Efficient Routing Algorithms for Multi-Channel Dynamic Spectrum Access Networks

Ranjan Pal
Department of Computer Science
University of California, Davis
Email: rnpal@ucdavis.edu

***Abstract*— This paper addresses the problem of spectrum-aware data-adaptive routing in multi-channel, single-radio (MC-SR) multi-hop dynamic spectrum access (DSA) networks. We develop a scalable and simple to implement data-adaptive routing scheme that routes varied amounts of data from a given source to a destination through fast routes. For a given amount of data, our routing scheme takes into account the capacity (per unit of time) of the links, the available spectrum, the link disruption probabilities as well as the link propagation time between nodes. We model our routing problem as a combinatorial optimization task with two objectives. Our solution methodology generates *Pareto* optimal paths in polynomial time as there do not exist paths in general which optimize both objectives simultaneously. We generalize our routing method for broadcasting and multicasting scenarios. To the best of our knowledge this work is the first analytical treatment of the problem of spectrum-aware routing in DSA networks.**

I. Introduction

Spectrum is amongst the most heavily regulated and expensive natural resource around the world. Although almost all the spectrum suitable for wireless communications has been allocated, preliminary studies and observations indicate that many portions of the radio spectrum are not used for a significant amount of time. An experiment conducted by the Shared Spectrum Company shows 62 percent of the white space below the 3 GHz band even in the most crowded places as Washington D.C which is both government and commerce intensive. The Federal Communications Commission (FCC) spectrum policy task force report concludes that in many bands, spectrum access instead of spectrum scarcity limits the potential growth of wireless services [3]. In such a case opportunistic or dynamic spectrum access can resolve the problem of spectrum scarcity.

A fundamental difference between DSA networks and traditional wireless networks is that in case of the former there is no statically allocated fixed spectrum for use. Therefore a secondary user in a DSA network using a particular frequency to communicate with its neighbour might have to give way to a primary user when it requires service on that channel. Due to this fundamental difference, data communication in DSA networks is always a challenge. In this paper we utilize spectrum opportunistically [9] to service demands of various secondary/unlicensed users in the network in the presence of primary users.

A. Our Contributions

- We propose a polynomial(in the number of vertices) time spectrum-aware, data-adaptive routing scheme in capacitated multi-channel, multi-hop dynamic spectrum access networks. Our scheme selects the Pareto fastest paths from a source to a destination by accounting the quantity of data to be transmitted, the channel capacities, spectrum availability, the link propagation time and the secondary user link occupancy probabilities as factors.
- We provide a fast, spectrum-aware data-adaptive data dissemination algorithm that addresses issues of broadcasting and multicasting in single-radio multi-hop DSA networks. Our algorithm runs in polynomial time and finds use in real-time and online applications such as internet chatting and gaming, video broadcasting and video conferencing. Our scheme accounts for general directed and weighted graphs and is an extension of the polynomial time algorithm stated above.

The rest of the paper is organized as follows. In section 2, we review some related work in this area. Section 3 describes the notations and basic network model. We formulate our problems in Section 4. In section 5 we propose and analyze our routing algorithms. The paper concludes in Section 6.

II. Related Work

Routing constitutes an important and an unexplored area of research in DSA networks [5]. We emphasize the need for routing algorithms in open spectrum management. In [2], a layered graph model is proposed for modeling network topology and routing in interference-based DSA networks. This model provides solutions for DSA networks with static link properties. Our routing strategy is more realistic in the sense that it considers the time-varying nature of the links as well as the intermittent connectivity in the network. Zheng et.al [6] propose decoupled and joint route selection and spectrum management methodologies. The route selection in the decoupled case is performed by using the shortest path algorithm. However, authors in [10] show that the shortest path algorithm may not yield optimal solutions when both the link propagation time and the channel capacities are taken into account. None of the above schemes target data-sensitive routing nor address data dissemination in DSA networks. In addition, our work provides an analytical treatment of routing which is missing in the cited works.

III. Notations and System Model

A. Notations

The various notations used in the paper are as follows.

n	The number of nodes in the network
V	$\{v_i \mid 1 \leq i \leq n\}$: the set of nodes in the network
N	The number of links in the network
E	$\{e_i \mid 1 \leq i \leq N\}$: the set of links in the network
Ψ	The number of orthogonal channels in the network
K	$\{k_i \mid 1 \leq i \leq \Psi\}$: the set of orthogonal channels
$t(e_i)$	Transmitting node of link e_i; $t(e_i)\,\epsilon\, V$ $\{e_i \mid 1 \leq i \leq N\}$
$r(e_i)$	Receiving node of link e_i; $r(e_i)\,\epsilon\, V$ $\{e_i \mid 1 \leq i \leq N\}$
p_k^i	The probability that channel k on link i is free for use by secondary user $t(e_i)$ at any particular time slot. $k\,\epsilon\, K$, $i\,\epsilon\, E$
p_i	Probability that link e_i is free for use by secondary user $t(e_i)$ at any particular time slot

$c_{pr}(e_i)$ Expected capacity of link e_i
T The set of all spanning trees of the graph $G = (V, E)$
v link propagation speed of the network
st A spanning tree of the network $G = (V, E)$; $st \epsilon T$
γ The amount of data to be shipped from the source to the sink
P The set of all possible paths from the source to the destination
γ_p γ amounts of data through path $p \epsilon P$
C $(C_1, C_2, \ldots\ldots, C_N)$: the maximal system capacity vector. C_i(a real number) denotes the maximal capacity of link e_i for each i = 1, 2,....,N
D $(D_1, D_2, \ldots\ldots, D_N)$: the link distance vector. D_i(a real number) denotes the distance of link e_i for each i = 1, 2,....,N
γ_{st} γ amounts of data through $st \epsilon T$
$d_{i,j}$ Distance of edge with endpoints i, $j| \, i, j \epsilon V$ and $d_{i,j} \epsilon \mathbf{D}$
$c_{i,j}$ Expected capacity of link with endpoints i, $j| \, i, j \epsilon V$ and $c_{i,j} \epsilon \mathbf{C}$
t_d Total average delay to ship data from source to sink
pr_d Propagation delay to ship data from source to sink
c_p Expected capacity of a path from source to destination
t_{st} Total average delay to broadcast data via spanning tree $st \epsilon T$
pr_{st} Propagation delay to broadcast data via spanning tree $st \epsilon T$
c_{st} Expected capacity at which broadcasting occurs in st where $st \epsilon T$
E_{st} The set of edges in the spanning tree st; $st \epsilon T$

B. System Model

We consider a fixed multi-hop wireless network with n nodes. The network is represented by a directed graph $G = (V, E)$ where V represents the set of nodes and E is the set of data links. The data links are unidirectional. Each node can communicate with a subset of other nodes in the network via these wireless links. We assume that each node is perfectly reliable. If a node u is able to transmit directly to node v in the network, we represent this fact by a directed edge $u \to v$ from node u to node v. Some specific model characteristics and assumptions are stated next.

1) Specific Model Characteristics and Assumptions:

- Each node is perfectly reliable.
- The nodes are equipped with only one radio and are sufficiently powered.
- Each radio has access to all the channels in the network. A node can transmit on any channel at a particular time instant.
- We assume that the system takes care of interference and so we do not model interference of flows amongst secondary users.
- We assume that the system adopts *Dynamic Link Channel Assignment*. Under this channel assignment scheme, the channel on which communication takes place between neighbors is decided at the beginning of each time slot.
- The maximal capacity of each channel on a link e_i is C_i
- The capacity (per unit time) of a link e_i as seen by $t(e_i)$ at any time instant is an element of the set $\{0, C_i\}$. A capacity of 0 indicates that primary users or secondary users other than $t(e_i)$ are active on all channels on that link. Therefore $t(e_i)$ has to wait until a channel becomes free. p_i is the probability that it finds a channel free for transmission on link e_i.
- On a particular link e_i, at any time slot, each channel has a 0-1 (inactive-active) probability distribution. Inactive status implies that a primary user or a secondary user other than $t(e_i)$ is occupying the channel. We assume that the distribution is known through some estimation techniques.
- The 0-1 probability distribution for each channel on a link is assumed to be the same. The distribution may be different for different links in the Dynamic Spectrum Access network.
- The capacities of various channels on different data links in the DSA network are statistically independent.
- The flow in the network satisfies the ***Ford-Fulkerson*** flow-conservation law [4] [1]. According to this law each, each unit of flow is transmitted through one and only one minimal path and no flow is created or destroyed during transmission via such a path.
- For a given amount of data, the whole of it is routed from a source to a destination through one single path.

IV. Problem Formulation

As stated in our model assumptions, a node in the network has access to all the available channels. At the beginning of every time slot it decides the channel(if any) through which it wants to communicate with its neighbour [*Dynamic Channel Assignment*]. This information is conveyed via means of a control channel. If it does not want to be involved in any activity (transmit/receive) during a time slot it remains quiet. A node can either have access to any non-busy channel $i \epsilon K$, on the link under consideration or might have to wait until a channel on the link becomes free. The **admittance probability** $(p_i)[Definition 1]$ for link i is given as

$$p_i = 1 - \prod_{k=1}^{\Psi}(1 - p_k^i)$$

where the values of p_k^i are assumed to be known via some estimation techniques. The expected/mean capacity of each link is given by $p_i C_i$. As a preprocessing step we calculate the admittance probabilities and the expected capacities for all the links in the network.

Definition 1: *The admittance probability for a link $i \epsilon E$ in a Dynamic Spectrum Access network is the probability that a channel from the set of available channels is free for communication by a secondary user $t(e_i)$ at any particular time slot.*

A. Case 1 : Single Source Single Destination

Let (s, d) be a (source, destination) pair where $s, d \epsilon V$.
In order to send γ units of data between s and d through path $p : (s, u_1, \ldots\ldots, u_k, d)$ where $u_i \epsilon V$ for $1 \leq i \leq k$ and $p \epsilon P$, the total average delay $t_d(\gamma_p)$ is given by

$$t_d(\gamma_p) = \text{the propagation delay} + \frac{\gamma}{capacity\, of\, the\, path\, p}$$

The propagation delay is

$$pr_d(\gamma_p) = (d_{s,u_1} + \Sigma_{i=1}^{k} d_{u_i,u_{i+1}} + d_{u_k,d})/v$$

The mean capacity of the path is

$$c_p(\gamma_p) = min\, \{c_{s,u_1},\, min_{1 \leq i \leq k-1}\, (c_{u_i,u_{i+1}}),\, c_{u_k,d}\}$$

The objective function for a given $\gamma \epsilon \Re^+$ is formulated as

$$\textit{Minimize}_{p \epsilon P}\{pr_d(\gamma_p) + \gamma/c_p(\gamma_p)\}$$

It is evident that the optimal solution to this optimization problem is one, which minimizes $pr_d(\gamma_p)$ and minimizes $c_p(\gamma_p)$ simultaneously. Therefore we can reformulate our objective function as follows.

$$\textit{Minimize}_{p \epsilon P}\{pr_d(\gamma_p)\} \qquad (P_1)$$

and

$$\textit{Minimize}_{p \epsilon P}\{\gamma/c_p(\gamma_p)\} \qquad (P_2)$$

We are required to find paths which satisfy both P_1 and P_2.

B. Case 2 : Data-Adaptive Broadcasting and Multicasting

1) Broadcasting: We use a spanning tree to broadcast data in a network. In order to broadcast γ units of data through a spanning tree $st \epsilon T$ the total average delay is given by

$$t_d(\gamma_{st}) = \text{the propagation delay + } \frac{\gamma}{capacity\, of\, the\, tree\, st}$$

The propagation delay is

$$pr_{st}(\gamma_{st}) = (\Sigma_{u_i,u_j \epsilon V^b | (u_i,u_j) \epsilon E_{st}} (d_{u_i,u_j}))/v$$

V^b is the set of vertices in the longest path from the root to any leaf node for the spanning tree st. In case the longest path is not unique, anyone can be chosen. The idea behind choosing the longest path is that broadcasting involves parallel transmission of information from a node at one level to all its neighbors in the next. In that case, the propagation time for broadcasting is the time taken to transmit information via the longest path from the root to a leaf node.

The mean capacity for broadcasting is

$$c_{st}(\gamma_{st}) = min_{u_i,u_j \epsilon V | (u_i,u_j) \epsilon E_{st}} (c_{u_i,u_j})$$

The objective function for a given $\gamma \epsilon \Re^+$ is formulated as

$$Minimize_{st \epsilon T}\{pr_{st}(\gamma_{st}) + \gamma/c_{st}(\gamma_{st})\}$$

The optimal solution to this optimization problem is one that minimizes $pr_{st}(\gamma_{st})$ and minimizes $c_{st}(\gamma_{st})$ simultaneously. Therefore we reformulate our objective function as follows.

$$Minimize_{st \epsilon T}\{pr_{st}(\gamma_{st})\} \qquad (P_3)$$

and

$$Minimize_{st \epsilon T}\{\gamma/c_{st}(\gamma_{st})\} \qquad (P_4)$$

Our aim is to find spanning trees which satisfy both P_3 and P_4.

2) Multicasting: The multicasting problem is formulated in a similar manner as the broadcasting case. We use a multicast tree to disseminate data. The formation of a multicast tree takes place in the following manner. The nodes which do not form a part of the multicasting set are removed from the network. All the edges coming into and going out of these nodes are then deleted to form a modified network G'. The spanning trees of the modified network are the multicast trees of the original network. The optimization problem can be formulated in a similar manner as the broadcasting case.

V. Methodology

A. Single-Source Single-Destination Routing

The routing problem for single source single destination routing is formulated as a bi-objective optimization task [section III]. The first objective is the shortest path problem whereas the second one is the maximum expected capacity path problem. In general, no path exists which is an optimal solution for both the above objectives.

Idea behind the algorithm : For a given γ we intend to find a set of paths which are dominant. A dominant path is one which is a privileged solution when dealing simultaneously with problems P_1 and P_2. No objective on a dominant solution can be improved without worsening the other one. The term "dominant" is a ***decision theoretic*** term used to describe a choice that is at least as good as the alternatives in all circumstances and better in some. There can be many dominant solutions to our bi-objective optimization problem. All these solutions are termed as ***Pareto*** optimal solutions. We now state some definitions and lemmas relevant to our algorithm.

Definition 1. *Let $p, q \epsilon P$. Then p is dominant over q (pDq) if and only if $pr_d(\gamma_p) \leq pr_d(\gamma_q)$ and $\gamma/c_p(\gamma_p) < \gamma/c_p(\gamma_q)$ or $pr_d(\gamma_p) < pr_d(\gamma_q)$ and $\gamma/c_p(\gamma_p) \leq \gamma/c_p(\gamma_q)$ holds for a given γ.*

Definition 2. *Let $P_{ND}(\gamma)$ be the set of non-dominant paths for a given γ. $P_{ND}(\gamma)$=$\{p \epsilon P \mid \exists q \epsilon P$ such that $qDp\}$. The set of dominant paths are denoted by $P_D(\gamma)$ where $P_D(\gamma) = P - P_{ND}(\gamma)$.*

Property 1. *For any dominated path there exists a dominant path with better values for both objective functions (or equal for one objective function and better for the other).*

Lemma 1. *Let $p^*_\gamma \epsilon P$ be an optimal path for a given $\gamma \epsilon \Re^+$. Then p^*_γ is a dominant path for P_1 and P_2.*

Proof. We use the "proof by contradiction" method to prove this theorem.
Let us assume that p^*_γ is a non-dominant solution. Then there exists a path $p_\gamma \epsilon P$ such that pDp^*_γ. Therefore by Property 1, $pr_d(\gamma_p) \leq pr_d(\gamma_{p^*_\gamma})$ and $\gamma/c_p(\gamma_p) < \gamma/c_p(\gamma_{p^*_\gamma})$ or $pr_d(\gamma_p) < pr_d(\gamma_{p^*_\gamma})$ and $\gamma/c_p(\gamma_p) \leq \gamma/c_p(\gamma_{p^*_\gamma})$. In this case p is a better solution than p^*_γ. This contradicts the assumption made that p^*_γ. is optimal. Therefore p^*_γ. is a dominant path. Q.E.D

Algorithm DOMPATH(γ)
begin
1. $P_D(\gamma) = \phi$
2. Find a $p^*(\gamma)$ for which $\gamma/c_p(\gamma_{p^*(\gamma)}) \leq \gamma/c_p(\gamma_{p(\gamma)})$ for any $p(\gamma) \epsilon\ argmin_{p(\gamma) \epsilon P}(pr_d(\gamma_{p^*(\gamma)})$; $p^*(\gamma) \epsilon\ argmin_{p(\gamma) \epsilon P}(pr_d(\gamma_{p^*(\gamma)}))$
3. If no $p^*(\gamma)$ exists **then**{**print** $P_D(\gamma)$; **exit**}
4. $P_D(\gamma) = P_D(\gamma) \bigcup p^*(\gamma)$
5. Remove from the $G = (V, E)$ all links (i, j) for which $\gamma/c_{i,j} > \gamma/c_p(\gamma_{p^*(\gamma)})$
6. goto Step 2
end

Theorem 1. *The running time of algorithm* DOMPATH(γ) *is O(EVlog(V) + E^2)*

Proof. In this algorithm, for each iteration of step 5 some links are removed from the original network. Therefore starting from the given network, a sequence of networks is generated. Step 2 of the algorithm computes a shortest path from the given source to the destination. Atmost E shortest paths can be computed. We use **Dijkstra's shortest path algorithm** to enumerate the shortest paths. Each execution of the Dijkstra's algorithm requires $O(V log V + E)$ time if we use a **Fibonacci heap** data structure for queuing operations [7]. In each iteration of step 5, atleast one arc is removed. Therefore, for atmost E iterations the running time of the algorithm is $O(EV log(V) + E^2)$. Q.E.D

B. Broadcasting and Multicasting

1) Broadcasting: In section 3 we have shown that optimal broadcast routing is achieved by solving the following bi-objective optimization problems $P3$ and $P4$. Problem P_3 aims to minimize the spanning time whereas P_4 is a maximum expected capacity spanning tree problem. There is no solution in general which optimizes both the objectives and so we find out Pareto optimal trees.

Idea behind the Algorithm : For given γ we want to find a set of spanning trees which are dominant. A dominant spanning tree is one which is a privileged solution when dealing simultaneously with problems P_3 and P_4. No objective on a dominant solution can be improved without worsening the other one. If we assume that the set of all spanning trees are not given in advance, this algorithm would first find all the spanning trees possible for the directed graph G. These trees would then be the input to the bi-objective problem. *We use Uno's algorithm to enumerate all spanning trees of a directed graph.* [8] The output of the algorithm would be the set of Pareto optimal spanning trees. We now state some definitions and lemmas relevant to our algorithm.

Definition 3. *Let $p, q \epsilon T$. Then p is dominant over q (pDq) if and only if $pr_d(\gamma_p) \leq pr_d(\gamma_q)$ and $\gamma/c_{st}(\gamma_p) < \gamma/c_{st}(\gamma_q)$ or $pr_d(\gamma_p) < pr_d(\gamma_q)$ and $\gamma/c_{st}(\gamma_p) \leq \gamma/c_{st}(\gamma_q)$ holds for a given γ.*

Definition 4. *Let $T_{ND}(\gamma)$ be the set of non-dominant spanning trees for a given γ. $T_{ND}(\gamma)$=$\{p \epsilon T \mid \exists q \epsilon T$ such that $qDp\}$. The set of dominant spanning trees are denoted by $T_D(\gamma)$ where $T_D(\gamma) = T - T_{ND}(\gamma)$.*

Property 2. *For any dominated spanning tree there exists a dominant spanning tree with better values for both objective functions (or equal for one objective function and better for the other).*

Lemma 2. *Let $p^*_\gamma \epsilon T$ be an optimal spanning tree for a given $\gamma \epsilon \Re^+$. Then p^*_γ is a dominant spanning tree for P_3 and P_4.*

The proof of the lemma is very similar to that of Lemma 1. The author can be contacted for further details.

Algorithm DOMTREE(γ)
begin
1. Enumerate all the spanning trees of G using Uno's algorithm
2. $T_D(\gamma) = \phi$
3. Find a $p^*(\gamma)$ for which $\gamma/c_{st}(\gamma_{p^*(\gamma)}) \leq \gamma/c_{st}(\gamma_{p(\gamma)})$ for any $p(\gamma) \epsilon \ argmin_{p(\gamma)\epsilon T}(pr_d(\gamma_{p^*(\gamma)}))$;
$p^*(\gamma) \epsilon \ argmin_{p(\gamma)\epsilon T}(pr_d(\gamma_{p^*(\gamma)}))$
4. If no $p^*(\gamma)$ exists **then**{**print** $T_D(\gamma)$; **exit**}
5. $T_D(\gamma) = P_T(\gamma) \bigcup p^*(\gamma)$
6. Remove from the $G = (V, E)$ all links (i, j) for which $\gamma/c_{i,j} > \gamma/c_{st}(\gamma_{p^*(\gamma)})$
7. goto Step 3
end

Theorem 2[Uno] : *The running time for enumerating all spanning trees in a directed graph $G = (V, E)$ is $O(E + \phi M(V, E))$ where ϕ denotes the total number of spanning trees of the graph and M denotes the time complexity of the data structure used to update the minimum spanning tree in an undirected graph with n nodes and N edges.*

Theorem 3 : *The running time for algorithm* DOMTREE(γ) *is $max\{(E + \phi M(V, E)), E^2 logE\}$.*

Proof. According to theorem 2, step 1 requires $O(E + \phi M(V, E))$ time. For each iteration of step 6 some links are removed from the original network. Therefore starting from the given network, a sequence of networks is generated. Step 3 of the algorithm computes a minimum spanning tree(MST) from the given network. Atmost E MST's can be computed. We use **Kruskal's minimum spanning tree algorithm** to enumerate the MST's. Each execution of the Kruskal's algorithm requires $O(ElogE)$ time. [7]. In each iteration of step 6, atleast one arc is removed. Therefore, for atmost E iterations the running time of the algorithm is $max\{(E + \phi M(V, E)), E^2 logE\}$ Q.E.D

2) Multicasting: The multicasting problem is very similar to the broadcasting case and evaluates to running the algorithm DOMTREE on the modified graph $G' = (V', E')$. Finding efficient routes for multicasting takes $O(\ max\{(E' + \phi M(V', E')), E'^2 logE'\})$ time.

VI. Conclusion and Future Work

In this paper we have proposed and analyzed an efficient polynomial-time spectrum-aware routing scheme in multi-channel, single-radio, multi-hop dynamic spectrum access networks which routes variable amounts of data from a source to the destination in a fast manner. Our algorithm is data-adaptive in the sense that it selects the best routes depending on the amount of data to be transported. We have also extended the above problem to account for broadcasting and multicasting scenarios in these networks. Due to the nature of our optimization problems, we generate Pareto optimal solutions instead of general optimal solutions. Our work provides the first analytical treatment of routing in DSA networks. As a part of future work we want to simulate the algorithms to evaluate the performance with respect to traditional and well known routing schemes. We also want to extend our work to incorporate different kinds of interference models in multi-channel environments.

VII. Acknowledgements

The author would like to thank professor Somprakash Bandyopadhay for his valuable time and suggestions.

References

[1] A.K.Ahuja, T.L Magnanti, and J.Orlin. *Network Flows*. Prentice Hall International.
[2] C.Xin. A novel-layered graph model for topology formation and routing in dynamic spectrum access networks. In *IEEE DySPAN*, 2005.
[3] FCC. *FCC spectrum policy task force report, ET Docket No. 02-135*. FCC.
[4] L.R. Ford and D.R Fulkerson. *Flows in Networks*. Princeton University Press.
[5] I.F.Akyildiz, W.Y.Lee, M.C.Vuran, and S.Mohanty. Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey. *Computer Networks*, 50:2127–2159, 2006.
[6] Q.Wang and H.Zheng. Route and spectrum selection in dynamic spectrum access networks. In *IEEE CNCC*, 2006.
[7] T.H.Cormen, C.L.Leiserson, R.Rivest, and C.Stein. *An Introduction to Algorithms*. MIT Press.
[8] T. Uno. An algorithm for enumerating all directed spanning trees in a directed graph. In *Lecture Notes in Computer Science*, 1996.
[9] X.Liu, E.Chong, and N.Shroff. Opportunistic transmission scheduling with resource-sharing constraints in wireless networks. *IEEE Journal on Selected Areas in Communications*, 19(10), 2001.
[10] Y.L.Chen and Y.H.Chin. The quickest path problem. *Computers and Operations Research*, 17:153–161, 1990.