvous in every channel is needed, and in this section, we describe an asynchronous CH system that satisfies this requirement. We describe a CH system called *asynchronous maximum overlapping CH (A-MOCH)* system which enables any pair of CH sequences to rendezvous in all available channels

(i.e., degree of overlapping is  $N$ , where  $N$  is the number of channels).

### B. Construction Method

The A-MOCH scheme generates CH sequences using *Latin Squares*. Pseudo-code for constructing an A-MOCH system is provided in Algorithm 3.

#### 1) The Latin Square:

**Definition 8:** A Latin Square (LS) is an  $n \times n$  table filled with  $n$  different numbers in such a way that each number  $i \in \mathbf{Z}_n$  occurs exactly once in each row and exactly once in each column.

Here is an example of Latin square when  $n = 3$ :

$$\begin{Bmatrix} 0, 1, 2, \\ 1, 2, 0, \\ 2, 0, 1 \end{Bmatrix}.$$

**Definition 9:** An Identical-Row Square (IRS) is an  $n \times n$  table filled with  $n$  different numbers in such a way that each row consists of a permutation of integers in  $\mathbf{Z}_n$  and all rows are identical.

Here is an example of identical-row square when  $n = 3$ :

$$\begin{Bmatrix} 1, 2, 0, \\ 1, 2, 0, \\ 1, 2, 0 \end{Bmatrix}.$$

**2) Construction of an A-MOCH System:** Using an  $N \times N$  Latin square or an  $N \times N$  identical-row square described above, every node constructs its CH sequence independently.

- **Default CH sequence  $\mathbf{u}$ .** If a node has nothing to send, it randomly selects a permutation of  $\{0, 1, \dots, N - 1\}$ , denoted as  $x = \{x_0, x_1, \dots, x_i, \dots, x_{(N-1)}\}$ , and constructs a default sequence as  $\mathbf{x} = \{(0, x_0), (1, x_1), \dots, (i, x_i), \dots, (N - 1, x_{(N-1)})\}$ . The node constructs its default CH sequence,  $\mathbf{u}$ , by repeating  $\mathbf{x}$   $N$  times and concatenating them into a single sequence.
- **Alternative CH sequence  $\mathbf{v}$ .** If a node is a sender that has data to transmit, it randomly selects a permutation of  $\{0, 1, \dots, N - 1\}$ , denoted as  $y = \{y_0, y_1, \dots, y_i, \dots, y_{(N-1)}\}$ , and constructs an alternative sequence as  $\mathbf{y} = \{(0, y_0), (1, y_1), \dots, (i, y_i), \dots, (N - 1, y_{(N-1)})\}$ . Then, it constructs its alternative CH sequence,  $\mathbf{v}$ , by concatenating the following  $N$  CH sequences:  $rotate(\mathbf{y}, 0)$ ,  $rotate(\mathbf{y}, 1)$ ,  $rotate(\mathbf{y}, 2)$ , ...,  $rotate(\mathbf{y}, i)$ , ...,  $rotate(\mathbf{y}, (N - 1))$ .

Every node generates an alternative or a default CH sequence, depending on whether it has data to transmit or not. If the channel indexes of a CH sequence are populated into an  $N \times N$  table as shown in Figure 4, it can be seen that the alternative CH sequence is generated from an  $N \times N$  Latin square and the default CH sequence is generated from an  $N \times N$  identical-row square. The following theorem states that  $H = \{\mathbf{u}, \mathbf{v}\}$  is an asynchronous CH system in which the number of rendezvous channels between the two CH sequences is  $N$ .

**Theorem 4:** Given  $N$  channels, Algorithm 3 constructs a CH system composed of the two sequences  $\mathbf{u}$  and  $\mathbf{v}$  of period  $N^2$ , which satisfies  $C(\mathbf{u}, \mathbf{v}) = N$  and the rotation closure property.

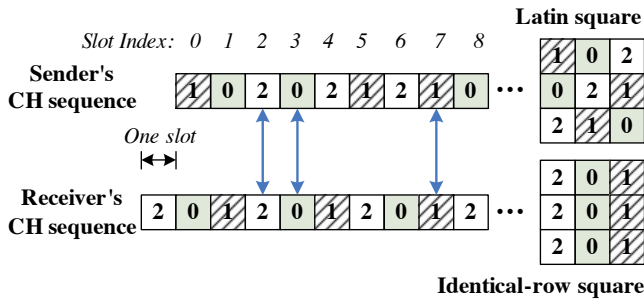


Fig. 4. An A-MOCH system,  $H$ , with  $D(H) = 3$  and  $T = 9$ , when  $N = 3$ . The receiver's sequence,  $\mathbf{u}$ , is generated from a  $3 \times 3$  identical-row square and the sender's sequence,  $\mathbf{v}$ , is generated from a  $3 \times 3$  Latin square. The receiver's clock is one slot ahead of that of the sender. The two sequences rendezvous in  $N$  distinct channels, i.e.,  $D(H) = N$ .

*Proof:* Suppose a CH system  $H$  is composed of a default CH sequence  $\mathbf{u}$  constructed by a receiver and an alternative CH sequence  $\mathbf{v}$  constructed by a sender. The channel indexes of the default CH sequence  $\mathbf{u}$  can be populated into an  $N \times N$  table to form an identical-row square, and the channel indexes of the alternative CH sequence  $\mathbf{v}$  can be populated into an  $N \times N$  table to form a Latin square (see the example shown in Figure 4). We prove this theorem in two steps.

**Number of rendezvous channels between  $\mathbf{u}$  and  $\mathbf{v}$ .** According to Definition 8, each number  $i \in \mathbf{Z}_N$  appears exactly once in each column of a Latin square. On the other hand, according to Definition 9, each column of the identical-row square contains  $N$  repetitions of a unique number  $j \in \mathbf{Z}_N$ . If the two squares are superimposed on top of one another, one can readily observe that there are  $N$  overlaps of  $N$  distinct numbers. An overlap occurs when the same number appears in the same position (i.e., same row and column positions) in the two squares. Thus,  $C(\mathbf{u}, \mathbf{v}) = N$ .

**Rotation closure property of  $H = \{\mathbf{u}, \mathbf{v}\}$ .** According to Definition 7, in order to prove that the CH system,  $H$ , has the rotation closure property, it is sufficient to show that  $C(\text{rotate}(\mathbf{u}, k), \mathbf{v}) = N, \forall k \in [0, N^2 - 1]$ .

Without loss of generality, let us suppose the receiver's clock is  $i$  slots ahead of the sender's clock. Suppose that their timeslot boundaries are aligned, but their CH period boundaries are misaligned (in the example shown in Figure 4, the receiver's clock is one slot ahead of the sender's clock). With respect to the sender's clock, receiver's sequence  $\mathbf{u}$  is equivalent to  $\text{rotate}(\mathbf{u}, i)$ .

Suppose that the operation  $\text{rotate}(\mathbf{u}, i)$  yields the CH sequence  $\mathbf{u}^*$ . If the channel indexes of this sequence are used to fill an  $N \times N$  table, we can see that the table forms an identical-row square. If this identical-row square is compared with the Latin square used in generating  $\mathbf{v}$ , we can see  $N$  overlaps of  $N$  distinct numbers. Thus,  $\mathbf{u}^*$  and  $\mathbf{v}$  have  $N$  distinct rendezvous channels within a sequence period. ■

**Discussions.** Two CH sequences in an A-MOCH system  $H$  of period  $N^2$  overlap in  $N$  channels, i.e., the degree of overlapping of an A-MOCH system is  $N$ . The pairwise rendezvous over  $N$  channels may occur during the last  $N$  timeslots of a CH period, and thus its MTTR is equal to  $N^2 - N + 1$ . Since there are only two CH sequences in  $H$ , the load of  $H$  is one. However, the possibility of rendezvous convergence is low because different pairs of sender and

### Algorithm 3 A-MOCH System Construction Algorithm

**Input:** the total number of channels  $N$ .

**Output:** the A-MOCH system  $H$ .

- 1:  $H = \emptyset$ .
- 2:  $\{x_0, x_1, \dots, x_i, \dots, x_{(N-1)}\} \leftarrow$  a random permutation of  $0, 1, \dots, N - 1$  chosen by the receiver.
- 3:  $\mathbf{x} = \{(0, x_0), (1, x_1), \dots, (i, x_i), \dots, (N - 1, x_{(N-1)})\}$  a default sequence constructed by the receiver.
- 4:  $\{y_0, y_1, \dots, y_i, \dots, y_{(N-1)}\} \leftarrow$  a random permutation of  $0, 1, \dots, N - 1$  chosen by the sender.
- 5:  $\mathbf{y} = \{(0, y_0), (1, y_1), \dots, (i, y_i), \dots, (N - 1, y_{(N-1)})\}$  an alternative sequence constructed by the sender.
- 6: **for**  $i = 0$  to  $N - 1$  **do**
- 7:      $\mathbf{z} = \text{rotate}(\mathbf{y}, i)$ .
- 8:     **for**  $j = 0$  to  $N - 1$  **do**
- 9:          $u_{(i \cdot N + j)} = x_j$ .
- 10:          $v_{(i \cdot N + j)} = z_j$ .
- 11:     **end for**
- 12: **end for**
- 13: The A-MOCH system  $H = \{\mathbf{u}, \mathbf{v}\}$ .

receiver are likely to construct different pairs of alternative and default CH sequences, and the rendezvous points (in terms of frequency and time) of those CH sequence pairs are different.

## VII. RENDEZVOUS PERFORMANCE IN OPPORTUNISTIC SPECTRUM ACCESS

In an opportunistic spectrum sharing environment (e.g., CR networks), we assume that every node employing the proposed channel hopping protocol is able to detect incumbent signals on its current channel using some sort of a *fast sensing* technique at the beginning of every timeslot<sup>3</sup>. In addition, we also assume that every secondary node is able to perform *perfect sensing*<sup>4</sup>. The rendezvous process between a pair of secondary nodes is affected by the appearance of incumbent signals because a secondary node should refrain from transmitting on a channel where incumbent signals are detected. In this section, we discuss the rendezvous performance of CH systems in the presence of incumbent traffic.

### A. Maximum Conditional TTR in Opportunistic Spectrum Access

Let  $A(x)$  denote the set of channels that are free of incumbent signals and available for node  $x$  and  $A(x) \subseteq \{0, 1, \dots, N - 1\}$ . For any two neighboring secondary nodes,  $x$  and  $y$ , their corresponding  $A(x)$  and  $A(y)$  may be different, and the following condition needs to be satisfied to achieve rendezvous between  $x$  and  $y$  without causing interference to incumbent users:

$$A(x) \cap A(y) \neq \emptyset.$$

<sup>3</sup>The fast sensing prescribed in IEEE 802.22 typically employs energy detection and performs sensing at speeds of under 1 ms per channel [7].

<sup>4</sup>There exists a body of research work that studied optimal strategies for dynamic spectrum access under sensing errors [13], [16]. However, such strategies are beyond the scope of this paper.



A channel in  $A(x) \cap A(y)$  is called a common *incumbent-free* channel to nodes  $x$  and  $y$ .

Given two channel hopping nodes,  $x$  and  $y$ , under a CH system, we define the *Maximum Conditional TTR (MCTTR)* of the CH system as the maximum time for the two nodes to rendezvous when  $A(x) \cap A(y) \neq \emptyset$ . The differences between MTTR and MCTTR are:

- MTTR denotes the maximum TTR between two channel hopping nodes when incumbent traffic is *absent*. The MTTR exists for any CH system.
- MCTTR denotes the maximum TTR between two channel hopping nodes when incumbent traffic is *present* and at least one incumbent-free channel is available for the two nodes. When there is only one channel that is free of incumbent signals for the two nodes, their CH sequences picked from a CH system have to overlap on every channel to guarantee that they can find the incumbent-free channel for rendezvous. Thus, MCTTR of a CH system is available only if the degree of overlapping of the CH system is  $N$  (the total number of channels).

### B. MCTTR of Synchronous QCH Systems in CR networks

In a synchronous QCH system (e.g., M-QCH or L-QCH), the maximum TTR for two nodes  $x$  and  $y$  is  $(X + 1) \cdot k$ , where  $X$  denotes the number of rendezvous channels occupied by incumbent users and  $k$  is the frame length. When  $A(x) \cap A(y) \neq \emptyset$ , the MCTTR of a synchronous QCH system (M-QCH or L-QCH) is  $kN$ . This implies that the two nodes are able to successfully rendezvous over an incumbent-free channel within a TTR bounded by  $kN$ .

### C. MCTTR of Asynchronous CH Systems in CR networks

For asynchronous CH systems, rendezvous in the presence of incumbent traffic is more challenging than that for synchronous CH systems, because the rotation closure property has to be maintained. The following theorem helps explain the optimality of A-MOCH in terms of MCTTR.

**Theorem 5:** Given two nodes  $x$  and  $y$  in an asynchronous CH system and  $A(x) \cap A(y) \neq \emptyset$ , the MCTTR of the asynchronous CH system is at least  $N^2$ , where  $N$  is the number of channels.

*Proof:* Given  $N$  channels, suppose  $H$  is an asynchronous CH system of period  $T$  and  $A(x) \cap A(y) \neq \emptyset$  for two channel hopping nodes  $x$  and  $y$ .

If the MCTTR of  $H$  exists, the degree of overlapping of  $H$  is equal to  $N$ . Since  $A(x) \cap A(y) \neq \emptyset$ , the two nodes  $x$  and  $y$  are able to find an incumbent-free channel for rendezvous within a CH period. In the worst case, the rendezvous would happen in the last timeslot of the CH period. Thus, the MCTTR of  $H$  is equal to its CH period  $T$ , and we prove this theorem by showing that  $T \geq N^2$ .

Suppose  $\mathbf{u}$  and  $\mathbf{v}$  are two CH sequences in the asynchronous CH system  $H$ . Let  $u^h$  denote the set of slot indexes in  $\mathbf{u}$  whose corresponding channel indexes are equal to  $h$ , where  $h$  is a channel index in  $[0, N - 1]$ , that is

$$u^h = \{i | u_i = h, i \in [0, T - 1]\}.$$

Similarly, we have  $v^h = \{i | v_i = h, i \in [0, T - 1]\}$ .

Since  $H$  is an asynchronous CH system that satisfies the rotation closure property,  $u^h$  and  $v^h$  compose a quorum system under the universal set  $U = \{0, \dots, T - 1\}$ , and  $Q = \{u^h, v^h\}$  satisfies the rotation closure property.

Theorem 2 in [15] states the following: Let  $Q$  be a quorum system under  $U = \{0, \dots, n - 1\}$ . If  $Q$  satisfies the rotation closure property, then any quorum in  $Q$  must have a cardinality that is no less than  $\sqrt{n}$ .

Thus, we have  $|u^h| \geq \sqrt{T}$  and

$$T = \sum_{h=0}^{N-1} |u^h| \geq N \cdot \sqrt{T}.$$

Therefore,  $T \geq N^2$ . ■

According to Theorem 5, the A-MOCH system is an optimal design in terms of MCTTR since its MCTTR is  $N^2$ , which is the minimum MCTTR value for any asynchronous CH system.

## VIII. DISCUSSIONS

### A. Comparisons

In this subsection, we compare the proposed CH systems with three existing CH schemes using four metrics: degree of overlapping, load, MTTR and MCTTR.

**Blind rendezvous (BR) channel hopping [23].** In this scheme, each node hops from one channel to another randomly. At a particular instant, a node occupies one of these channels with probability  $1/N$ , where  $N$  is the total number of channels. When two nodes occupy the same channel at the same time, rendezvous occurs. The BR scheme does not guarantee a bounded TTR between any two CH sequences.

**Slotted Seeded Channel Hopping (SSCH) [2].** Each node is allowed to have one or multiple (channel, seed)-pairs to determine its CH sequences. SSCH allows  $(N - 1)$  seeds. Each sequence period includes a parity slot at which time instant all nodes with the same seed are guaranteed to rendezvous on a channel indicated by the seed value. Thus, the load of the SSCH system is  $\frac{1}{N-1}$ . When each node selects one (channel, seed)-pair, the resulting sequence period is  $(N + 1)$  timeslots, and each pair of sequences rendezvous exactly once within a period. Thus, the MTTR of SSCH is  $(N + 1)$ . By design, SSCH is a synchronous CH system, although results in [2] show that it can tolerate moderate clock skew. The amount of clock skew used in [2] to evaluate SSCH is very small relative to one slot duration.

**Sequence-based rendezvous (SeqR) [8].** Each sequence generated by the SeqR scheme has a period of  $N(N + 1)$  slots. This scheme builds the initial sequence,  $\mathbf{u}$ , by first selecting a permutation of elements in  $\mathbf{Z}_N$ . Then it repeats the selected permutation  $(N + 1)$  times in the sequence  $\mathbf{u}$  using the following method: the permutation is used contiguously  $N$  times, and once the permutation is interspersed with the other  $N$  permutations. For example, when  $N = 3$ , one can select a permutation such as  $\{0, 2, 1\}$ . Then the channel indexes of the initial sequence  $\mathbf{u}$  would be  $\{\mathbf{0}, 0, 2, 1, \mathbf{2}, 0, 2, 1, \mathbf{1}, 0, 2, 1\}$ . Note that the elements in the permutation  $\{0, 2, 1\}$  is interspersed with the three replications of the same permutation. By applying the operation  $rotate(\mathbf{u}, i), \forall i \in [1, N(N + 1) - 1]$  to the initial sequence,  $\mathbf{u}$ , a number of new sequences can be generated, thereby creating a total of  $N(N + 1)$  sequences.

TABLE I  
A comparison of CH schemes.

	Degree of overlapping	Load	MTTR	MCTTR
BR	0	-	-	-
SSCH	1	$\frac{1}{N-1}$	$N+1$	-
M-QCH	$N$	$\frac{2}{3}$	3	$3N$
L-QCH	$N$	$\geq \frac{1}{\sqrt{7}}$	$k$	$kN$
SeqR	1	$\frac{1}{N}$	$N(N+1)$	-
A-QCH	2	$\approx \frac{1}{2}$	$k$	-
A-MOCH	$N$	1	$N^2 - N + 1$	$N^2$

Collectively, this set of sequences forms an asynchronous CH system that satisfies the rotation closure property. The sequence period is  $N(N+1)$  and  $C(\mathbf{u}, \mathbf{v}) \geq 1, \forall \mathbf{u}, \mathbf{v} \in H$ . Its MTTR is  $N(N+1)$  and its load is  $\frac{1}{N}$ .

SSCH, M-QCH, and L-QCH are synchronous CH systems that require the slot and period boundaries to be aligned. In contrast, SeqR, A-QCH, and A-MOCH are asynchronous CH systems that have no such requirements. A comparison of all the CH schemes discussed in this paper are summarized in Table I. In terms of degree of overlapping, M-QCH, L-QCH and A-MOCH are superior to others since these schemes can maximize the degree of overlapping to  $N$  channels. Note that the metrics load and MTTR are not applicable to the BR scheme; the metric MCTTR is not applicable to BR, SSCH, SeqR, and A-QCH due to the limited degree of overlapping in these schemes.

### B. Implementing CH Systems in Opportunistic Spectrum Access Networks

In this subsection, we discuss some of the important issues that may arise when implementing a CH system in opportunistic spectrum access networks.

1) *Channel Heterogeneity*: In this paper, we only consider *logical* channels at the MAC layer and do not consider PHY-layer channel characteristics. However, channel heterogeneity may undermine the performance of CH protocols when implementing these protocols in real-world communication systems. The factors that cause heterogeneous channel conditions include channel fading effects due to multi-path or shadowing, dynamic incumbent user traffic patterns, etc. Any of these factors would affect the reliability of rendezvous channels in CH protocols.

One method for addressing the heterogeneity of channel conditions is to prioritize the channels in terms of channel reliability under various channel conditions and then construct the CH sequences by selecting the set of rendezvous channels with the higher reliability. However, prioritizing channels is a very difficult proposition because physical-layer channel conditions such as fading may vary drastically with time, geographical position, and/or radio frequency. Moreover, the incumbent traffic on licensed channels is unpredictable. Note that the modeling of incumbent traffic patterns is currently an active area of research, and there is no universally accepted traffic model for it [10]–[13].

Another way to increase the reliability of the rendezvous is to spread out the rendezvous points across as many channels as possible (which is equivalent to maximizing the degree of overlapping), and this is the approach that was emphasized in this paper. By using spectrum sensing techniques, a secondary node can determine whether a given channel is appropriate for rendezvous. If incumbent signals are detected in the channel, then the node moves onto the next channel in the sequence. By maximizing the degree of overlapping, the likelihood of rendezvous failure due to incumbent signal appearance is minimized.

2) *Control Packet Broadcast*: Difficulty in supporting control message broadcast is one of the intrinsic drawbacks of all parallel rendezvous protocols, including CH protocols [2], [25]. In other words, CH protocols do not natively support broadcast. As suggested in [2], one method for supporting broadcast in CH protocols is to enable the nodes to employ retransmission at the cost of additional overhead: a sender retransmits broadcast messages a number of times so that the messages reach all of the neighboring nodes. Since any two nodes' CH sequences are able to rendezvous in at least one channel within a CH period, only a few retransmissions are needed for the broadcast messages to reach all of the nodes [2].

One approach to alleviating the control overhead of retransmission-based broadcast is to augment a CH protocol with other control message exchange techniques that more readily support broadcast such as the cluster-based control channel techniques [4], [17] or the control channel hopping techniques [6]. For instance, a channel hopping protocol can be employed to establish initial pair-wise rendezvous channels between two neighboring nodes. Then the initial rendezvous channels can be used to exchange control information needed to establish a local/cluster control channel.

3) *Data Exchange after Rendezvous*: The proposed CH scheme facilitates the exchange of control information via parallel rendezvous that enable multiple pairs of nodes to establish links simultaneously on distinct channels. However, the CH scheme does not dictate how the nodes coordinate the exchange of data information once the control information has been exchanged (via rendezvous). There are a number of known techniques for coordinating the exchange of data packets in a multi-channel environment. In a scheme known as (*temporary*) *common hopping* [2], [25], a transmitting node alters its hopping sequence so that it matches that of the receiving node while it is transmitting data packets, and then returns back to its original sequence once the transmission has finished.

4) *Hidden Terminal Problem*: Like most multi-channel MAC protocols [2], [24], [25], the proposed CH systems adopt the technique of exchanging RTS/CTS packets to avoid the *hidden terminal* problem. When multiple nodes hop onto a common channel at the same time, each node has to send an RTS to reserve the channel in the current slot before transmitting a control or data packet.

## IX. PERFORMANCE EVALUATION

We simulate the proposed CH schemes in ns-2 (version 2.31) [27] and use three MAC-layer reference protocols for comparison: IEEE 802.11b, SSCH, and the SeqR protocol.

The data rate is 11 Mbps by default. Note that IEEE 802.11b and SSCH were not designed for use in CR networks. However, they serve as good benchmarks in evaluating the performance of QCH. Furthermore, the design criteria of CH schemes for conventional multi-channel networks are almost identical to those of CH schemes for CR networks, and SSCH is one of the most well-known schemes of the former type. In the simulations, secondary nodes can opportunistically access  $N = 3$  channels. The channel switching delay is chosen as  $80 \mu\text{s}$ , which is well supported by existing technology [9]. The duration of a time slot is 200 ms unless otherwise specified. Every node uses Ad hoc On-Demand Distance Vector Routing (AODV) [22] as the routing protocol. At the transport layer, UDP is used in the simulations by default. The traffic generator uses Constant Bit Rate (CBR) flows with a flow rate of 11 Mbps and a packet size of 512 bytes. The transmission range of every node is 250 m. We simulated two networks: a static network with ten single-hop flows in a  $100 \text{ m} \times 100 \text{ m}$  square area and a random network with five multi-hop flows in a  $1000 \text{ m} \times 1000 \text{ m}$  square area. In the simulations, we study the time-to-*rendezvous* (TTR) value between two nodes and the throughput in each CH protocol under varying conditions, including random incumbent traffic, clock skew, node mobility, and multi-hop flow networks.

The following protocols were simulated.

- SSCH: Each node randomly chooses one (channel, seed)-pair to construct its CH sequence. For example, if a node selects the pair (0, 1), then its CH sequence has a period of  $(N + 1)$  slots, and the channel indexes in its sequence are  $\{0, 1, 2, 1\}$ . The last slot of a period is the parity slot, and the channel index of this slot is equal to the value of the node's seed.
- SeqR: This is the protocol proposed in [8]; it was briefly described in Section VIII-A.
- QCH: We simulate two synchronous QCH systems (M-QCH, and L-QCH) and an asynchronous one (A-QCH).
- A-MOCH: This is an asynchronous CH system that is not based on any quorum system.

We assume that every node randomly picks one sequence from a QCH system and performs channel hopping in accordance with the sequence. Once the sending-receiving node pair rendezvous on a channel, the pair performs common hopping to exchange data packets. The sender follows the receiver's sequence.

**Incumbent traffic generation.** In the simulations, we generated incumbent traffic as follows. In every time slot, the incumbent transmitter decides whether to transmit or not by flipping a coin. If the incumbent transmitter decides to transmit, it randomly selects a number of transmission channels and transmits packets in the current time slot. All of the secondary nodes are within the transmission range of the incumbent transmitter. A single incumbent transmitter was simulated. A channel is tagged as "unavailable" while incumbent traffic is present on it. All secondary nodes should refrain from transmitting on unavailable channels during the period of incumbent transmission. Note that all nodes that perform channel hopping are secondary nodes.

### A. Time-synchronous Networks

In the first set of simulations, we assume that the clocks of all nodes are synchronized—i.e., the boundaries of channel hopping periods and timeslots are aligned.

1) *Impact of MTTR:* We first simulated a single-hop flow to show the effect of TTR on channel access delay and the effect of channel switching overhead on throughput. The results are shown in Figure 5. We can see that the starting times of the traffic delivery for the simulated protocols are different, which coincides with the discrepancy of the protocols' channel access delays due to the variation in TTR values. Note that the throughput of each CH protocol is lower than that of 802.11b, which we can attribute to the channel switching overhead.

Next, we simulated a network with ten single-hop *disjoint* flows in a  $100 \text{ m} \times 100 \text{ m}$  square area. Two flows are considered disjoint if they do not share either endpoint. The average TTR for three CH schemes (when there is no incumbent traffic) is shown using the leftmost group of bars in Figure 6. As can be seen, M-QCH has the lowest average TTR compared to SSCH and SeqR—this is expected since M-QCH has the lowest MTTR value among the three CH protocols.

2) *Impact of Degree of Overlapping:* As expected, our simulation results indicate that a CH scheme's degree of overlapping has a clear impact on its TTR value when the incumbent transmitter is active. The average TTR for three CH schemes in the presence of incumbent traffic is shown using the center and rightmost groups of bars in Figure 6. M-QCH has a clear advantage over SSCH and SeqR in terms of TTR, because M-QCH's degree of overlapping is greater than that of the other two schemes in this simulation. This advantage becomes more evident in the presence of incumbent traffic since a pair of nodes using M-QCH can rendezvous on other channels if the current rendezvous channel is occupied by incumbent signals. In contrast, a pair of nodes using either SSCH or SeqR can rendezvous only on one channel (here, we are referring to the initial rendezvous). This implies that the nodes may not be able to achieve the initial rendezvous until the incumbent vacates the rendezvous channel. Note that in SSCH, initial rendezvous is needed to exchange data, such as each other's sequence, that is required to rendezvous in multiple channels.

Next, we set up ten *non-disjoint* flows in a  $100 \text{ m} \times 100 \text{ m}$  square area, where every node serves as both a transmitter and a receiver in multiple flows. In other words, this scenario includes multiple simultaneous flows with common endpoints. We assume temporary common hopping, i.e., each transmitter node has to change its hopping sequence and follow the receiver's sequence after a rendezvous has occurred to bootstrap communications. If the receiver node also acts as a transmitter in another flow, it must also follow the sequence of its intended receiver after a rendezvous. Thus, some nodes in this network have to switch between CH sequences continuously, and this scenario puts stress on a CH protocol's ability to establish links. Temporary common hopping prescribes that a transmitter should return back to its original sequence once the transmission has finished—this avoids the global synchronization of the CH sequences over the entire network (i.e., a scenario in which every node uses the same sequence). We compare the per-flow throughput of M-QCH, SSCH, and

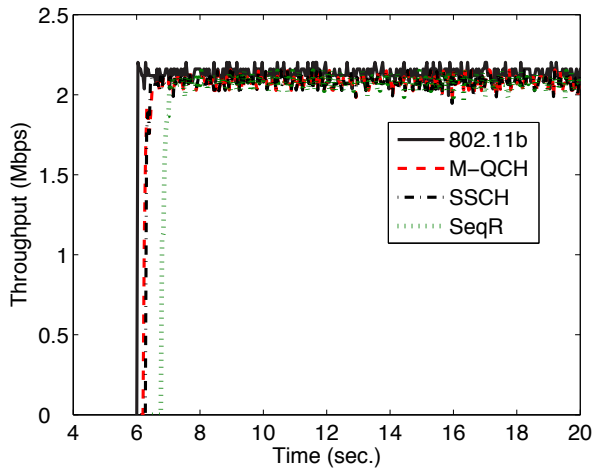


Fig. 5. Effect of time-to-rendezvous.

SeqR in Figure 7. When an incumbent signal is detected, M-QCH replaces the incumbent-occupied channel in its sequence with any incumbent-free channel. The results in the figure show that M-QCH outperforms the other two protocols, since M-QCH is faster in re-establishing links—this, of course, is due to the fact that M-QCH enables rendezvous in a greater number of distinct channels per sequence period.

3) *Impact of Node Mobility*: In this simulation experiment, we investigated the impact of node mobility on the performance of a QCH scheme. Specifically, we studied M-QCH's performance in a multi-flow random network. A random network was set up by placing 500 nodes randomly in a  $1000\text{ m} \times 1000\text{ m}$  square area. We simulated two cases: when the nodes are static and when they are mobile. In the case of mobile nodes, each node's movement follows the random way point mobility model: a node's maximum speed is 10 m/s, minimum speed is 5 m/s, and a node's maximum pausing time is 10 s. We randomly chose ten nodes from the network and set up five UDP flows among them. Obviously, maintaining the established links between node pairs in a multi-hop flow is more challenging compared to maintaining them in a single-hop flow. In our simulations, we adopt a simple flow-based channel assignment strategy [28]: nodes in the same flow simply follow the same CH sequence as the source node of the flow. Ten independent simulation runs were conducted for each result. The results are shown in Figure 8. As expected, the figure shows that the throughput gap between M-QCH and 802.11b decreases when nodes are mobile. M-QCH's throughput loss can be attributed to: (1) the overhead of re-establishing links and routes when a node moves out of communication range of its neighbors in the same flow, and (2) the control overhead incurred when a node in non-disjoint flows changes its CH sequence to synchronize with one of its neighbor's sequence for data transmission.

4) *Impact of the Load*: In this set of simulation experiments, we investigated the effect of load (as defined in Section III-C2) on network performance. In general, the load of a CH system determines the number of concurrent co-channel transmissions in each time slot. If the load is low, the number of concurrent co-channel transmissions in each slot is small, which means that the traffic is more evenly distributed

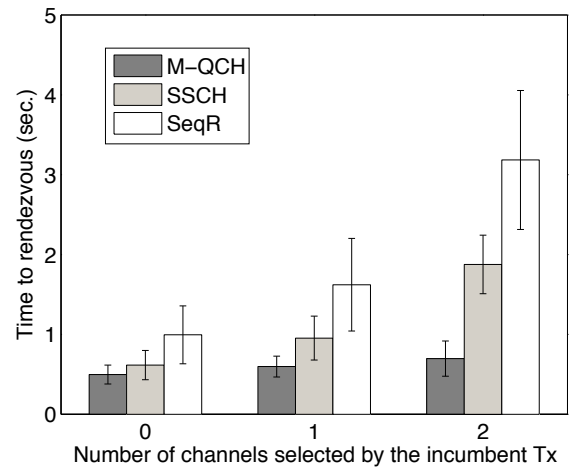


Fig. 6. Time-to-rendezvous with incumbent traffic.

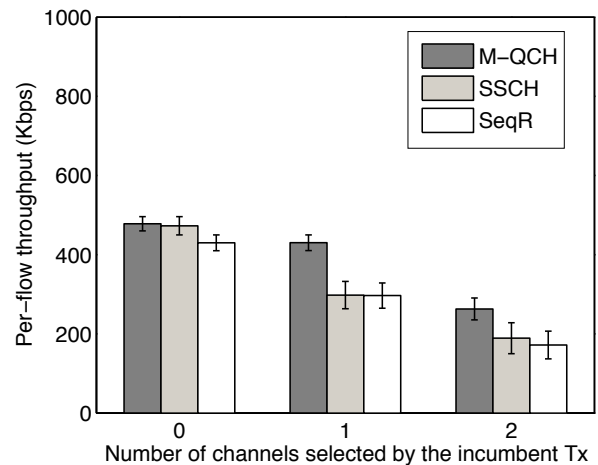


Fig. 7. Throughput of non-disjoint flows with incumbent traffic.

among different channels in every time slot. In general, a more even distribution of traffic among channels implies higher network throughput. In the simulations, we varied the number of disjoint flows in a  $100\text{ m} \times 100\text{ m}$  square area, and the results are shown in Figure 9. In the figures, we can see that L-QCH and SeqR outperform the other schemes since the two schemes have the lowest load compared to the other schemes. It is interesting to note that M-QCH's performance is inferior to that of the other schemes when the number of flows is small (because M-QCH has the highest load); however, when the number of flows is large, the system throughput for SSCH is lower than that of the other schemes. This phenomenon can be attributed to the limiting effect of the parity slot prescribed by SSCH when the network is close to saturation: nodes using SSCH can only utilize  $(N - 1)$  channels specified by  $(N - 1)$  seed values in the parity slot. In contrast, nodes using the other CH protocols can fully utilize all  $N$  channels in any time slot. From Figure 9, we can conclude that CH schemes that have lower load values are generally advantageous in terms of being able to support higher throughput; when the network is nearly saturated, the system throughput is closely related to the number of channels that can be fully utilized by each CH scheme.

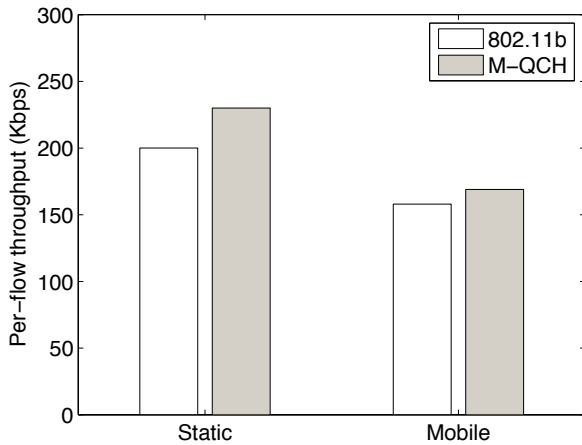


Fig. 8. Effect of node mobility.

### B. Time-asynchronous Networks

Here, we simulate time-asynchronous networks where clock skew is introduced between the sender's clock and the receiver's clock.

1) *Impact of Clock Skew*: In this simulation experiment, we investigated the effect of clock skew (i.e., misalignment of the slot boundaries) on the TTR of asynchronous CH schemes. Synchronous CH schemes can only tolerate a minimal amount of clock skew. If the clock skew is non-trivially large, synchronous CH schemes cannot guarantee rendezvous between any pair of sequences. In contrast, asynchronous CH schemes can make such a guarantee irrelevant of the amount of clock skew. We simulated a network with ten single-hop disjoint flows with two asynchronous CH systems—A-QCH and SeqR—in a 100 m×100 m square area. The sequence period of A-QCH was set to 9. We introduced a clock skew of up to one time slot length between each sender and receiver pair such that the sender is always ahead of the receiver by the clock skew amount. The average TTR for both schemes is shown in Figure 10. Both A-QCH and SeqR have a bounded TTR regardless of the amount of clock drift, because the set of sequences generated by both schemes satisfy the rotation closure property. A-QCH's average TTR is slightly lower than that of SeqR. This can be explained by the fact that the simulated A-QCH system's MTTR value is 9 whereas the SeqR's MTTR value is 12.

2) *Rendezvous Performance in Opportunistic Spectrum Access*: To observe the average TTR in time-asynchronous CR networks, we performed a set of simulation experiments comparing A-QCH and A-MOCH. We simulated a secondary ad hoc network with ten disjoint single-hop flows in the presence of incumbent traffic. A randomly selected clock skew was inserted between the sender's clock and the receiver's clock. The secondary nodes opportunistically access  $N = 11$  channels. Every node constructs its CH sequence independently, and uses perfect fast sensing at the beginning of every timeslot to determine whether the current channel can be used for rendezvous.

**Timeslot duration.** The length of an IEEE 802.22 time

frame is 10 ms and the timeslot length in SSCH [2] is 10 ms<sup>5</sup>. Hence, in the simulations, we set the timeslot length to 10 ms.

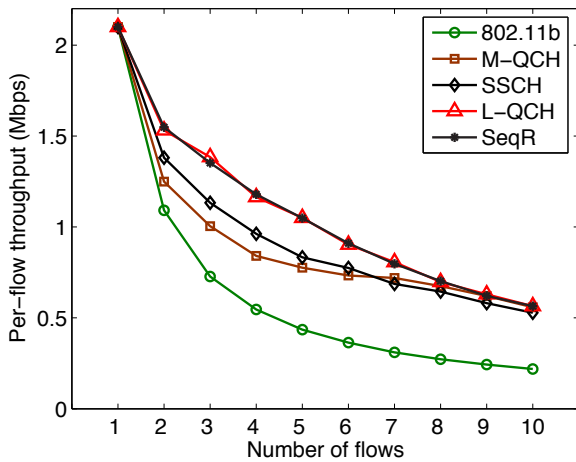
**Incumbent traffic pattern.** In most existing work on incumbent traffic modeling, it is assumed that an incumbent transmitter follows a “busy/idle” or “on/off” traffic pattern on a licensed channel [10], [12], [13]. The idle period ( $L$ ) and the busy period ( $B$ ) are treated as two independent random variables, defined by the distributions  $f_L(\cdot)$  and  $f_B(\cdot)$  with means  $\bar{l}$  and  $\bar{b}$ , respectively. The busy period is often assumed to follow an exponential distribution or have a fixed length which varies according to the type of wireless application being modeled. Some recent work assumed that the idle period follows an exponential distribution [10], [12]. However, actual measurement studies indicate that this may not be a realistic assumption for all types of incumbent traffic [11]. In [13], a number of distributions are used to model the incumbent idle period and their impacts on spectrum access strategy are compared. The proposition of modeling the incumbent traffic idle period remains a challenging open problem. In our simulations, we simulated  $X$  incumbent transmitters operating on  $X$  channels that are randomly chosen in each simulation run. Two types of “busy/idle” patterns were simulated for the incumbent transmitters: (1) the busy period follows an exponential distribution with a mean of  $\bar{b} = 2N$  timeslots and (2) the busy period has a fixed length of  $2N$  timeslots. The idle period follows an exponential distribution with a mean of  $\bar{l} = N$  timeslots. The simulation result is illustrated in Figure 11.

As expected, when the incumbent traffic is absent, A-QCH's TTR is less than that of A-MOCH, because the MTTR of A-MOCH is greater than that of A-QCH. However, as the number of incumbent transmitters is increased, the TTR value of A-QCH overtakes that of A-MOCH. This phenomenon can be explained by the fact that the degree of overlapping of A-QCH is only two. With such a small degree of overlapping, a communicating node pair needs to hop for a longer period of time before being able to find an incumbent-free rendezvous channel when incumbent signals are present in some of the channels. Moreover, the time needed to achieve the rendezvous is *unbounded*. In contrast, A-MOCH enables a communicating node pair to achieve the rendezvous within a bounded TTR when at least one channel is free of incumbent signals. These observations are valid for both the exponentially-distributed incumbent busy period and the fixed incumbent busy period. We can conclude that a CH system's degree of overlapping and MCTTR significantly affects the *expected* TTR of a communicating node pair.

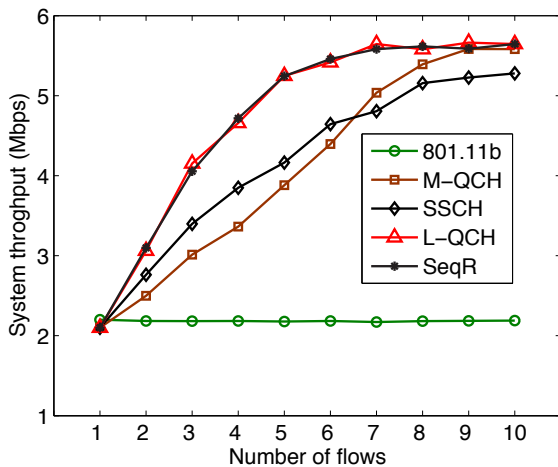
## X. RELATED RESEARCH

CR related research has received great attention recently. A major thrust in this research area is the development of spectrum sensing techniques capable of accurately detecting the existence of incumbent users or spectrum opportunities [1]. Once secondary users learn the available spectrum using spectrum sensing, they need to coordinate with each other to allocate the spectrum resources and dynamically change

<sup>5</sup>In IEEE 802.22, a super frame's length is 160 ms, and it contains 16 time frames [7].



(a) Per-flow throughput;



(b) System throughput.

Fig. 9. Effect of load on throughput of disjoint flows.

the allocation when primary users reclaim any spectrum. A MAC protocol for a CR network—a.k.a. a cognitive MAC protocol—needs to make provisions to support spectrum sharing. Existing cognitive MAC protocols can be divided into two categories.

The first category takes a centralized approach. That is, a centralized entity in a CR network controls the spectrum allocation and access rules for the network. The centralized entity can be physically centralized, such as a base station in an 802.22 network [7]. The advantage of this approach is its design simplicity and ease in achieving optimal spectrum access efficiency or fairness. However, for certain applications, the centralized approach may not be appropriate.

In the distributed approach, secondary users build up peer-to-peer ad hoc communications with each other based on CR. Most cognitive MAC protocols are derived from conventional multi-channel MAC protocols. Here, the term “conventional” MAC is used to describe a MAC designed for non-CR networks. See [19] for a comprehensive survey on conventional multi-channel MAC protocols. The schemes proposed in [4], [14], [20], [29] are all derived from conventional multichannel MAC protocols that rely on some form of a *dedicated* control

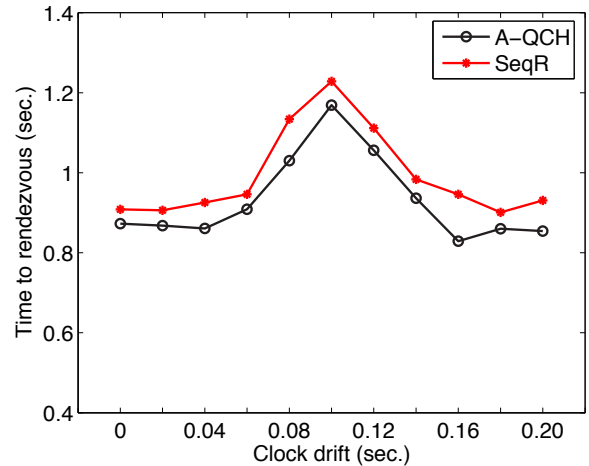


Fig. 10. Time-to-rendezvous vs. clock skew.

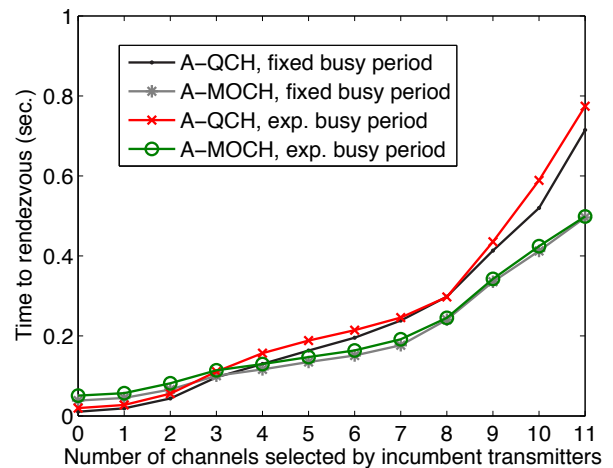


Fig. 11. Time-to-rendezvous vs. number of channels selected by incumbent transmitters.

channel. The HC-MAC [14] protocol considers the hardware limitations of a secondary user with a single transceiver. It focuses on the problem of how to optimize the proportion of time that each node allocates between spectrum sensing and spectrum access. The HD-MAC protocol [29] utilizes distributed coordination to elect a control channel for each group of secondary users that are in the same vicinity. Therefore, HD-MAC does not rely on a *global* common control channel. A similar idea is proposed in [4] for cognitive radio-based mesh networks. In [6], the scheme proposed features a dynamic control channel that can switch among channels depending on spectrum availability.

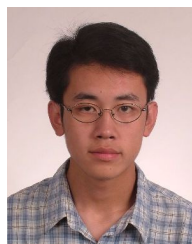
## XI. CONCLUSION

We presented a systematic approach, based on *quorum systems*, for designing and analyzing channel hopping (CH) protocols that enable control channel establishment in CR networks. A noteworthy feature of the proposed *Quorum-based Channel Hopping* (QCH) system is that it can establish control channels in multiple frequency channels so that the secondary network is less vulnerable to the unpredictable appearance of incumbent signals. We proposed two synchronous optimal designs of the QCH system: the first optimal design minimizes

the MTTR of the CH system and the second optimal design guarantees the even distribution of the rendezvous points in terms of both time and frequency. Minimizing the MTTR ensures short expected TTR which decreases channel access delay. An even distribution of rendezvous points alleviates the rendezvous convergence problem and increases the network capacity. We have also proposed two asynchronous CH systems. Both CH systems guarantee multiple rendezvous channels without requiring global clock synchronization.

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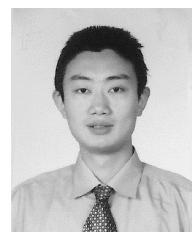
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