CHAPTER 7. PART III. SPECTRUM SHARING (MORE GAME THEORY)

What is OLIGOPOLY?

* An oligopoly is a market form in which a market or industry is dominated by a small number of sellers (oligopolists).

(e.g., AT&T, Verizon, T-Mobile, Sprint control 89% of the cellular market in the USA).

* Lack of competition can lead to higher costs for consumers.

* Because of few sellers, each oligopolist is likely to be aware of the actions of the others.

* Decisions of one firm influence, and are influenced by, the decisions of other firms.

* Strategic planning by oligopolists needs to take into account the likely responses of the other market participants.

OLIGOPOLY

- Oligopolistic competition can give rise to a wide range of different outcomes.
- In some situations, the firms may employ restrictive trade practices (collusion, market sharing etc.) to raise prices and restrict production in much the same way as a monopoly.
- Where there is a formal agreement for such collusion, this is known as a cartel.

e.g., OPEC (international price of oil).

COLLUSION

- Firms often collude in an attempt to stabilize unstable markets, so as to reduce the risks inherent in these markets for investment and product development.
- There are legal restrictions on such collusion in most countries.
- There does not have to be a formal agreement for collusion to take place
- e.g., in some industries there may be an acknowledged market leader which informally sets prices to which other producers respond, known as price leadership.

COLLUSION

- In other situations, competition between sellers in an oligopoly can be fierce, with relatively low prices and high production.
- This could lead to an efficient outcome approaching perfect competition.
- The competition in an oligopoly can be greater when there are more firms in an industry than if, e.g., the firms were only regionally based and did not compete directly with each other.

WHAT IS COLLUSION ?

- An agreement between two or more parties, sometimes illegal and therefore secretive, to limit open competition by deceiving, misleading, or defrauding others of their legal rights, or to obtain an objective forbidden by law typically by defrauding or gaining an unfair advantage.
- It is an agreement among firms or individuals to divide a market, set prices, limit production or limit opportunities.
- It can involve "wage fixing, kickbacks, or misrepresenting the independence of the relationship between the colluding parties”.
- In legal terms, all acts affected by collusion are considered void.

COLLUSION IS ILLEGAL !!

- Collusion is largely illegal in the US, Canada and most of the EU due to competition/ antitrust law, but implicit collusion in the form of price leadership and tacit understandings still takes place.

- Examples in the USA:
  Market division and price-fixing among manufacturers of heavy electrical equipment in the 1960s, including General Electric.

- An attempt by Major League Baseball owners to restrict players' salaries in the mid-1980s.

- Price fixing within food manufacturers providing cafeteria food to schools and the military in 1993.

- Market division and output determination of livestock feed additive, called lysine, by companies in the US, Japan and South Korea in 1996.

COLLUSION - BARRIERS

- **Number of Firms:** As the number of firms in an industry increases, it is more difficult to successfully organize, collude and communicate.

- **Cost and Demand Differences between Firms:** If costs vary significantly between firms, it may be impossible to establish a price at which to fix output.

- **Cheating:** There is considerable incentive to cheat on collusion agreements; although lowering prices might trigger price wars, in the short term the defecting firm may gain considerably. This phenomenon is frequently referred to as "**chiseling**".

- **Potential Entry:** New firms may enter the industry, establishing a new baseline price and eliminating collusion (though anti-dumping laws and tariffs can prevent foreign companies entering the market).

- **Economic Recession:** An increase in average total cost or a decrease in revenue provides incentive to compete with rival firms in order to secure a larger market share and increased demand.

OLIGOPOLY and GAME THEORY

Oligopoly theory makes heavy use of game theory to model the behavior of oligopolies:

- Stackelberg's duopoly. In this model the firms move sequentially (see Stackelberg competition).
- Cournot's duopoly. In this model the firms simultaneously choose quantities (see Cournot competition).
- Bertrand's oligopoly. In this model the firms simultaneously choose prices (see Bertrand competition).

STACKELBERG MODEL (1934)

(Heinrich Freiherr von Stackelberg (1905-1946))

- Stackelberg leadership model is a strategic game in economics in which the leader firm moves first and then the follower firms move sequentially.
- In game theory terms, the players of this game are a *leader* and a *follower* and they compete on quantity.
- Stackelberg leader is sometimes referred to as the Market Leader.

STACKELBERG MODEL

* Leader must know *ex ante* that the follower observes his action.

* Follower must have no means of committing to a future non-Stackelberg follower action and the leader must know this.

* Indeed, if the 'follower' could commit to a Stackelberg leader action and the 'leader' knew this, the leader's best response would be to play a Stackelberg follower action.

STACKELBERG MODEL

* Firms may engage in Stackelberg competition if one has some sort of advantage enabling it to move first.

* More generally, the leader must have commitment power.

* Moving observably first is the most obvious means of commitment: once the leader has made its move, it cannot undo it - it is committed to that action.

* Moving first may be possible if the leader was the incumbent monopoly of the industry and the follower is a new entrant.

* Holding excess capacity is another means of commitment.

COURNOT MODEL

(Antoine Augustin Cournot (1801-1877))

- An economic model used to describe an industry structure in which companies compete on the amount of output they will produce, which they decide on independently of each other and at the same time.
- Inspired by observing competition in a spring water duopoly.

COURNOT MODEL

(Antoine Augustin Cournot (1801-1877))

It has the following features:

- There is more than one firm and all firms produce a homogeneous product, i.e., there is no product differentiation;
- Firms do not cooperate, i.e. there is no collusion;
- Firms have market power, i.e. each firm's output decision affects the good's price;
- The number of firms is fixed;
- Firms compete in quantities, and choose quantities simultaneously;
- The firms are economically rational and act strategically, usually seeking to maximize profit given their competitors' decisions.

COURNET MODEL

- An essential assumption of this model is the "not conjecture" that each firm aims to maximize profits, based on the expectation that its own output decision will not have an effect on the decisions of its rivals.
- Price is a commonly known decreasing function of total output.
- All firms know, the total number of firms in the market, and take the output of the others as given.
- Each firm has a cost function.

COURNET MODEL

- Normally the cost functions are treated as common knowledge.
- Cost functions may be the same or different among firms.
- Market price is set at a level such that demand equals the total quantity produced by all firms.
- Each firm takes the quantity set by its competitors as a given, evaluates its residual demand, and then behaves as a monopoly.

BERTRAND MODEL (1883)

(Joseph Louis François Bertrand (1822-1900))

- A model of competition used in economics.
- It describes interactions among firms (sellers) that set prices and their customers (buyers) that choose quantities at the prices set.
- Cournot argued that when firms choose quantities, the equilibrium outcome involves firms pricing above marginal cost and hence the competitive price.
- Bertrand argued that if firms chose prices rather than quantities, then the competitive outcome would occur with price equal to marginal cost.

BERTRAND MODEL

- Also, there can be no equilibrium with firms setting different prices.
- Firms setting the higher price will earn nothing (the lower priced firm serves all of the customers).
- Hence the higher priced firm will want to lower its price to undercut the lower-priced firm.

BERTRAND MODEL

- Hence the *only* equilibrium in the Bertrand model occurs when both firms set price equal to unit cost (the competitive price).
- Note that the Bertrand equilibrium is a *weak* Nash-equilibrium.
- The firms lose nothing by deviating from the competitive price:

  it is an equilibrium simply because each firm can earn no more than zero profits given that the other firm sets the competitive price and is willing to meet all demand at that price.

CRITICAL ANALYSIS OF BERTRAND MODEL

- Bertrand model rests on some very extreme assumptions.
- For example, it assumes that consumers want to buy from the lowest priced firm.
- There are various reasons why this may not hold in many markets: non-price competition and product differentiation, transport and search costs.

CRITICAL ANALYSIS OF BERTRAND MODEL

- For example, would someone travel twice as far to save 1% on the price of their vegetables?
- Bertrand model can be extended to include product or location differentiation but then the main result - that price is driven down to marginal cost - no longer holds.

BERTRAND vs COURNOT

- Although both models have similar assumptions, they have very different implications:
- Since the Bertrand model assumes that firms compete on price and not output quantity, it predicts that a duopoly is enough to push prices down to marginal cost level, meaning that a duopoly will result in perfect competition.
- Neither model is necessarily "better"
- Accuracy of the predictions of each model will vary from industry to industry, depending on the closeness of each model to the industry situation.

STACKELBERG vs COURNOT

- Both models are similar because in both competition is on quantity.
- However, as seen, the first move gives the leader in Stackelberg a crucial advantage.
- There is also the important assumption of perfect information in the Stackelberg game: the follower must observe the quantity chosen by the leader, otherwise the game reduces to Cournot.
- With imperfect information, the threats described above can be credible.
- If the follower cannot observe the leader's move, it is no longer irrational for the follower to choose, say, a Cournot level of quantity (in fact, that is the equilibrium action).

GAME THEORETIC CONSIDERATIONS

- If the leader played a Stackelberg action, (it believes) that the follower will play Cournot.
- Hence it is non-optimal for the leader to play Stackelberg.
- In fact, its best response (by the definition of Cournot equilibrium) is to play Cournot quantity.
- Once it has done this, the best response of the follower is to play Cournot.

GAME THEORETIC CONSIDERATIONS

- Consider the following strategy profiles:

  the leader plays Cournot; the follower plays Cournot if the leader plays Cournot and the follower plays non-Stackelberg if the leader plays Stackelberg and if the leader plays something else,

  the follower plays an arbitrary strategy.

- This profile is a Nash equilibrium.

- As argued above, on the equilibrium path play is a best response to a best response.

GAME THEORETIC CONSIDERATIONS

- However, playing Cournot would not have been the best response of the leader were it that the follower would play Stackelberg if it (the leader) played Stackelberg.

- In this case, the best response of the leader would be to play Stackelberg.

- Hence, what makes this profile (or rather, these profiles) a Nash equilibrium is the fact that the follower would play non-Stackelberg if the leader were to play Stackelberg.

Competitive Pricing for Spectrum Sharing in Cognitive Radio Networks: Dynamic Game, Inefficiency of Nash Equilibrium, and Collusion

D. Niyato,and E. Hossain,

IEEE J. ON SELECTED AREAS IN COM, JANUARY 2008.

Key Points:

- Competitive Spectrum pricing model
- Few primary services offer spectrum access opportunities to secondary services.
- Formulated as an oligopoly market (few firms compete with each other in terms of price to gain the highest profit)
- PRIMARY SERVICE: Cost of sharing the spectrum is modeled as a function of QoS degradation.
- SECONDARY SERVICE: A spectrum demand function is established based on the utility function wich depends on the channel quality.

Competitive Pricing for Spectrum Sharing in Cognitive Radio Networks: Dynamic Game, Inefficiency of Nash Equilibrium, and Collusion

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- Equilibrium Pricing Scheme in which each of the primary service providers aims to maximize its profit under quality of service (QoS) constraint for PUs.
- This situation is formulated as an OLIGOPOLY.

Competitive Pricing for Spectrum Sharing in Cognitive Radio Networks: Dynamic Game, Inefficiency of Nash Equilibrium, and Collusion

- *Formulated as a Bertrand Game Model, the impacts of several system parameters are analyzed*
- Spectrum substitutability, i.e., the ability of the secondary service to switch among the operating frequency spectra offered by different primary services, considered.
- Distributed algorithms for price adaptation presented to obtain the solution for this dynamic game where primary services cannot observe the profit of each other.

Additional Points

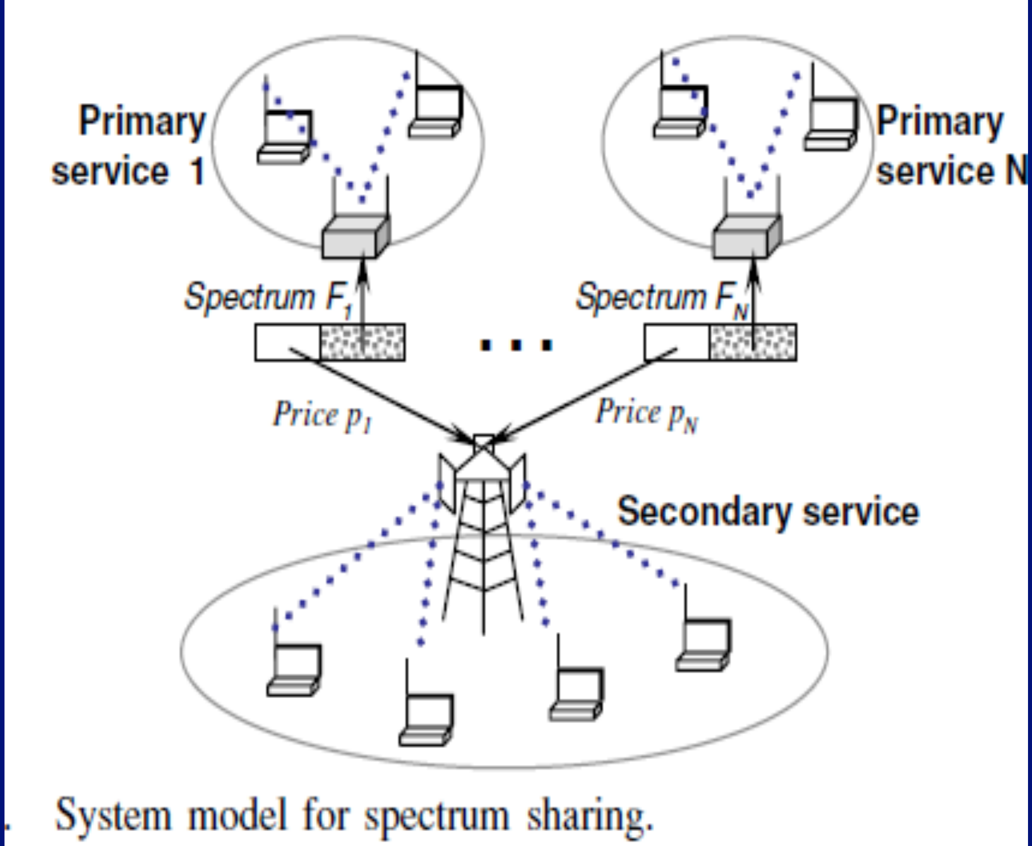
- Collusion established among the primary services helps in gaining higher profit than that for the Nash equilibrium.
- Punishment mechanism may be applied to the deviating primary service provider.
- Repeated game among primary service providers is formulated to show that the collusion can be maintained

System Model & Assumptions

- N # of primary services operating
- on different frequency spectrum F_i

* Primary service i serving M_i local connections wants to sell portions of the available spectrum F_i

(e.g., time slots TDMA-based wireless access system) at price p_i (per unit spectrum or BW) to the secondary service



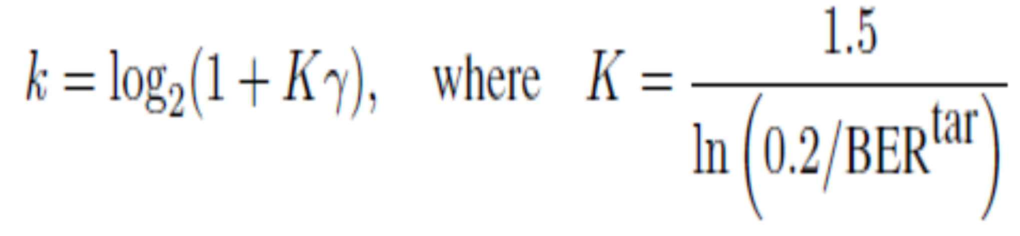
System model for spectrum sharing.

SUs utilize adaptive modulation for transmissions on the allocated spectrum in a time-slotted manner

System Model & Assumption

* Transmission rate can be dynamically adjusted based on the channel quality.

* The spectral efficiency k of transmission by a SU can be obtained



$$k = \log_2(1 + K\gamma), \quad \text{where} \quad K = \frac{1.5}{\ln\left(0.2/\mathrm{BER}^{\mathrm{tar}}\right)}$$

where γ is the SNR at the receiver and ($\mathrm{BER}^{\mathrm{tar}}$) is *the target BER*

Oligopoly Price Competition

* Oligopoly-Small number of firms (i.e., oligopolists) dominate a particular market.

* Firms compete with each other non-cooperatively and independently to achieve their objectives (i.e., maximize profit) by controlling the quantity or the price of the supplied product

* Decision of each firm is influenced by other firms' actions and action of one firm may be observed by other firms.

Bertrand Game

- Bertrand game model for price competition is applied to analyze and obtain equilibrium pricing scheme –

  * Spectrum demand of secondary service
  * Price charged by the PU
  * Cost of primary service ⇔ Degradation of its QoS
  * Profit to primary service

SPECTRUM PRICING COMPETITION AND SOLUTION

1. Utility function to quantify the spectrum demand of the secondary service

2. Primary service cost for offering spectrum access to the secondary service is formulated

3. This cost function is based on the degradation in the QoS performance for the local connections.

A. Bertrand game formulation is proposed
B. Nash equilibrium is considered as the solution of this game

SPECTRUM PRICING COMPETITION AND SOLUTION

- Utility function U(b) is to quantify the spectrum demand of the secondary service (quadratic utility function):

$$\mathscr{U}(\mathrm{b}) = \sum_{i=1}^{N} b_i k_i^{(s)} - \frac{1}{2}\left(\sum_{i=1}^{N} b_i^2 + 2\nu \sum_{i\neq j} b_i b_j\right) - \sum_{i=1}^{N} p_i b_i$$

where

* b is the set consisting of the size of shared spectrum from all primary services, i.e., b = $\{b_1, \ldots, b_i, \ldots, b_N\}$,

* p_i is the price offered by primary service i.

* $k_i^{(s)}$ is the the spectral efficiency by a SU using freq. spectrum F_i which is owned by primary i.

* Spectrum substitutability taken into account through the parameter v

- v ∈ [−1.0, 1.0] defined as

v=0 means SU cannot switch between frequencies; v=1 means SU can switch

SPECTRUM PRICING COMPETITION AND SOLUTION

Spectrum Demand Function:

To derive the demand function for spectrum F_i *the* secondary service,
we differentiate U (b) *with respect to* b_i
and given the prices of all primary services and then by solving, we get

$$\mathscr{D}_i(\mathrm{p}) = \frac{(k_i^{(s)} - p_i)(\nu(N-2)+1) - \nu\sum_{i\neq j}(k_j^{(s)} - p_j)}{(1-\nu)(\nu(N-1)+1)}$$

Revenue and Cost Functions for Primary Service

* If a portion of the frequency spectrum (in time domain and/or frequency domain) is shared with the secondary service,

degradation in the QoS performance of the PUs may occur.

* For primary service i, the revenue function R_i *and the* cost function C_i can be

- defined as follows:

$$\mathscr{R}_i = c_1 M_i, \quad \mathscr{C}_i(b_i) = c_2 M_i \left(B_i^{req} - k_i^{(p)} \frac{W_i - b_i}{M_i} \right)^2$$

* c_1 and c_2 -constant weights for the revenue and cost functions

* B_i^{req} is the BW requirement for a primary connection

* W_i is the size of spectrum, M_i is the no. of primary connections

* $k_i^{(p)}$ is the spectral efficiency of wireless transmission for primary service i.

Bertrand Game Formulation

* Players – Primary Services

* Strategy of each player - price per unit of spectrum (denoted by p_i) *which is* non-negative.

* Payoff for each primary service i (denoted by P_i) - profit (i.e., revenue minus cost) due to selling spectrum to the secondary service.

- Solution of this game is Nash equilibrium and is calculated for the special case of two primary services
  (i.e., i = 1 *and* j = 2*),* the set of equations in the paper

Dynamic Bertrand Game

- Primary service may not be able to observe the profit gained by other primary services and their current strategies may be unknown
- Hence, each primary service must learn the behavior (i.e., strategy on choosing price to be offered to the secondary service) of other players from the history.
- Therefore, for a primary service, a distributed price adjustment algorithm is required which would gradually reach the Nash equilibrium for the pricing solution

Stability Analysis of the Dynamic Game

- Dynamic algorithms are presented for two cases according to the Dynamic Bertrand model

  I. Strategies of other primary services are observable

  II. Strategies of other primary services cannot be observed

- Stability is important for both the dynamic algorithms (i.e., for Case I and Case II) to ensure steady state of the Nash equilibrium

Optimal Pricing to Maximize Total Profit of Primary Services

- Optimal price of all primary services obtained by Dynamic game model by solving these equations below

$$\frac{\partial \sum_{j=1}^{N} \mathscr{P}_j(\mathrm{p})}{\partial p_i} = 0.$$

* Optimal values of the prices p_i, which give the highest total profit, are different from those at the Nash equilibrium

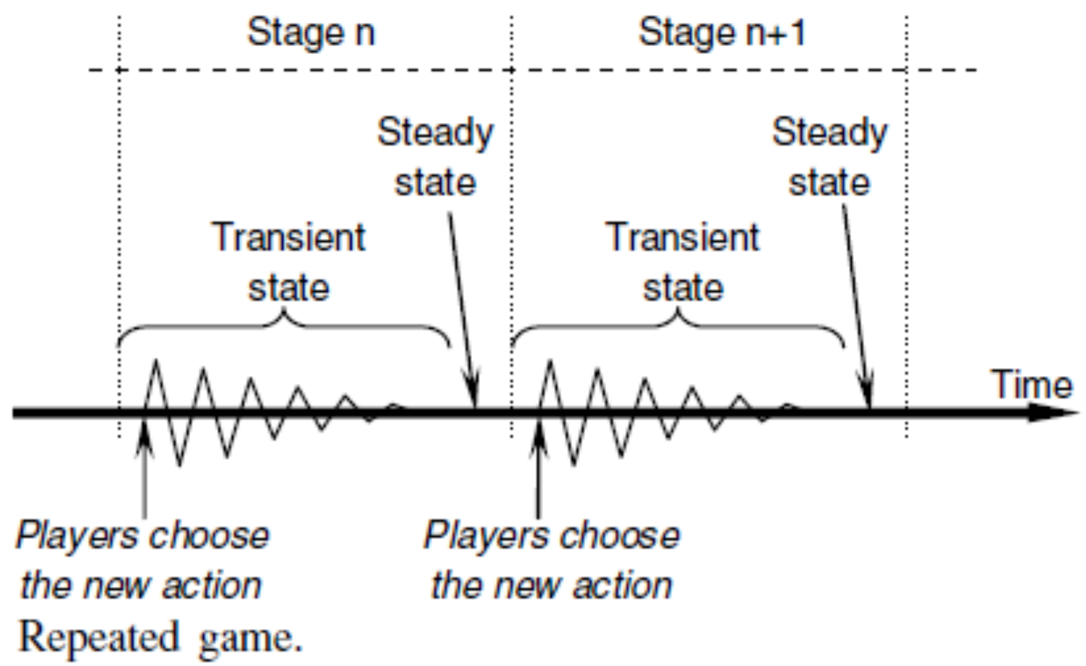
* Thus, primary services may prefer to cooperate to achieve the highest profit

Collusion and Repeated Game

- To model this, we formulate a repeated game which captures the behavior of the primary services when the pricing game is infinitely repeated.
- Since the game is repeated, a punishment mechanism can be applied to deter any primary service from deviating from the optimal price

Collusion and Repeated Game

- Dynamic Bertrand game can be defined as a repeated game for which each stage is defined from the time that the players change the parameters of price adaptation to the time that the steady state is reached



Repeated game.

Results

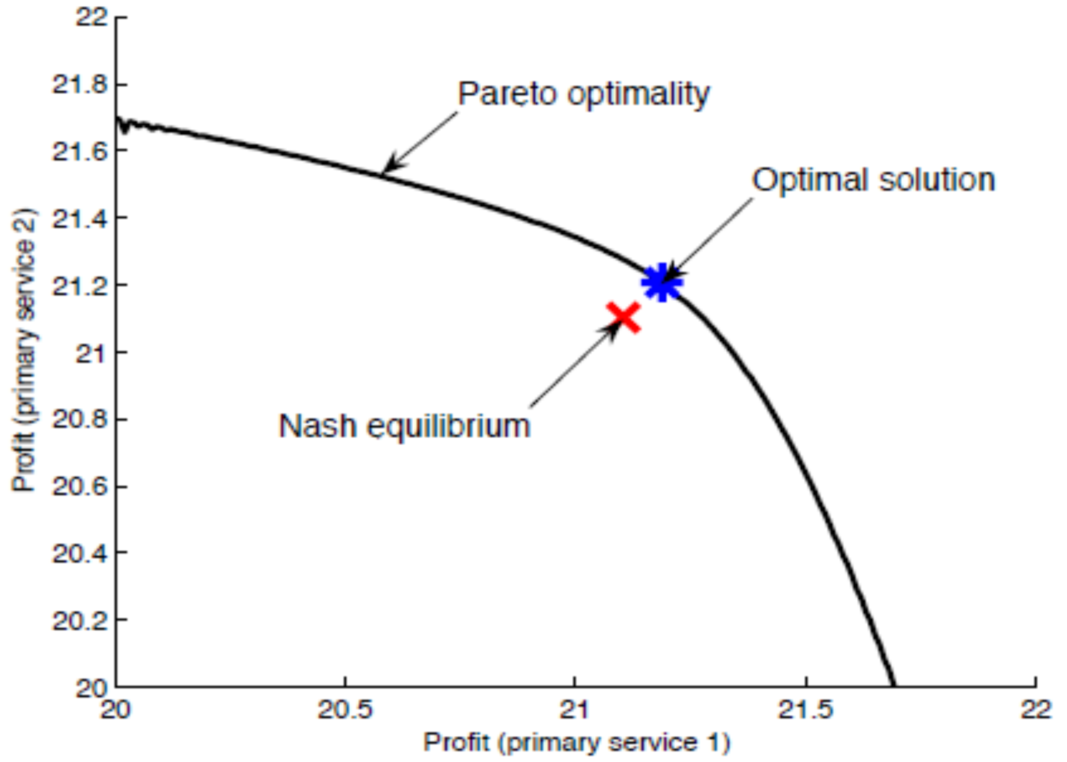


Fig. 11. Pareto optimality, Nash equilibrium, and optimal solution.

Results

Variations in profit corresponding to Pareto optimality (i.e., when one player cannot increase its payoff without decreasing other players payoffs).

Nash equilibrium is not Pareto optimal while the optimal solution is.

Nash equilibrium is inefficient (i.e., total profit is not maximized).

However, Nash equilibrium provides a “stable” solution

On the other hand, if a collusion among primary services can be established at the optimal price, each of the primary services can achieve a profit higher than that of Nash equilibrium.

Results

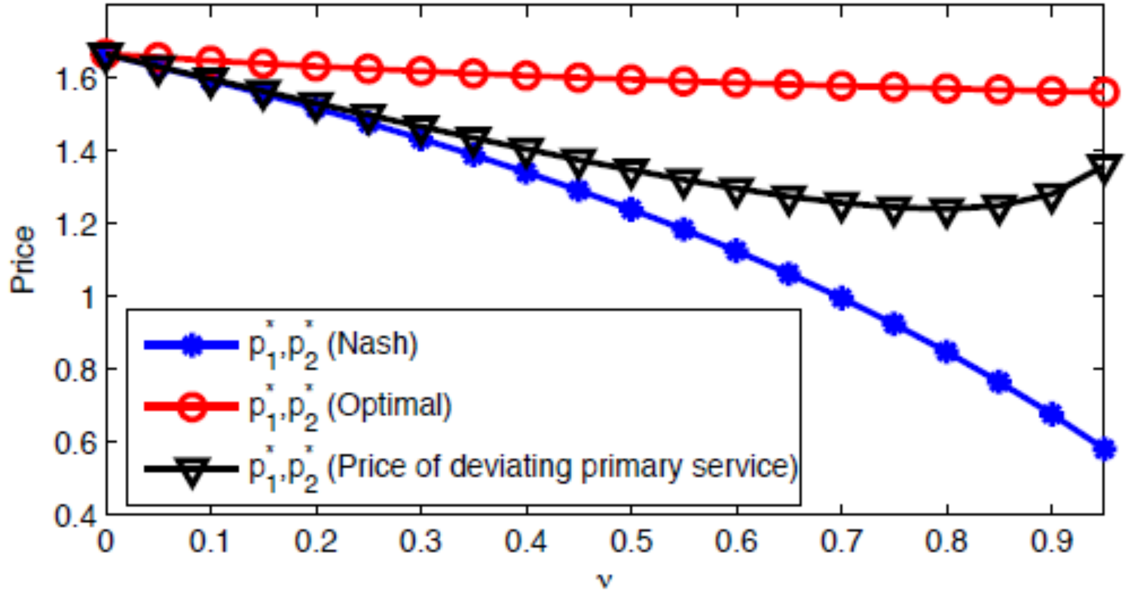


Fig. 12. Price at the Nash equilibrium, the optimal price, and the price of deviating primary service.

* Prices at Nash equil. and optimal prices resulting in highest profit.

* If both primary services make an agreement for a collusion, optimal price will be offered to the secondary device.

* One primary service can unilaterally deviate to achieve higher profit than that it can gain at the optimal price.

Results

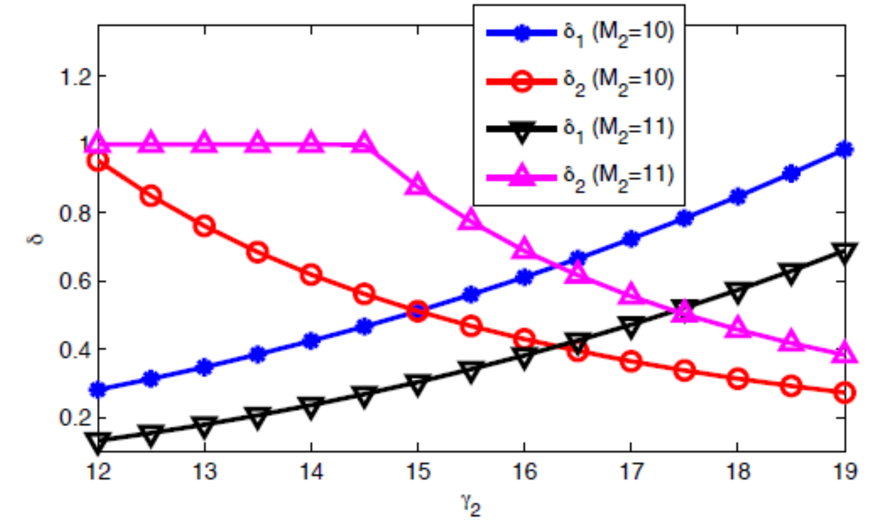


Fig. 13. The smallest value of the weight to maintain collusion.

* Variations in the lower bound of delta$_i$ under different channel qualities.

* When channel quality in the spectrum offered by primary service 2 becomes higher, the corresponding weight delta$_i$ becomes smaller while that of primary service 1 becomes larger.

* A better channel quality results in smaller profit obtained due to deviation from the optimal price, primary service 2 has less motivation to deviate.

* Delta$_i$ can be reduced to maintain collusion.

* A larger value of delta is requried for primary service one to maintain the collusion.

* When # of primary connections 2 increases the price increases while the profit decreases (at small SNR)

Conclusions

- Environment in which multiple primary services compete with each other to offer spectrum access opportunities to the secondary service analyzed the problem as a Bertrand game and obtained the Nash equilibrium which provides the optimal pricing (i.e., maximizes the total profit of all the primary services).

- For the primary services, the cost of sharing the spectrum with the secondary service has been calculated as a function of the QoS performance degradation of the primary connections

Conclusions

- Nash equilibrium is inefficient to achieve the highest total profit for all of the primary services.
- Therefore, any primary service can deviate to gain higher profit
- The optimal price to gain the highest profit can be obtained if all of the primary services can make an agreement to establish a collusion.

A Revenue Enhancing Stackelberg Game for Owners in Opportunistic Spectrum Access

by A. O. Ercan, J. Lee, S. Pollin, J. Rabaey

Proceedings of IEEE Dyspan, October 2008.

Key Points:

Stackelberg Game presented and economical aspect of opportunistic spectrum access (OSA) model and interference to PUs by SUs considered in the OSA model

A Revenue Enhancing Stackelberg Game for Owners in Opportunistic Spectrum Access

A. O. Ercan, J. Lee, S. Pollin, J. Rabaey

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Key Points:

Shown through simulations that

1. Spectrum owner can enhance his revenue by allowing OSA under certain conditions
2. Revenue enhancement results from the subscription fees of the SUs and better utilization of the spectrum
3. Enhancement is available for a large range of user preferences

OSA and Stackelberg Game

- M_p (potential PUs) and M_s (potential SUs)
- m_p (subscription fee), m_s (subscription fee) and p_{tol} (interference prob)

- Each PU buys service from PO with a prob p_p

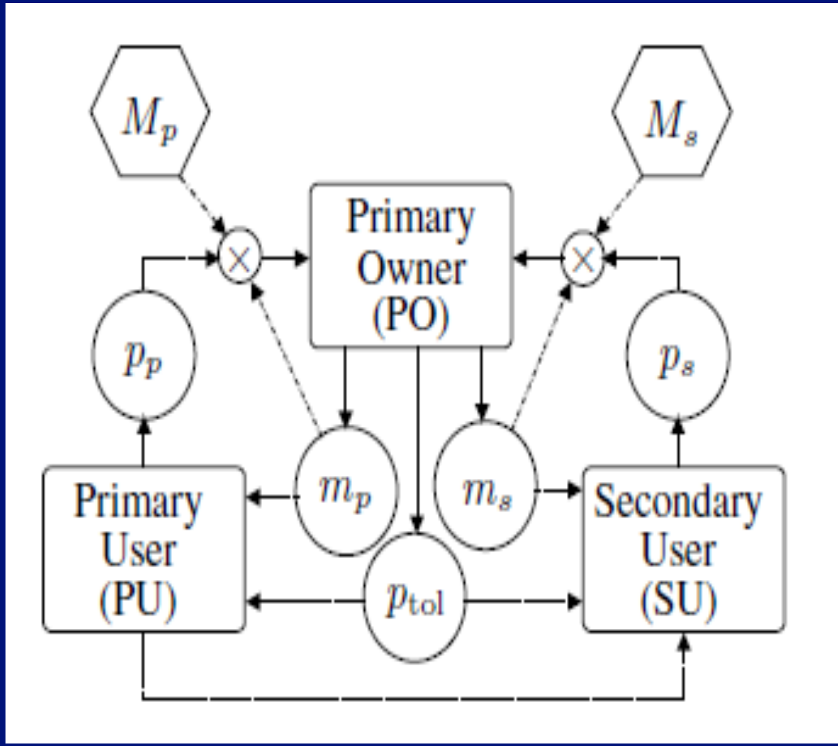
- Each SU buys service from PO with a prob p_s

- Actual number of SUs served by the channel is assumed to be $M_s p_s$

Revenue (per second per channel) of PO is given:

$$R = M_p p_p m_p + M_s p_s m_s$$

OSA and Stackelberg Game



- Three player Stackelberg game
- PO is the leader and PUs & SUs are followers
- Solid lines→ Players actions
- Dashed lines→ Inputs to the players that affect their actions.

* M_p and M_s are given and fixed.

- PO sets the values of m_p, m_s and p_{tol}
- PUs and SUs set p_p and p_s, respectively, in response to PO's actions

  - Actions of PUs and SUs in turn determine revenue of PO
  - As PO knows the best actions of PUs & SUs (p_p and p_s) in response to his actions (m_p, m_s and P_{tol}), PO sets them such that his revenues are maximized

Primary User Model

- PU generates calls independently at rate q
- When channel is used by a PU, channel is blocked.
- If not blocked, PU uses channel for an exponentially distributed period with rate p
- Busy and idle periods of the channel are exponentially distributed random variables with means B = 1/p and I = $1/(M_p p_p)$, respectively
- With these mean utilization, U_p of channel by a PU becomes

$$U_p = \frac{q}{p + M_p p_p q}.$$

Secondary User Model

- SU network uses the channel with no collision with the PUs is given by

$$U_s = p_a \mathrm{P}(\mathcal{H}_0)(1 - p_{\mathrm{FA}}) \exp(-T_u/I) \frac{T_u}{T_u + T_s},$$

where H_0 and H_1 *are the hypotheses that the channel is idle* and busy, respectively,

- p_D is the detection probability, *and*
- p_{FA} is the false alarm probability *during the sensing phase,*
- $I = 1/(M_p p_p)$ is defined before as the mean idle time of the channel
- The time is divided into slots of length "sensing time" plus the "utilization time" ($T_s + T_u$), which are both deterministic

Primary Owner Model

- Primary Owner (PO) owns the channel and is the leader of the Stackelberg game
- Owner knows that the followers (PUs and SUs) will choose their optimal acceptance probabilities p_p and p_s according to earlier stated equations
- Therefore, PO adjusts the tolerated interference probability p_{tol} and the subscription fees m_p and m_s to maximize its revenue *R*

$$\begin{aligned} R &= M_p p_p m_p + M_s p_s m_s \\ &= M_p p_p m_p + \frac{CU_s}{K\alpha}, \end{aligned}$$

Constraints Model

OBJECTIVE:

Maximize the average SU utilization of the channel U_s and the revenue R of the PO subject to conditions as stated below

$$\begin{aligned} \underset{T_u, p_a}{\text{maximize}} \quad & U_s \\ \text{subject to} \quad & p_{\text{int}} \leq p_{\text{tol}} \\ \text{and} \quad & 0 \leq p_a \leq 1. \end{aligned}$$

$$\begin{aligned} \underset{p_{\text{tol}}, m_p}{\text{maximize}} \quad & R \\ \text{subject to} \quad & 0 \leq p_{\text{tol}} \leq 1 \\ \text{and} \quad & 0 \leq m_p. \end{aligned}$$

Simulations

- Paper shows that the PO can increase its revenues by allowing OSA under the Stackelberg game model
- Default values of parameters used in simulations are:

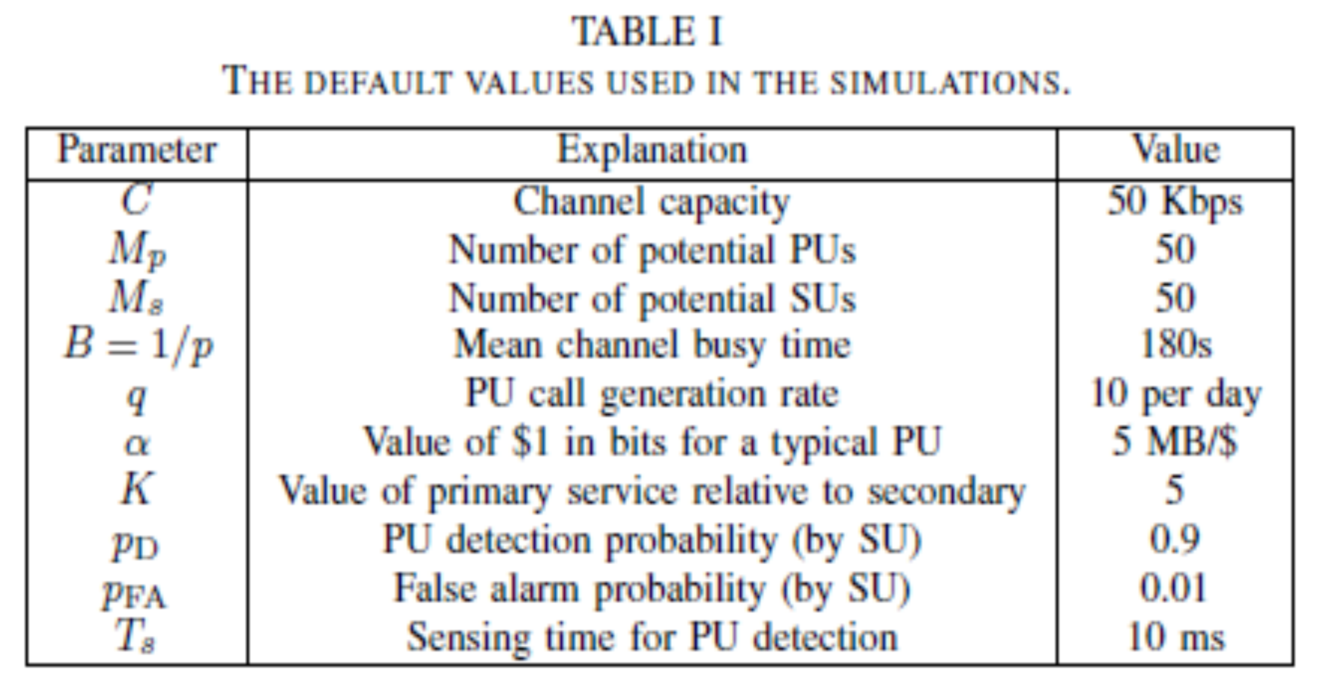


TABLE I

THE DEFAULT VALUES USED IN THE SIMULATIONS.

Parameter	Explanation	Value
C	Channel capacity	50 Kbps
M_p	Number of potential PUs	50
M_s	Number of potential SUs	50
$B = 1/p$	Mean channel busy time	180s
q	PU call generation rate	10 per day
α	Value of \$1 in bits for a typical PU	5 MB/\$
K	Value of primary service relative to secondary	5
p_{D}	PU detection probability (by SU)	0.9
p_{FA}	False alarm probability (by SU)	0.01
T_s	Sensing time for PU detection	10 ms

Results

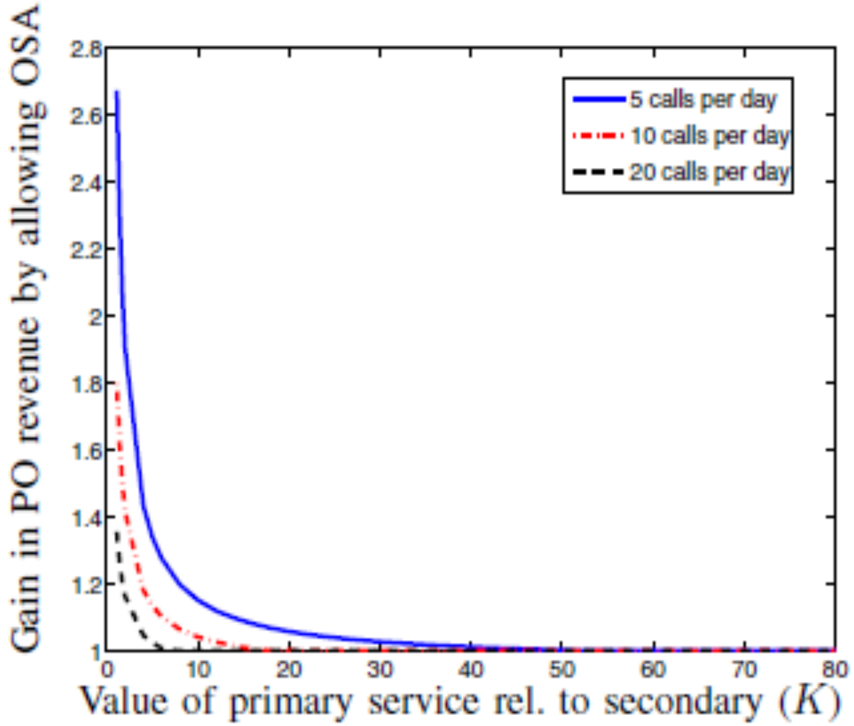


Fig. 4. Gain in the PO revenue vs. K for 3 different levels of PU channel usage statistics. Other parameters are given in Table I.

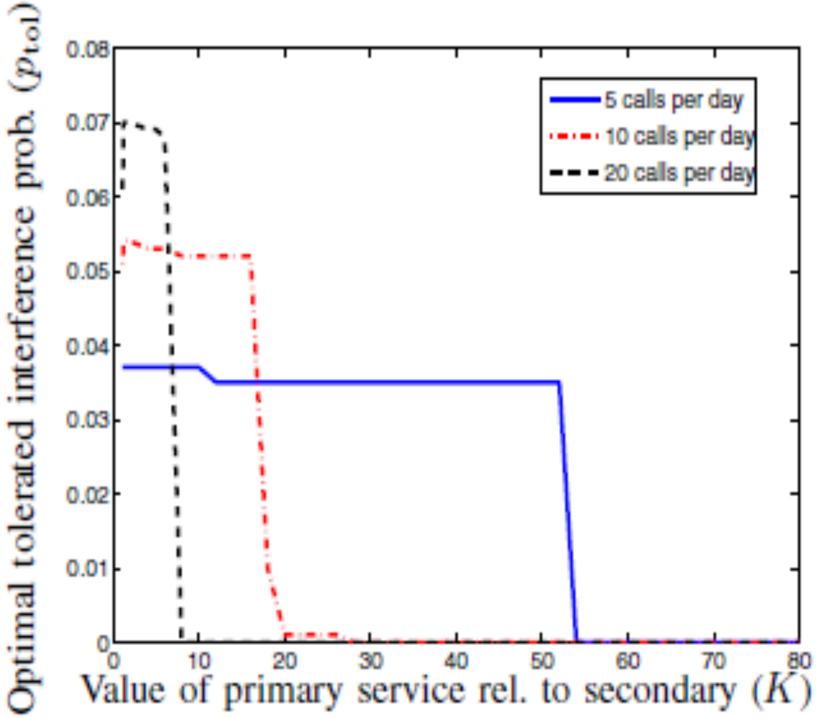


Fig. 5. Optimal tolerated interference level p_{tol} vs. K for 3 different levels of PU channel usage statistics. Other parameters are given in Table I.

Results

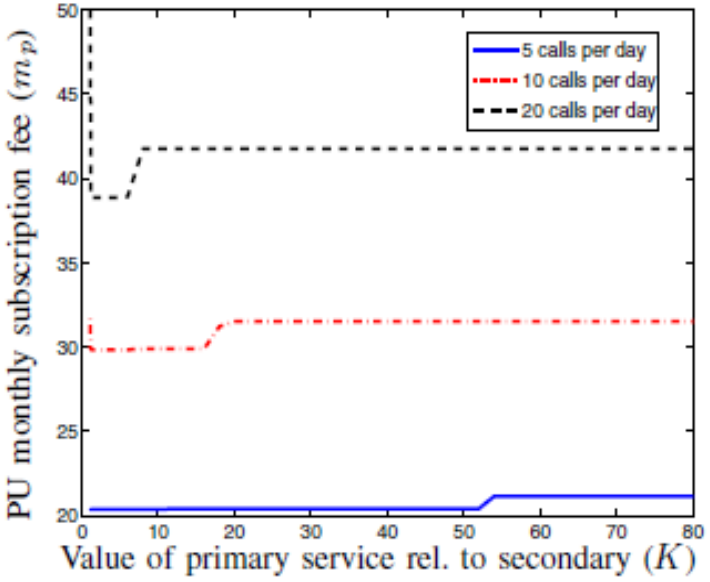


Fig. 6. Optimal PU subscription fee vs. K for 3 different levels of PU channel usage statistics. Other parameters are given in Table I.

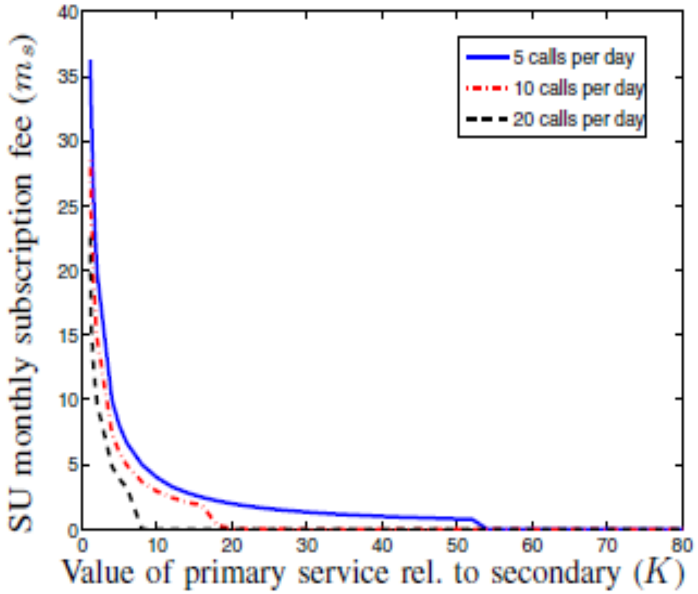


Fig. 7. Optimal SU subscription fee vs. K for 3 different levels of PU channel usage statistics. Other parameters are given in Table I.

Results

- Incentives through revenue enhancement for the spectrum owners to adopt this model
- By allowing OSA with a non-zero tolerated interference probability to the PUs, the spectrum owner can enhance her revenue
- The enhancement of the revenue comes from the subscription fee of the SUs and the fact that the spectrum is utilized better

Results

Optimal values for the actions of the PO and the resulting revenues for cases with and without OSA

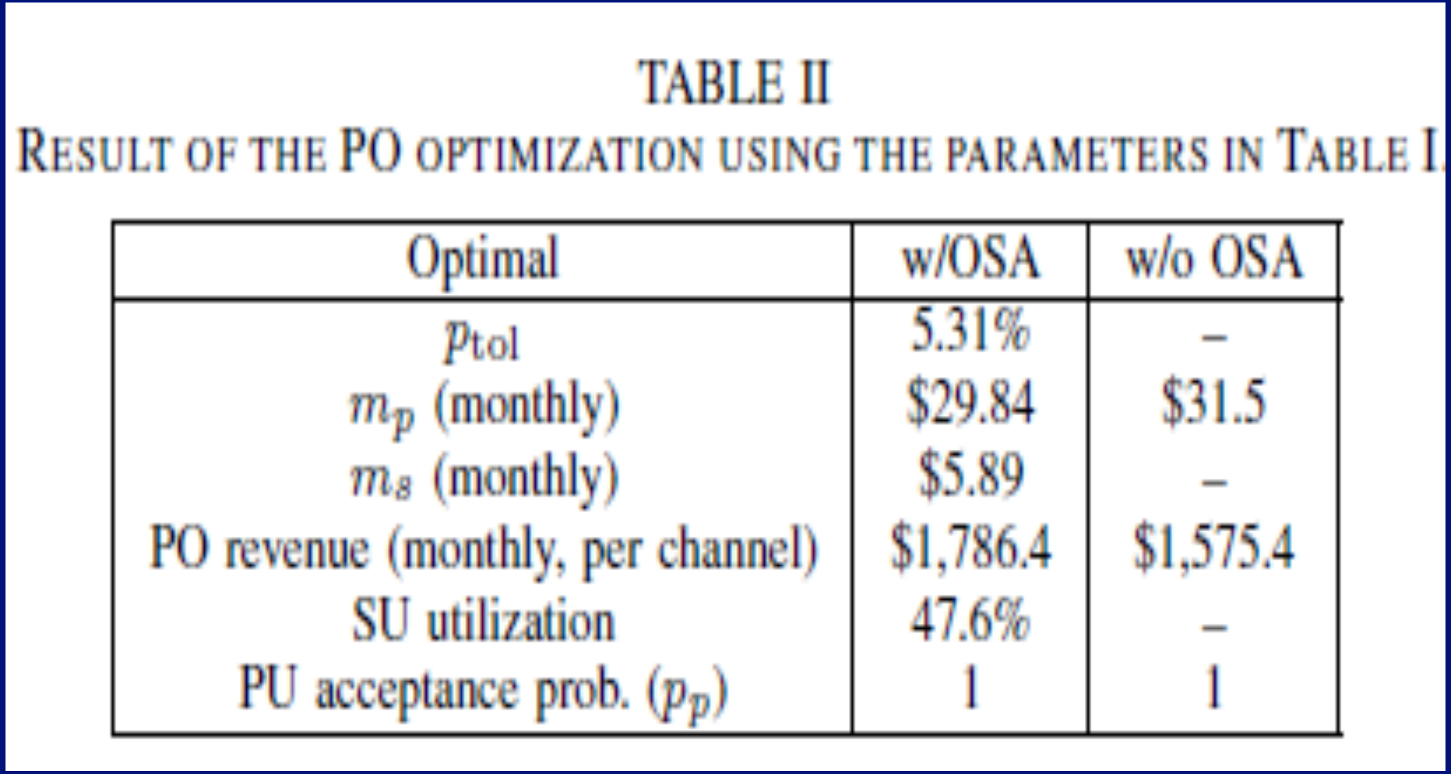


TABLE II

RESULT OF THE PO OPTIMIZATION USING THE PARAMETERS IN TABLE I.

Optimal	w/OSA	w/o OSA
p_{tol}	5.31%	–
m_p (monthly)	\$29.84	\$31.5
m_s (monthly)	\$5.89	–
PO revenue (monthly, per channel)	\$1,786.4	\$1,575.4
SU utilization	47.6%	–
PU acceptance prob. (p_p)	1	1

Discussion of Results and Drawbacks of Model

- The model proposed to adopt OSA, it is still overly simplified
- Single channel assumed
- Metrics suitable for a wide variety of applications should be used rather than just assuming the average throughput
- Analysis of the Pareto-optimal not considered
- Effect of competition among multiple spectrum owners not accounted for

Why Cooperative Spectrum Sharing?

- Higher rates and spectrum utilization than non-cooperative alternatives
- Some degree of cooperation is *always* required
  - no communication possible if a node is always jamming any possible transmission!
- Typically devices follow operators policies, making cooperation easy to implement
- Cost: coordination complexity, control overhead

Why Relaying?

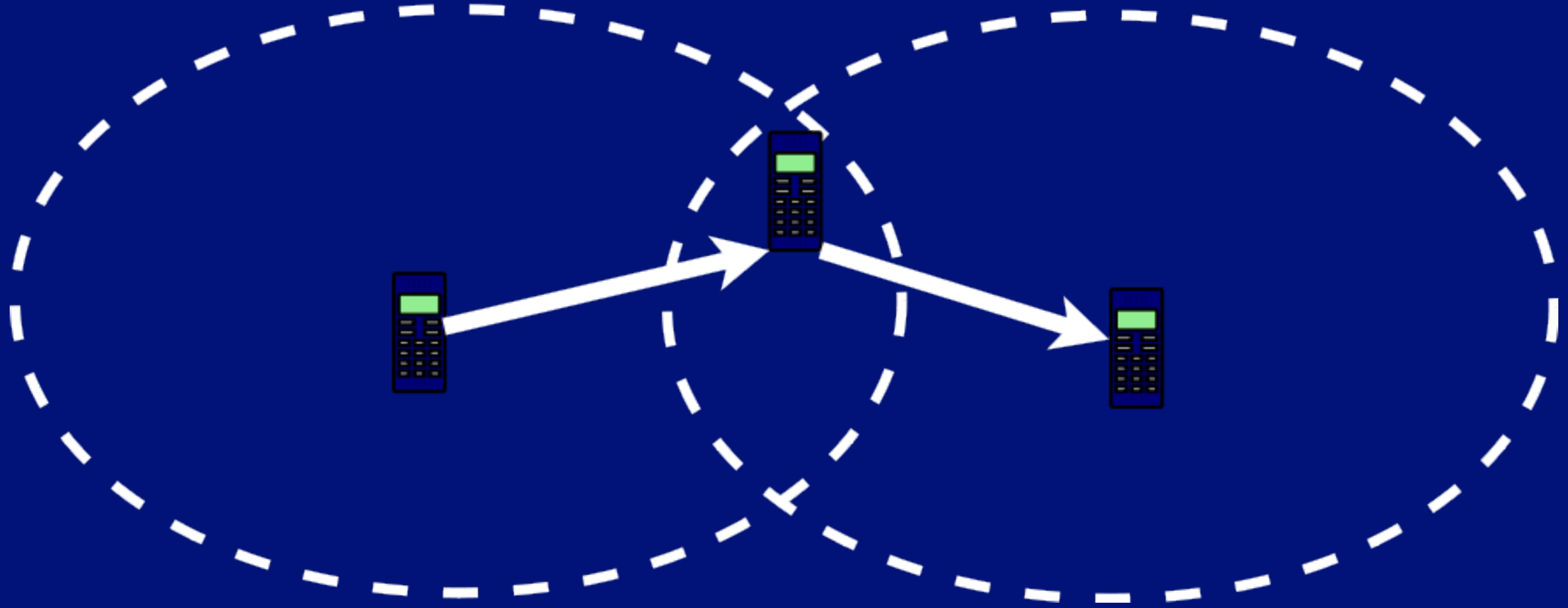
- Research on spectrum sharing largely based on game theory
- Recently, new approaches appearing, among the most promising, cooperative relaying
  - Significant capacity improvements possible
- Cooperative relaying complements (can be used along with) other approaches

Cooperative Relay in Wireless Networks

- In wireless networks, relay offers several advantages
  - Multi-hop routing (spatial bridging)
  - Bridging nodes with disjoint available channels (spectrum bridging)
  - Improved network resilience through diversity

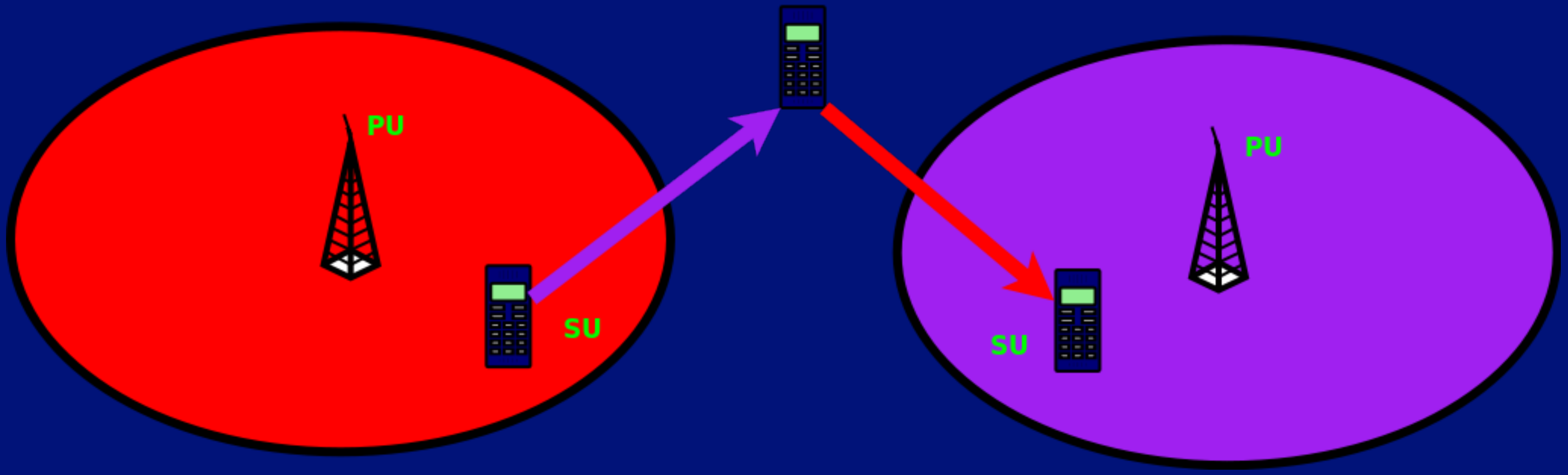
Relay Spatial Bridging

- Typical use for multi-hop ad hoc networks



Relay Spectrum Bridging

- System with two available channels, color coded
- Relay allows communication of previously isolated nodes



Relaying for Spectrum Sharing

- Always cooperative technique

- Intra-network spectrum sharing
  - Use relays among SUs for better utilization of existing spectrum holes

- Inter-network spectrum sharing
  - Help other SUs or PUs to make more spectrum holes available

Amplify-and-Forward Relaying

Y. Han, A. Pandharipande and S.H. Ting,
"Cooperative Spectrum Sharing via Controlled Amplify-and-Forward Relaying",
IEEE PIMRC, Sept 2009.

- Inter-network spectrum sharing
- Cooperative transmission
- SU and PU use same spectrum, coding...
  - SU simultaneously improves PU transmission and gets spectrum access

Amplify-and-Forward: Idea

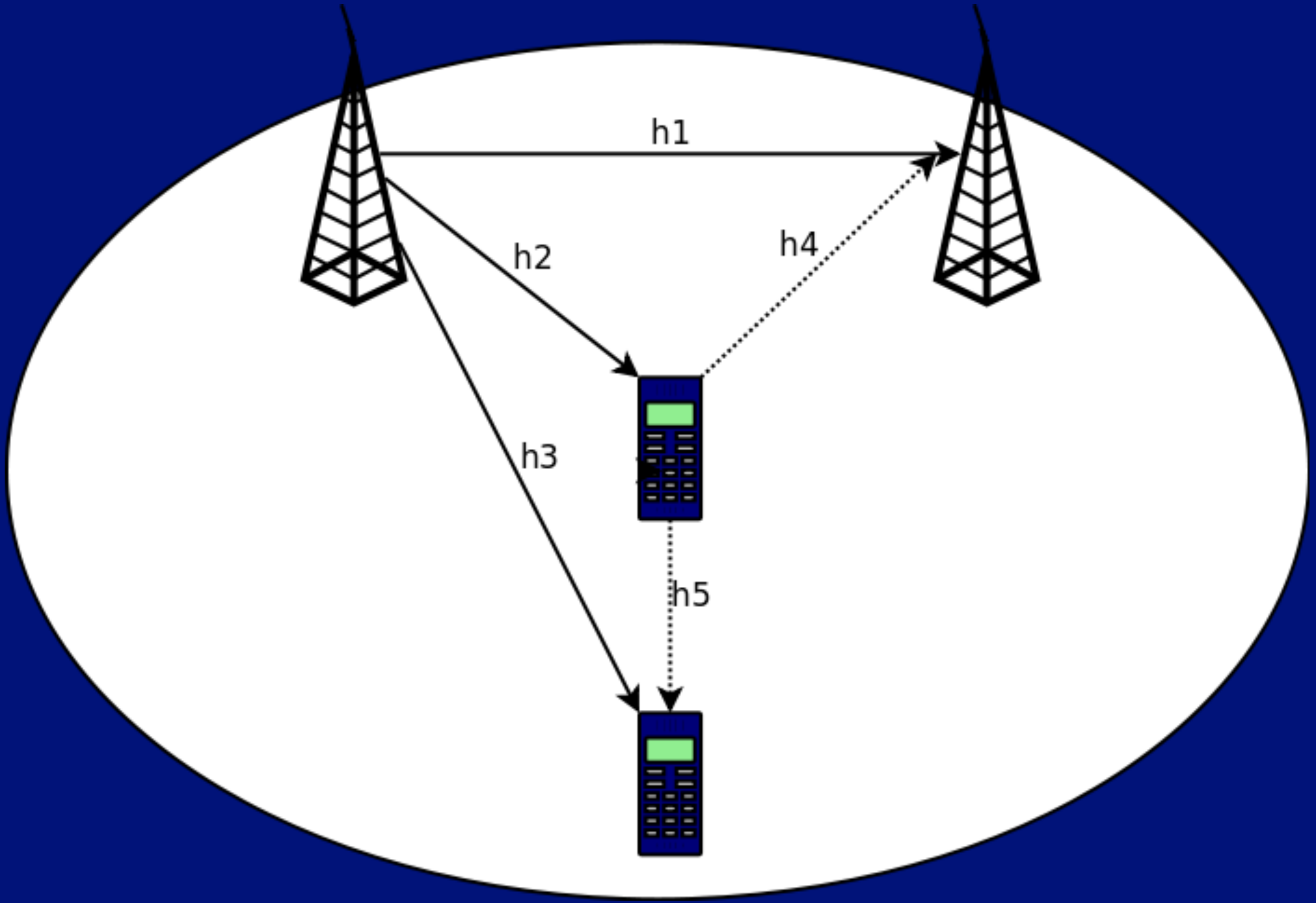
- Primary transmitter (PT) and receiver (PR), secondary transmitter (ST) and receiver (SR)

- Two phase transmission
  - Phase 1: PT transmits x_p
  - Phase 2: ST transmits $\alpha x_{p'} + (1-\alpha)\,x_s$

- PR: two phase equivalent to SIMO channel, signal recovered using maximum ratio combination

- SR recovers signal by interference cancellation

Amplify-and-Forward: Idea



Achievable Rate for PU

PT tx power

- No relay

$$R_n = \log_2\left(1 + \frac{P_p\gamma_1}{\sigma^2}\right)$$

- Relay

$$R_p = \frac{1}{2}\log_2\left(1 + \frac{P_p\gamma_1}{\sigma^2} + \frac{P_p\gamma_2\gamma_4 g^2}{\lambda}\right)$$

$$\gamma_i = |h_i|^2$$

$$\lambda = P_s(1-\alpha)\gamma_4 + g^2\gamma_4\sigma^2 + \sigma^2$$

$$g = \sqrt{\frac{P_s\alpha}{P_p\gamma_2 + \sigma^2}}$$

ST tx power, ratio

Achievable Rate for PU

$$R_p \geq R_n??$$

- Possible in a range of α
  - Computable knowing all channel coefficients
  - Possible to obtain lower bound using just PR coefficient
    - Needed feedback from PR to ST
  - Average lower bound using only statistical info

Achievable Rate for SU

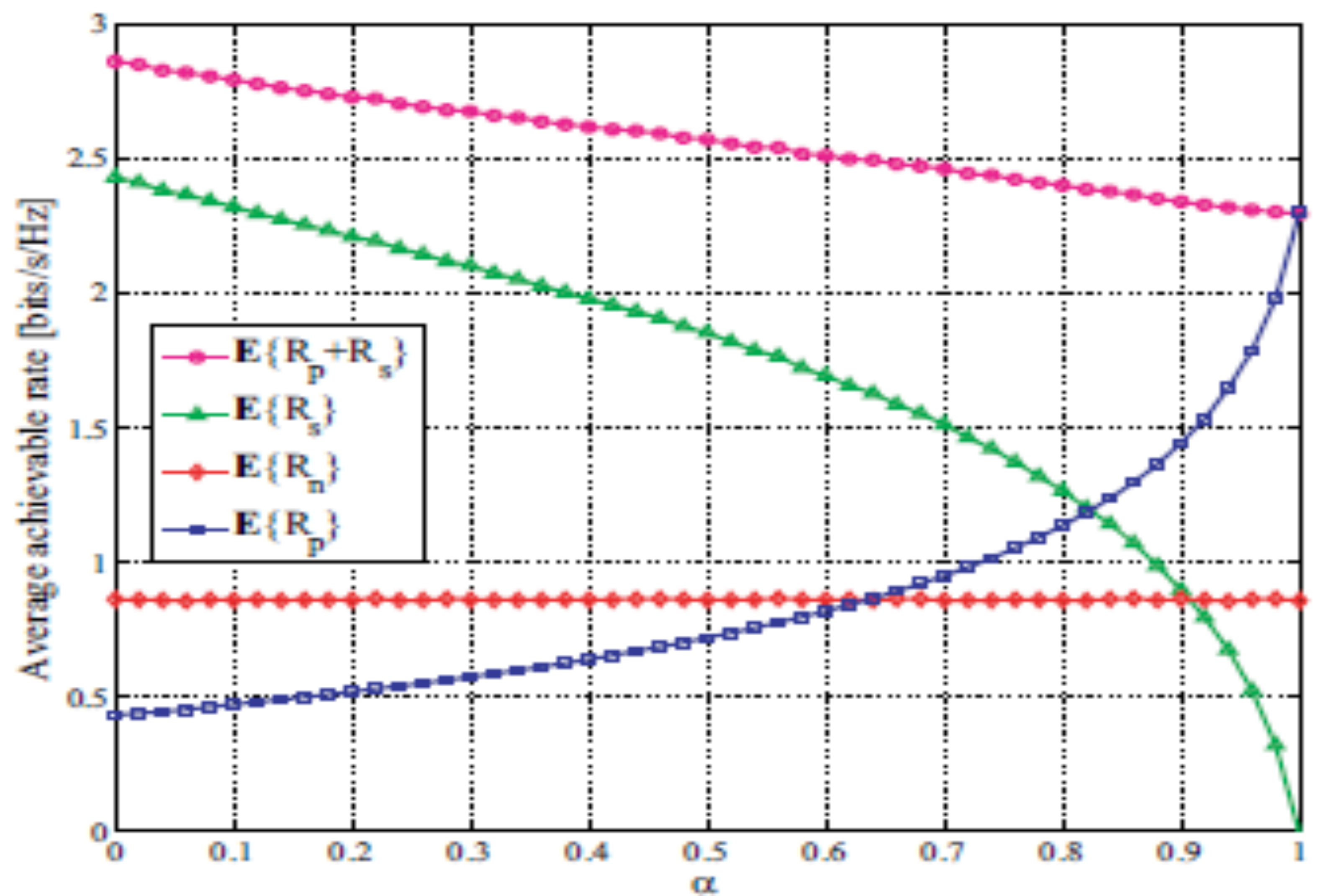
- Minimum of
  - Achievable rate PT-SR

$$R_{s1} = \frac{1}{2}\log_2\left(1 + \frac{P_p\gamma_3}{\sigma^2}\right)$$

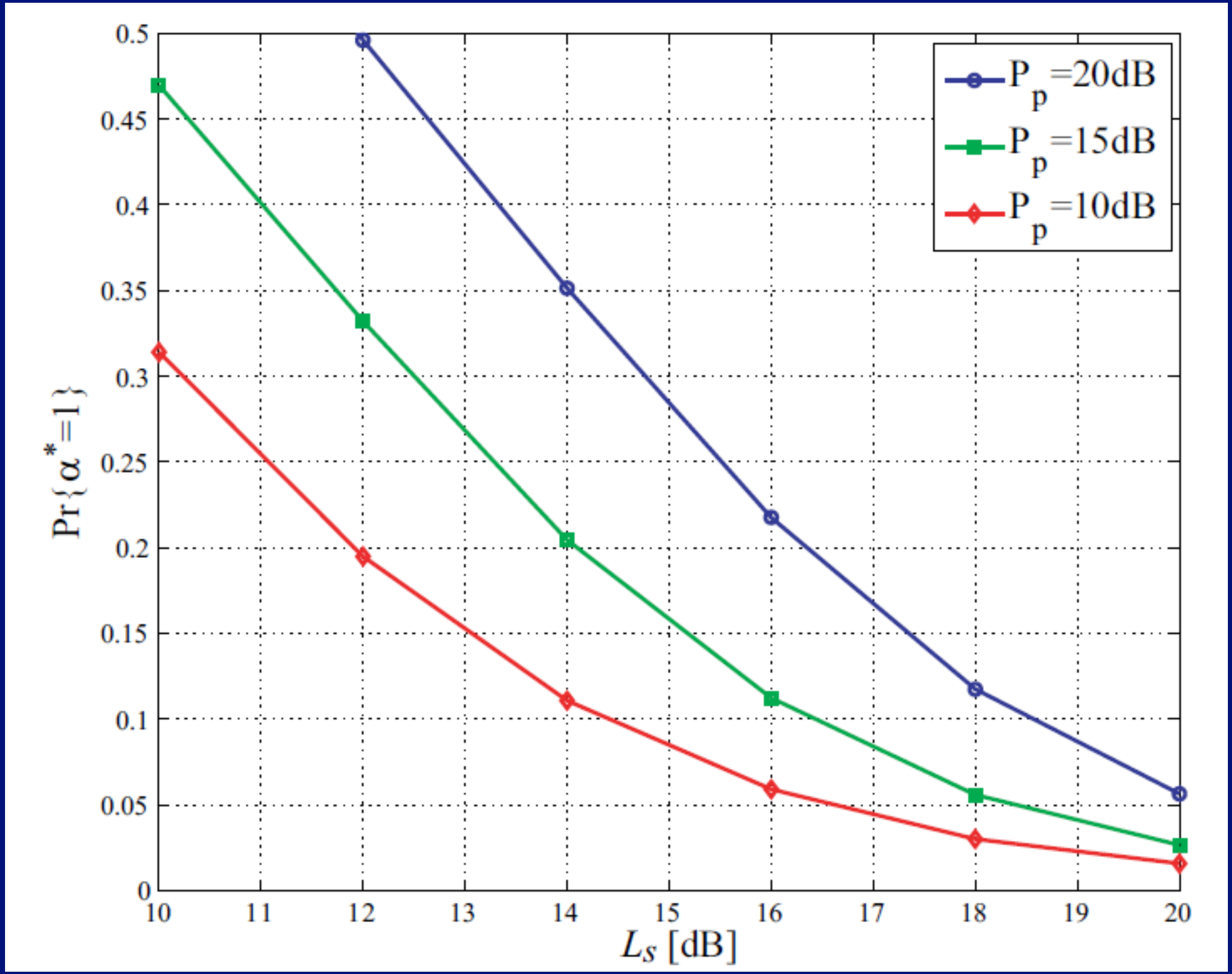
  - Achievable rate ST-SR

$$R_{s2} = \frac{1}{2}\log_2\left(1 + \frac{P_p(1-\alpha)\gamma_2\gamma_5}{\alpha\gamma_5\sigma^2 + \frac{P_p}{P_s}\gamma_2\sigma_2}\right)$$

Amplify-and-Forward: Results



Amplify-and-Forward: Results



Decode-and-Forward Relaying

Y. Han, A. Pandharipande and S.H. Ting,
"Cooperative Decode-and-Forward Relaying for Secondary Spectrum Access",
IEEE Trans. on Wireless Communications, Oct 2009.

- Same idea as amplify-and-forward
- Maintain or improve PU outage probability
- Accounts for wrong reception at SU transmitter
- Accounts specifically for distance

Amplify-and-Forward: Conclusions

- Pros
  - Use of PU spectrum without affecting it
  - Possible improvement of PU performance

- Cons
  - PU must be modified
  - Suitability for voice, video

Relays Extended

- Low complexity Decentralized Fair Resource Allocation Algorithm aiding far away CR
- Multiuser Non Selfish Symbiotic CR to enhance overall throughput
- Stackelberg Game for Optimal Selection of Cooperative Relay Set S

Decentralized Fair Resource Allocation for Relay Assisted Cognitive Radio

- Dynamic Spectrum Access: ideal for rapidly growing demands

Obstacles in Implementation:

- Direct Transmission from BS to cell edge users requires large power
- Small Access Opportunity for edge users affecting their fairness

Relay Assisted CR Systems

Current Design and Operations

- Most works contain centralized solution and global knowledge of the system CSI
- Fair Considerations:

  RS-assisted CR fairness still not fully addressed
- Imperfect CSIT and Sensing Measurement

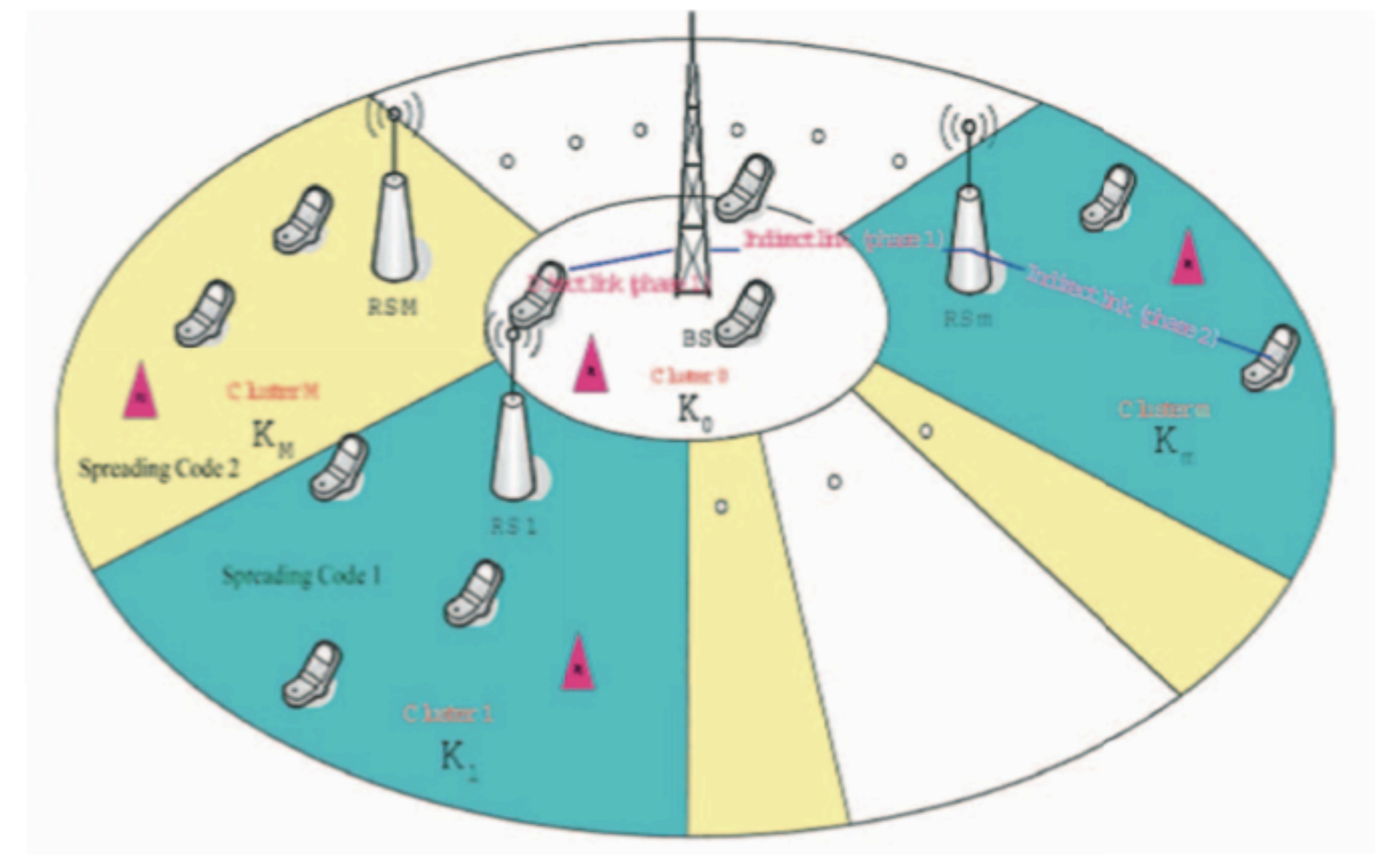
Relay Assisted CR Systems

R. Wang; V.K. N. Lau, Y. Cui, H. Kaibin, B. Chen and X. Yang;
"Decentralized Fair Resource Allocation for Relay-Assisted Cognitive Cellular Downlink"
IEEE ICC, June 2009.

Architecture

- Single Cell Cluster with 1 Base Station with k mobile users
- M users becomes RS
- Cell divided into M+1 clusters: 0th cluster and M clusters served by half duplex RS
- K_m users in a cluster
- Two Phased Transmission

System Model: Architecture



System Model: Architecture

- SU opportunistically access licensed network

Relays use DaF strategy

- Phase 1: BS to 0^{th} cluster and RS
- Phase 2: RS to cluster users on subband w/o PU
- Frequency selective and divided into N independent subbands using OFDM
- Neighboring Relay Stations use orthogonal spread sequence

System Model: Channels

- Channels modeled as block fading
- Gains characterized with path loss and microscopic fading
- CSIT estimation imperfect due to noise and duplexing delay

$\hat{H}_{m,n,k} = H_{m,n,k} + \Delta H_{m,n,k}$ (CSIT error model)

System Model: Dynamic Spectrum Access

- SU senses for unused subband by PU
- RSI denoted by $S_{m,n,k}=\{0,1\}$

  state of the kth user on nth subband in mth cluster
- 1: Available and 0: Unavailable
- Perfect Sensing with non-zero probabilities of false alarm and mis-detection

Joint Control of Rate, Power and Subband Allocation

The system resource allocation must satisfy

- $\sum^{N}_{n=1}\sum^{Km}_{k=1} p_{m,n,k} \leq P_m$ (peak power constraint)
- $\sum^{N}_{n=1}\sum^{Km}_{k=1} a_{m,n,k} \leq 1$(% allocated subbands)
- $0<P_{out}(r_{m,n,k})<1$
  (per link packet error probability adjusted data rate)

Resource Allocation System Policies

a) Transmit power becomes $P_0=\{p_{0,n,k}(\hat{H},\hat{S},Tm)\}$,

b) Subband sharing becomes $A_0=\{a_{0,n,k}(\hat{H},\hat{S}, Tm)\}$ and

c) Transmit data rate at the BS becomes $R_0=\{r_{0,n,k}(\hat{H},\hat{S}, Tm)\}$.

d) Information bit for relay $D=\{ d_{m,n,k} ,(\hat{H},\hat{S})\}$

Formulating the Problem

- Instantaneous mutual information between the m_{th} transmitter and the k_{th} receiver in the n_{th} subband

  $$C_{m,n,k}=g a_{m,n,k}\log_2((1+p_{m,n,k}l_{m,k}|H_{m,n,k}|^2)/a_{m,n,k})$$

- Instantaneous goodput is given by

  $$U_{m,n,k}=S_{m,n}r_{m,n,k}I(r_{m,n,k}<=C_{m,n,k})$$

- Set of goodput weights for different users $\{w_{m,k}\}$
- Average weighted goodput (policies to be optimized for) $G'(A,P,D)$

Joint Control of Rate, Power and Subband Allocation: Solution

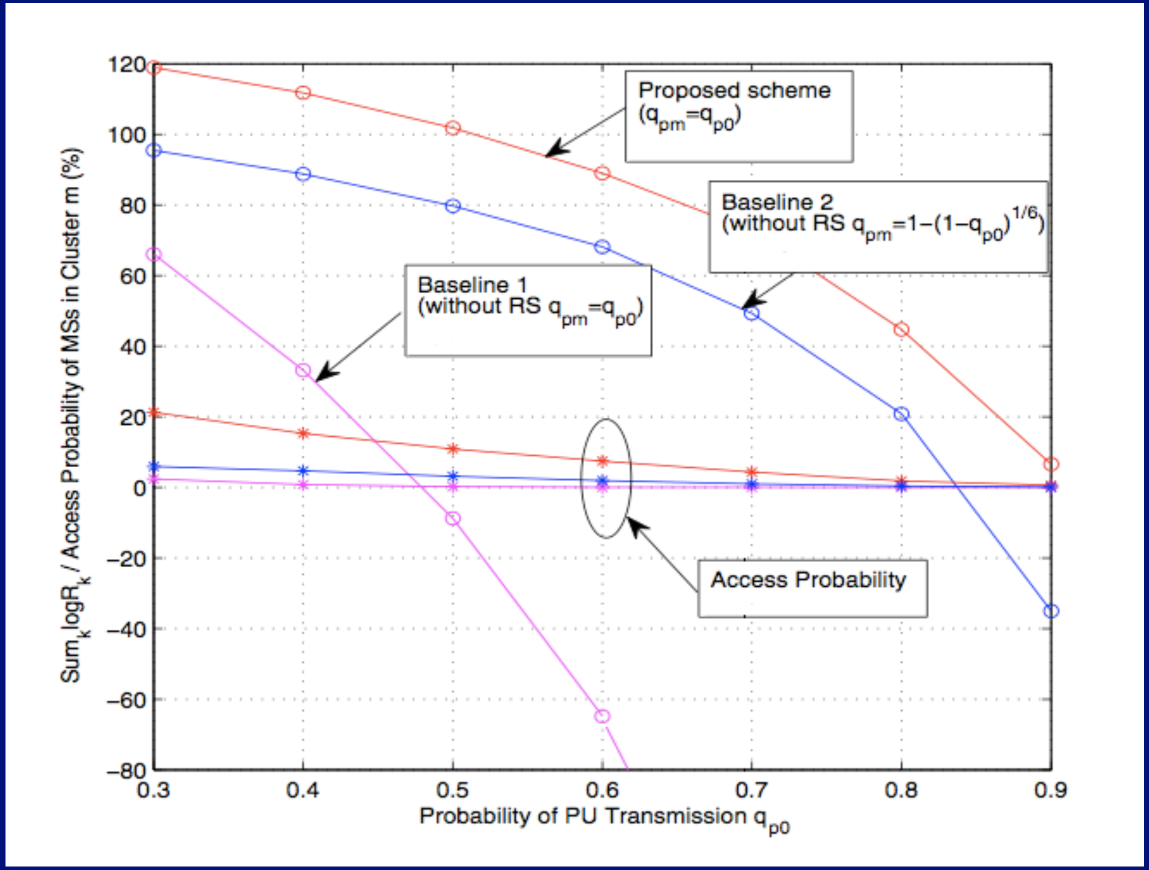
- Optimization required at both the RS and BS
- Computational load shared between RS and BS
- **Step 1:** For m={0,…,M} mobiles in cluster m deliver the 1-bit RSI to the cluster controller (BS or RS)
- **Step 2:** The m_{th} RS feeds back the function G'm**(r) to the BS

Joint Control of Rate, Power and Subband Allocation: Solution

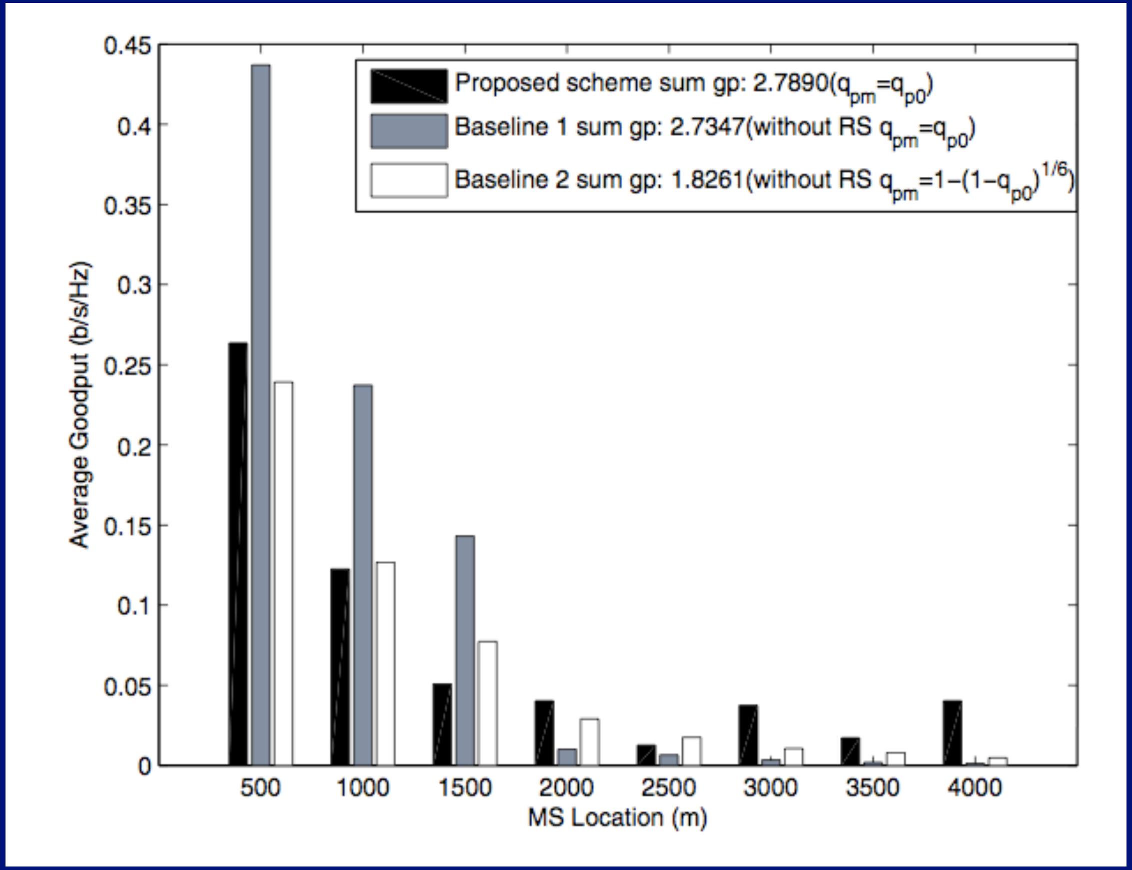
- **Step 3:** From local CSI($\hat{H}$), RSI($\hat{S}$) and $G'm^{**}(r)$, BS determines the power, rate and subband allocation of mobiles in cluster 0 as well as the RSs

- **Step 4:** The m_{th} RS that decodes the information from BS successfully will determine the power, rate, subcarrier allocation to the MSs in cluster based on the local CSI($\hat{H}m$) and RSI.

Access Probability, Fairness/Throughput



Average Goodput of MSs at Various Distances



Existing Directions for Dynamic Spectrum Access

1. PU unaware of the existence of SU

   SU can access the channel only if their transmission does not cause interference for the PU

2. PU aware that SU using their allocated Freq Band

   PU has higher priority in transmission access and retains right to improve revenue by charging SU

Why not the PU leverage the SU?

What if SUs have better channel conditions and other PUs cannot help:

Cooperative Cognitive Radio Network

Stackelberg Game for Cooperative CR Networks

J. Zhang and Q. Zhang,
"Stackelberg Game for Utility-Based Cooperative Cognitive Radio Networks",
Proc. of ACM MOBIHOC, 2009.

- PU aware of the existence of SU, select some to be a cooperative relay
- SU cooperating with primary transmissions choose their payment mode
- PU target maximizing their utility depends on transmission rate and revenue from SU
- SU target how less they must pay PU to achieve maximum transmission rate.

Solution Steps

1. **CCRN:**
   PU involve SU as relay, SU achieves access to the channel, and a payment mechanism for SU to pay PU.

2. **Stackelberg Game:**
   Hierarchical framework where PU-SU becomes leader-follower.
   Existence of unique NE, analytical result and corresponding constraints to select optimal cooperative relay set S.

3. **Implementation Protocol:** based on analytical results

4. **Simulation Results:**
   Numerical results show that both PU and SU achieve better performance in terms of transmission rate.

System Model

- PT communicates with PR

- Unlicensed users $\{ST, SR\}_{i=1}^{K}$ looking to exploit transmission opportunity

PT decides to use entire slot or use cooperation by using

- α of the slot for transmission from PT to PR ($0 \leq \alpha \leq 1$)
- β to further divide the slot ($0 \leq \beta \leq 1$)

System Model

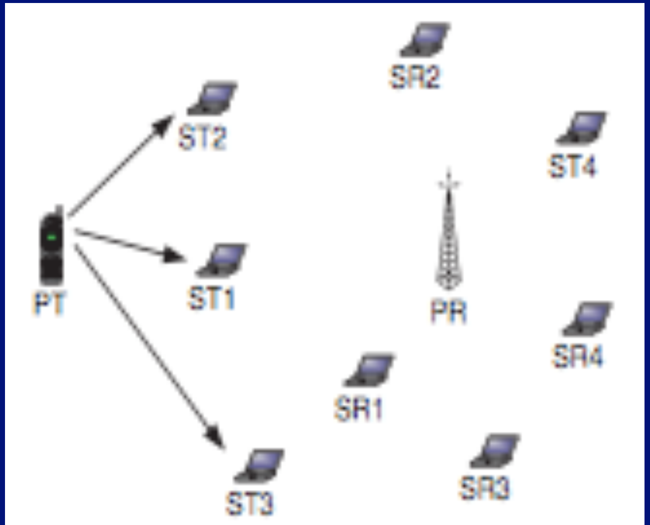
Time is divided into

- $\alpha\beta$ for Primary Transmission,
- $\alpha(1-\beta)$ for Cooperative Transmission and

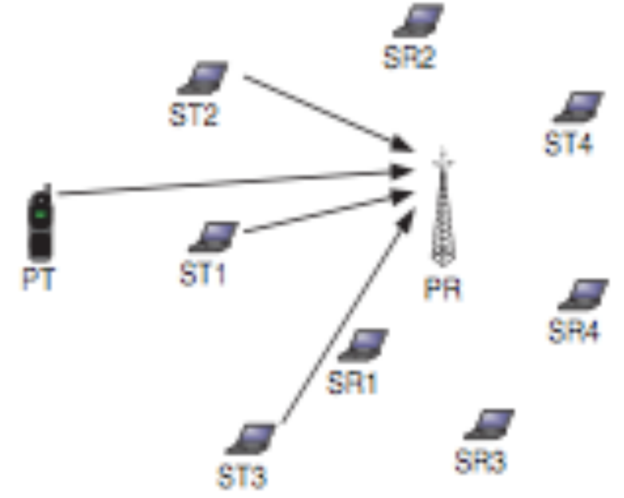
- $(1-\alpha)$ for Secondary Transmission using TDMA for a time proportional to the payment c_i made to the PU.

$$T_{i=} (1-\alpha)\, c_i / \Sigma_{j=1}^{k} c_j$$

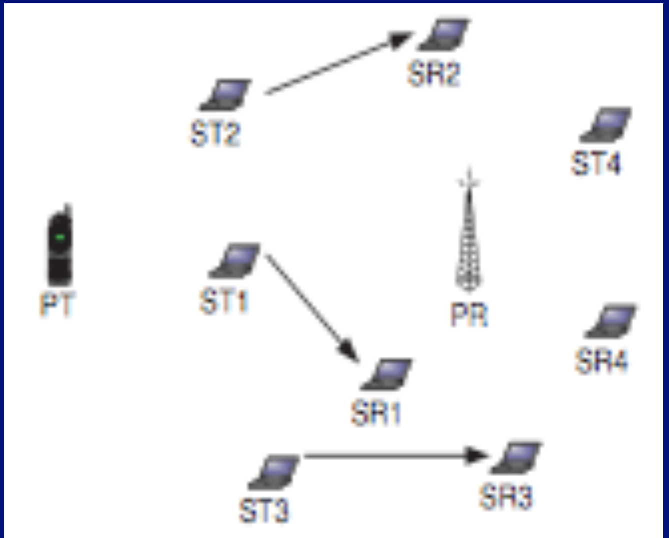
Primary, Cooperative and Secondary Transmission



(a) Primary transmission



(b) Cooperative transmission



(c) Secondary transmission

Stackelberg Game

- Both PU and SU are rational and selfish
- PU being licensed gets to decide the parameters α,β and S
- Secondary transmitter in S decides the payment it is willing to make with predecided α,β
- Utility functions $U_P=w_P U_R(R_P(\alpha,\beta,S))+\Sigma_{i\varepsilon S}c_i$ where $U_R(R_P(\alpha,\beta,S))$ is data rate utility and measure of PUs degree of satisfaction.

SU Payment Selection

- SU Payment Selection to determine b/w competing SU by maximizing its utility by selecting its payment forming a non-cooperative payment selection game (NPG)
- Payment vector: Nash Equilibrium of NPG and the unique equilibrium of NPG used by PU to select S.
- Leader (PU) uses analytical result to optimize strategy (α,β,S) so as to maximize revenue as per U_p
- This will affect the decision made by Stackelberg followers (SU).

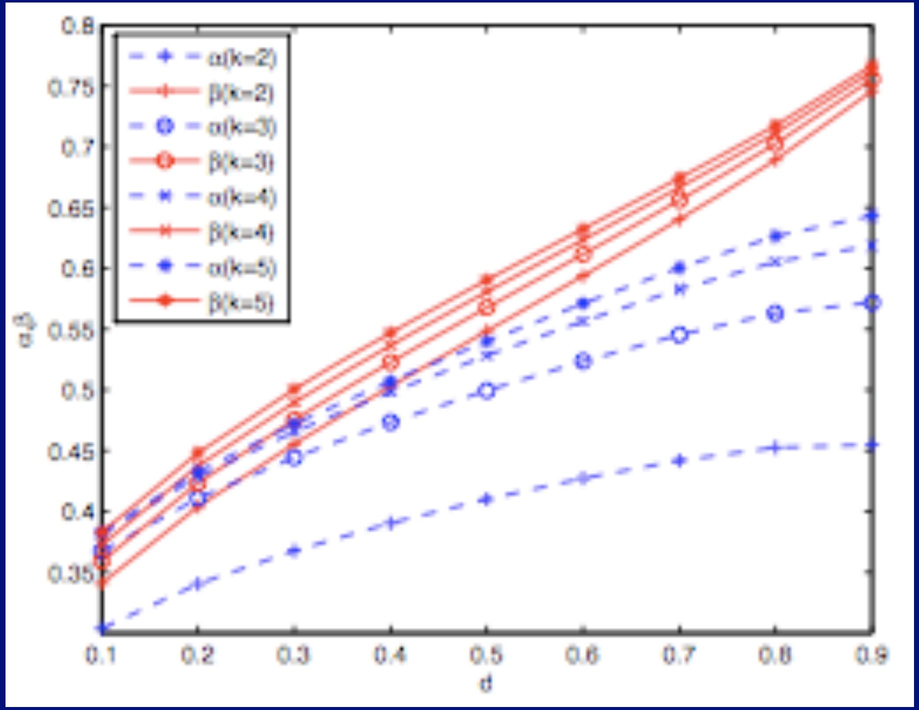
Implementation

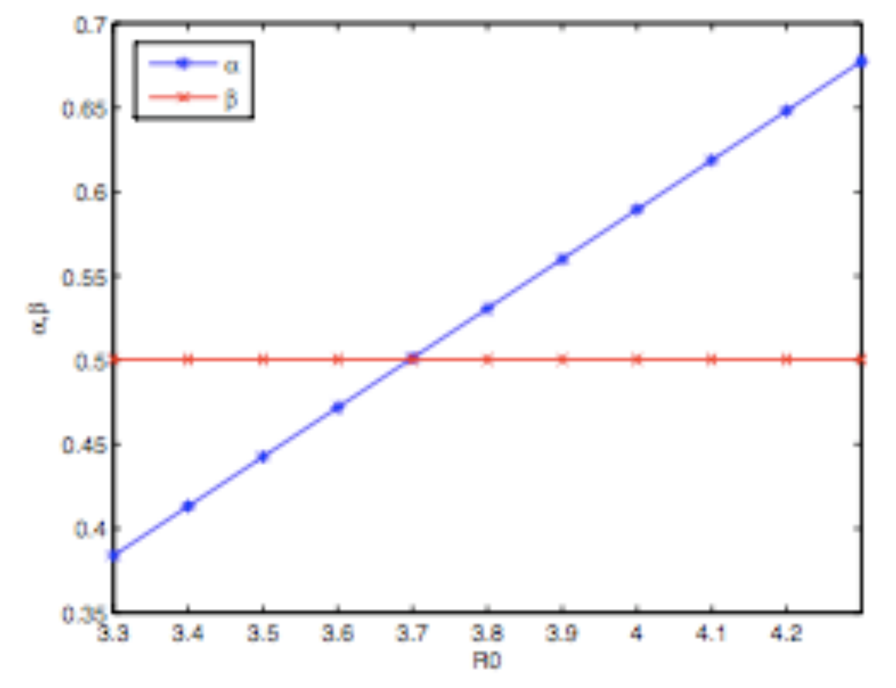
- Optimal Slot Division Parameters α^* β^* depends on $w_p, w_{s,}$ and dynamically changing $R_{PS,i}$, $R_{SP,i}$, $R_{P,I}$,
- PT periodically collects information from PR and each $ST_{i,}$
- For each S, $\alpha^*(S)$, $\beta^*(S)$ and overall utility $U_P^*(S)$ can be calculated to find Optimal Relay Set that maximizes PU's utility function.
- PT piggybacks the value of α^*, β^* and S^* to SU
- Each SU does not need to know other channel's condition allowing distributed implementation of the protocol.

Simulation Results

Optimal α^*, β^* v/s Normalized Distance d for varying number of relays

Optimal α^*, β^* v/s Required Primary Rate R_0





Simulation: Primary Users Utility Function

Primary users Utility Function of different schemes v/s Normalized distance d

Primary users Utility Function of different schemes v/s number of relays k