



# CHAPTER 7. SPECTRUM SHARING



# Intranetwork Spectrum Sharing: Distributed - Non-Cooperative

1. Device Centric Approach:  
H. Zheng and L. Cao,  
"Device-centric Spectrum Management,"  
Proc. IEEE DySPAN, Nov. 2005.
2. Belief Assisted Pricing  
J. Zhu and Ray Li,  
Proc. IEEE SECON, Sept. 2006.



# Belief Assisted Pricing

Z. Ji and K. J. R. Liu,

"Belief-Assisted Pricing for Dynamic Spectrum Allocation in Wireless Networks with Selfish Users,"

Proc. IEEE SECON, Sept. 2006.

- Consider the spectrum sharing as multistage dynamic game
- Belief-assisted dynamic pricing approach
  - Optimize the overall spectrum efficiency, but keep the participating incentives of the users.



# Belief Assisted Pricing

- Coordinate the spectrum allocation among PUs and SUs through a trading process to maximize the payoffs of both PUs and SUs.
- Develop a BELIEF SYSTEM to assist greedy users to update their strategies adaptive to
  - \* spectrum demand and
  - \* supply changes.





# System Model

- Assume all users are selfish and rational, i.e., their objectives are to maximize their own payoffs without causing damage to other users
- They can cheat whenever they think that they can increase their payoff by cheating



# System Model

- Selfishness of both PUs and SUs will prevent them from revealing their private information (e.g., acquisition costs or reward payoffs)
- **GOAL:** A Novel Spectrum Allocation Approach

Not only optimize the spectrum efficiency but also extract private information from selfish users to assist the optimization of spectrum allocation.



# Pricing Game Model

Assume  $J$  PUs, i.e.,  $P = \{P_1, P_2, P_3, \dots, P_J\}$

$K$  SUs, i.e.,  $S = \{S_1, S_2, S_3, \dots, S_K\}$

## ■ For PUs:

Let  $A_i = \{a_i^j\} \quad j \in \{1, 2, \dots, n_i\}$  represent the channels authorized to PU  $P_i$

where  $a_i^j$  represents the channel index in the spectrum pool and

$n_i$  is the total number of channels which belong to user PU  $P_i$ .

A is the set of all channels in the spectrum pool

and

$C_i = \{c_i^j\} \quad j \in \{1, 2, \dots, n_i\}$  represent the acquisition costs of PU  $P_i$ 's channel

where the  $j$ -th element represents the acquisition costs of the  $j$ -th channel in  $A_i$



# Pricing Game Model

- For SUs, the payoff vector is:

$V_i = \{v_i^j\}$   $j \in \{1, 2, \dots, N\}$ , represents payoff of SUs  $S_i$ .

if this user successfully leases the  $j$ -th channel in the spectrum pool.

$N$  : total # of channels



# Pricing Game Model

If PU  $P_i$  reaches agreements of leasing all or part of its channels to SUs, the payoff function of this PU is:

$$U_{P_i}(\varphi_{A_i}, \alpha_i^{A_i}) = \sum_{j=1}^{n_i} (\varphi_{a_i^j} - c_i^j) \alpha_i^{a_i^j}$$

$$\varphi_{A_i} = \{\varphi_{a_i^j}\} \quad \text{for } j \in \{1, 2, \dots, n_i\}$$

$\varphi_{a_i^j}$  : payment that PU  $P_i$  obtains from SU by leasing channel  $a_i^j$  in the spectrum pool

$\alpha_i^{a_i^j} \in \{0, 1\}$  indicates if the  $j$ th channel of PU  $P_i$  has been allocated to SU or not



# Pricing Game Model

The payoff function of SU is

$$U_{s_i}(\varphi_A, \beta_i^A) = \sum_{j=1}^N (v_i^j - \varphi_j) \beta_i^j$$

$\beta_i^j \in \{0, 1\}$  : indicates if SU  $s_i$  successfully leased the  $j$ -th channel in the spectrum pool or not

Strategies for PUs and SUs are defined by alpha and beta



# An Auction Approach

- Selfish users will not reveal their private information to others unless some mechanisms have been applied to guarantee that it is not harmful to disclose the private information.
- Non-cooperative game with incomplete information is complex and difficult to study.
- Auction Theory can be applied to analyze the pricing game.



# Auction Approach

- In auction games, the principles (**auctioneers**) determine resource allocation and prices on the basis of bids from the agents (**bidders**).
- PUs (**auctioneers**) attempt to sell the unused channels to the SUs.
- SUs (**bidders**) who compete with each other to buy the permission of using PUs' channels, by which they may gain extra payoffs for future use.
- Multiple sellers and buyers coexist, which indicates the double auction scenario.





# Static Pricing Game and Competitive Equilibrium

- **Assumptions:** Available channels from the PUs are leased for usage for certain time period  $T$ .  
Also, that the acquisition cost of the PUs and reward payoffs of the SUs remain unchanged

**Goal-** Maximize payoff functions.

Since it is a non-cooperative game with incomplete information, we analyze it from the Game theory point of view.



# Static Pricing- Game Theory Approach

- Pareto optimal outcomes may not be sustained considering the selfishness of the players.
- Further, considering the double auction scenarios (*i.e.*, **maximum payoffs for both PUs and SUs**) of the pricing game, Competitive Equilibrium (CE) is a well-known theoretical prediction of the outcomes.



# Competitive Equilibrium (CE)

- It is the price at which  
the number of buyers willing to buy =  
= the number of sellers willing to sell.
- CE can also be interpreted as where  
the supply and demand match  
(i.e., max payoffs for PUs and SUs)



# Competitive Equilibrium (CE)

DEFINITION: **Supply Function**

Relationship between the acquisition costs of PUs and the # of corresponding channels.

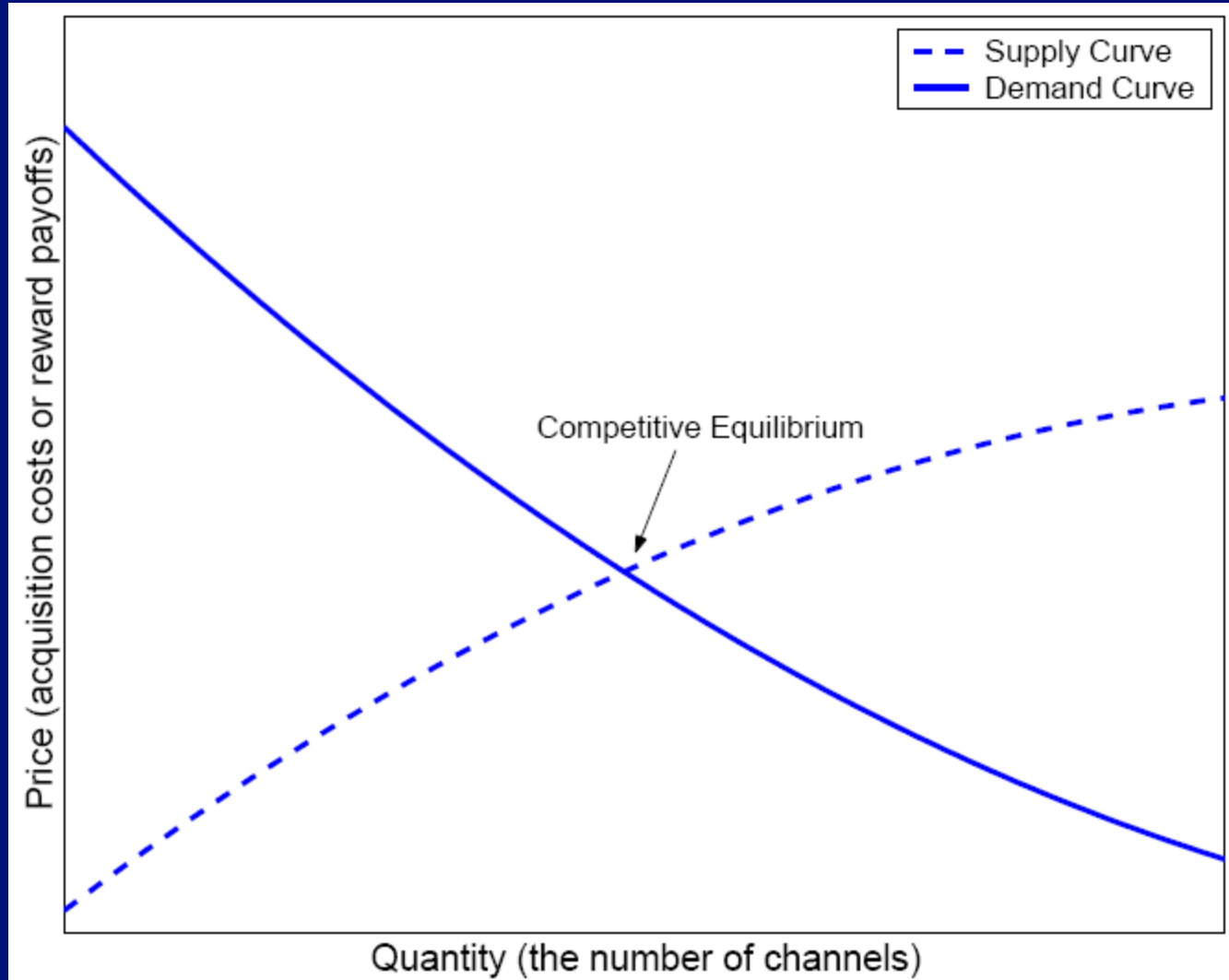
DEFINITION: **Demand Function**

Relationship between the reward payoffs of SUs and the # of corresponding channels.

REMARK: CE is proved to be PARETO Optimal in stationary double action scenarios. To achieve CE in traditional continuous bid/ask interactions among players will involve a great amount of message exchanges and require powerful centralized control which are not applicable to wireless networks (limited BW)



# Competitive Equilibrium- Analysis





# Classical Multistage (Repeated) Game

Considering network dynamics due to mobility, channel variations or wireless traffic variations,

the SUs may have different reward payoffs of acquiring certain channels from PUs at different time stages.

→ Need a multistage game model



# Classical Multistage Game

Objective functions for PUs and SUs are:

$$\tilde{O}(P_i) = \max_{\varphi_{A_i,t}, \alpha_{i,t}^{A_i}} E_{c_i^j, v_i^j} \left[ \sum_{t=1}^{\infty} \gamma^t \cdot U_{P_i,t}(\varphi_{A_i,t}, \alpha_{i,t}^{A_i}) \right]$$

$$\tilde{O}(S_i) = \max_{\varphi_{A_i,t}, \beta_{i,t}^{A_i}} E_{c_i^j, v_i^j} \left[ \sum_{t=1}^{\infty} \gamma^t \cdot U_{S_i,t}(\varphi_{A_i,t}, \beta_{i,t}^{A_i}) \right]$$

$c_i^j, v_i^j$  : considered as random variables in dynamic scenarios.

$\gamma$ : discount factor of the multi-stage game.

$t$ :  $t^{\text{th}}$  stage of multi-stage game





# Problems of Classical Multistage Game

- This model requires information of other users
  - Probability density functions of RVs  $c_{i,j}$  and  $v_{i,j}$
  - Utility functions of other users

## MAJOR DIFFICULTY:

How to efficiently and quickly update the spectrum strategies adapt to the changing network conditions only based on local information

→ not easy to get all necessary information for multistage games





# Problems of Classical Multistage Game

A BELIEF ASSISTED DYNAMIC PRICING APPROACH → DEVELOPED

→ RESPONDS QUICKLY TO NETWORK DYNAMIC CHANGES WITH LIMITED OVERHEAD and MEETS THE CE OUTCOMES



# New Solution: Belief Assisted Dynamic Pricing - Definition of Beliefs

- **Bid Price**- Price offered by SU to PU
- **Ask Price**- Price at which PU is willing to sell to the SU
- It is more efficient to define **one common belief function** based on the publicly observed bid/ask prices than generating specific belief of every other player's private information.



# Belief Assisted Dynamic Pricing

## - Definition of Belief Functions

- We consider the PUs and SUs' beliefs as the ratio of their bid/ask being accepted at different price levels
- The ratio of asks from PUs at price  $x$  that have been accepted by SUs is

$$\tilde{r}_P(x) = \frac{\mu_A(x)}{\mu(x)} \quad \mu_A: \# \text{ of accepted asks at } x; \mu: \# \text{ of asks at } x$$

- The ratio of bids from SUs at price  $y$  that have been accepted by PUs is

$$\tilde{r}_S(y) = \frac{\eta_A(y)}{\eta(y)} \quad \eta_A: \# \text{ of accepted bids at } y, \eta: \# \text{ of bids at } y$$



# Belief Assisted Dynamic Pricing -Characteristics of Double Auction

We need to consider using the historical bid/ask information to build up empirical belief values. (i.e.,  $\tilde{p}_{\text{past}}$ )

- If an ask  $\tilde{x}$  is rejected and  $\tilde{x} < x$ , then the new ask  $x$  will also be rejected;
- If an ask  $\tilde{x}$  is accepted and  $\tilde{x} > x$ , then the new ask  $x$  will also be accepted;
- If a bid  $\tilde{y}$  was made by SU and  $\tilde{y} > x$ , then the ask  $x$  will be accepted because bid  $\tilde{y}$  is higher than what PUs ask for  $x$ .



# Belief Assisted Dynamic Pricing- PU's Beliefs

PU's beliefs for each potential ask at price  $x$  are given by

$$\hat{r}_p(x) = \begin{cases} 1 & x = 0 \\ \frac{\sum_{w \geq x} \mu_A(w) + \sum_{w \geq x} \eta(w)}{\sum_{w \geq x} \mu_A(w) + \sum_{w \geq x} \eta(w) + \sum_{w < x} \mu_R(w)} & x \in (0, M) \\ 0 & x \geq M \end{cases}$$

ask  $w (> x)$  is accepted  $\rightarrow x$  is accepted

bid  $w (> x)$  is made  $\rightarrow x$  is accepted

ask  $w (< x)$  is rejected  $\rightarrow x$  is rejected

$x=0$  means PUs give it free to SUs; All SUs will accept it with prob. 1.

$x \geq M$ : too expensive and SUs will not accept it.

$M$  is a large value so that the asks greater than  $M$  will not be accepted

$\mu_R(w)$ : # of asks at  $w$  that has been rejected at  $w$



# Belief Assisted Dynamic Pricing- SU's Beliefs

SUs' beliefs for each potential bid at price  $y$  are given by

$$\hat{r}_s(y) = \begin{cases} 0 & y = 0 \\ \frac{\sum_{w \leq y} \eta_A(w) + \sum_{w \leq y} \mu(w)}{\sum_{w \leq y} \eta_A(w) + \sum_{w \leq y} \mu(w) + \sum_{w \geq y} \eta_R(w)} & y \in (0, M) \\ 1 & y \geq M \end{cases}$$

bid  $w (< y)$  is accepted  $\rightarrow$   $y$  is accepted  
 ask  $w (< y)$  is made  $\rightarrow$   $y$  is accepted  
 bid  $w (> y)$  is rejected  $\rightarrow$   $y$  is rejected

$\eta_R(w)$ : # of bids at  $w$  that has been rejected at  $w$



# Belief Assisted Dynamic Pricing - Optimization

Define:

ox: current lowest (outstanding) ask ( $\rightarrow$  need to decrease by PUs)

oy: highest (outstanding) bid ( $\rightarrow$  need to increase by SUs)

Modified Belief Functions are:

$$\bar{r}_S(y) = \hat{r}_S(y) \cdot I_{[oy, M)}(y)$$

$$\bar{r}_P(x) = \hat{r}_P(x) \cdot I_{[0, ox)}(x)$$

$$I_{(a,b)}(x) = \begin{cases} 1 & \text{if } x \in (a,b) \\ 0 & \text{otherwise} \end{cases}$$





# Belief Assisted Dynamic Pricing - Optimization

- By using the belief function  $\tilde{r}_p(x)$ , the payoff maximization of selling the  $i$ th PU's  $j$ th channel at price  $x$  can be written as

$$\max_{x \in (0, \infty)} E[U_{P_i}(x, j)]$$

$U$  represents the payoff introduced by allocating the  $j$ th channel when the ask is  $x$

then

$$E[U_{P_i}(x, j)] = (x - c_i^j) \cdot \bar{r}_p(x)$$





# Belief Assisted Dynamic Pricing - Optimization

- The payoff maximization of leasing the  $j$ th channel in the spectrum pool at price  $y$  can be written as

$$\max_{y \in (o_y, o_x)} E[U_{S_i}(y, j)]$$

$U$  represents the payoff introduced by leasing the  $j$ th channel in the spectrum pool when the bid is  $y$

then

$$E[U_{S_i}(y, j)] = (v_i^j - y) \cdot \bar{r}_S(y)$$

REMARK: Solving max values PU and SUs can make optimal decision of spectrum allocation at every stage conditional on dynamic spectrum demand and supply.



# Belief Assisted Dynamic Pricing- Algorithm

## 1. Initialize the user's belief and bids/asks

- PUs: initialize with asks  $x$  as large values close to  $M$  & their Beliefs as small values less than 1, i.e.,  $\rightarrow$  almost 0 (see eq. for  $\hat{r}_p$ )
- SUs: start with bids  $y \sim 0$  Beliefs  $\rightarrow$  almost 0 (see eq. for  $\hat{r}_s$ )

## 2. Belief update based on local information

- Update the PU's and SU's beliefs:  $\tilde{r}_p(x)$  and  $\tilde{r}_s(y)$



# Belief Assisted Dynamic Pricing- Algorithm

## 3. Optimal bid/ask update

- Optimal ask for each PU by solving
- Optimal bid for each SU by solving

$$\max_{x \in (o_y, o_x)} E[U_{P_i}(x, j)]$$

$$\max_{y \in (o_y, o_x)} E[U_{S_i}(y, j)]$$

## 4. Update leasing agreement and spectrum allocation

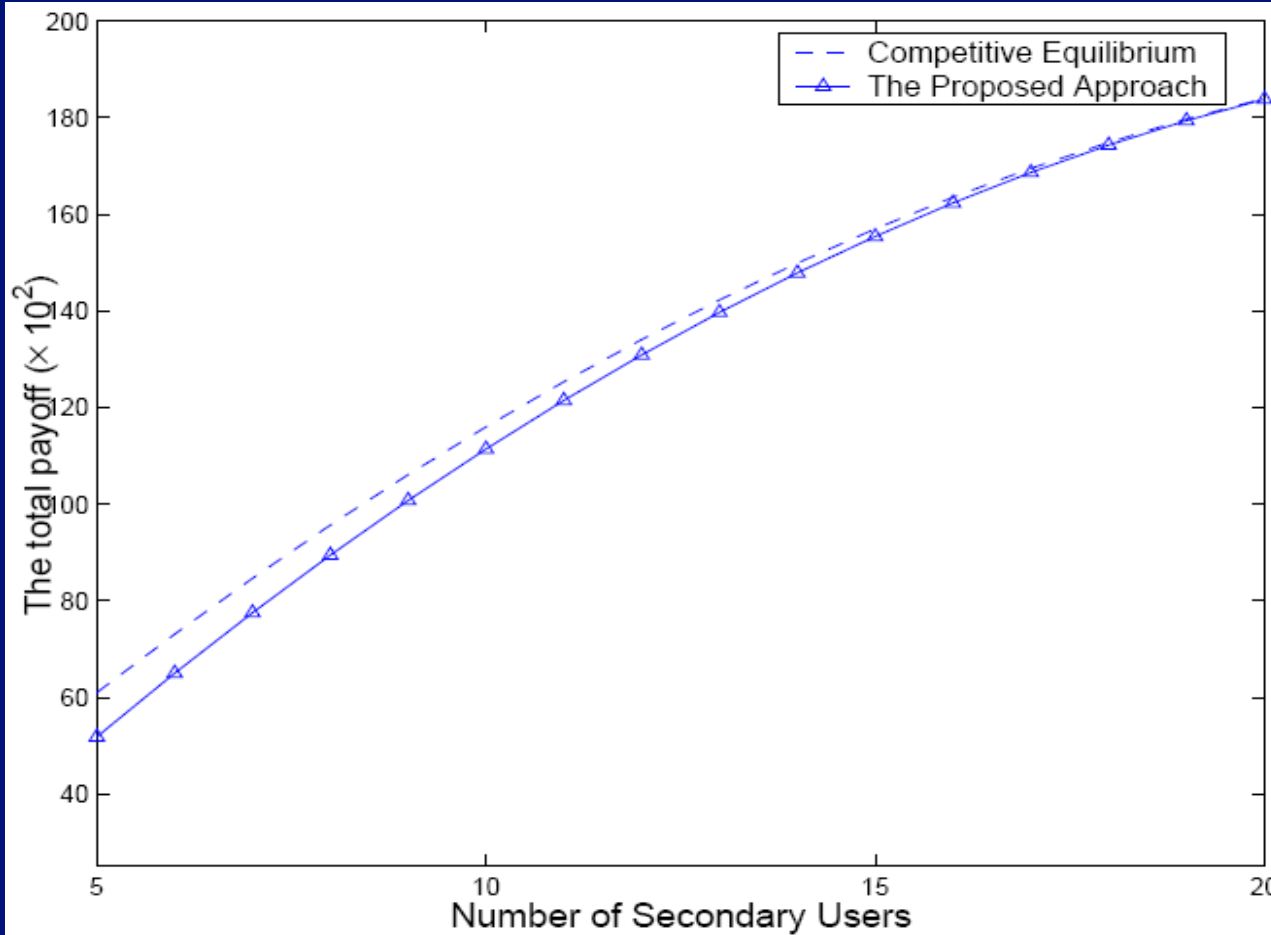
- If outstanding bid  $\geq$  outstanding ask, sign the leasing agreement between corresponding users.
- Update the spectrum pool by removing the assigned channels from the list

## 5. Iteration:

- If the pool is not empty, Go back to step 2



# Comparison with CE





# Advantages

- Performance is close to theoretical optimal solutions.
- When the number of SUs increases, this solution approaches the optimal CE because the belief function reflects the spectrum demand and supply more accurately when more users are involved in spectrum sharing.
- This solution substantially decreases the pricing communication overhead (no co-operation).



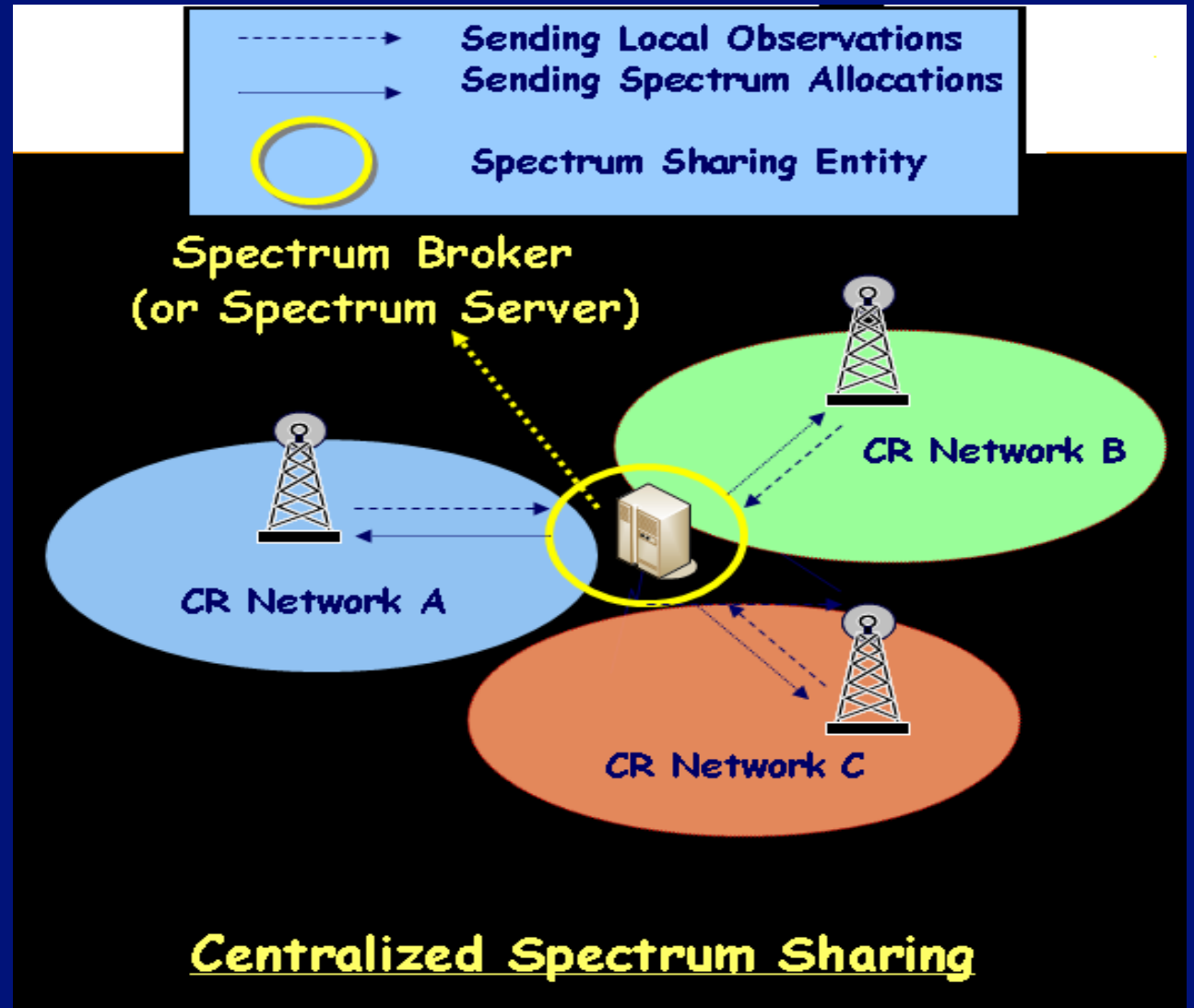
# Disadvantages

- Updating the beliefs creates overhead.



# Internetwork Spectrum Sharing

- Multiple systems are deployed in overlapping locations and spectrum bands
- A centralized system controls the spectrum allocation and access procedures





# Internetwork Spectrum Sharing

## 1. Spectrum Policy Server-based Approach

O. Ileri, D. Samardzija, and N. Mandayam  
IEEE DySPAN 2007

## 2. "Allocation through Auction" Scheme

S. Gandhi, C. Buragohain, L. Cao, H. Zheng, S. Suri,  
IEEE DySPAN 2007





# Spectrum Policy Server (SPS) SCHEME

O. Ileri, D. Samardzija, N. Mandayam,

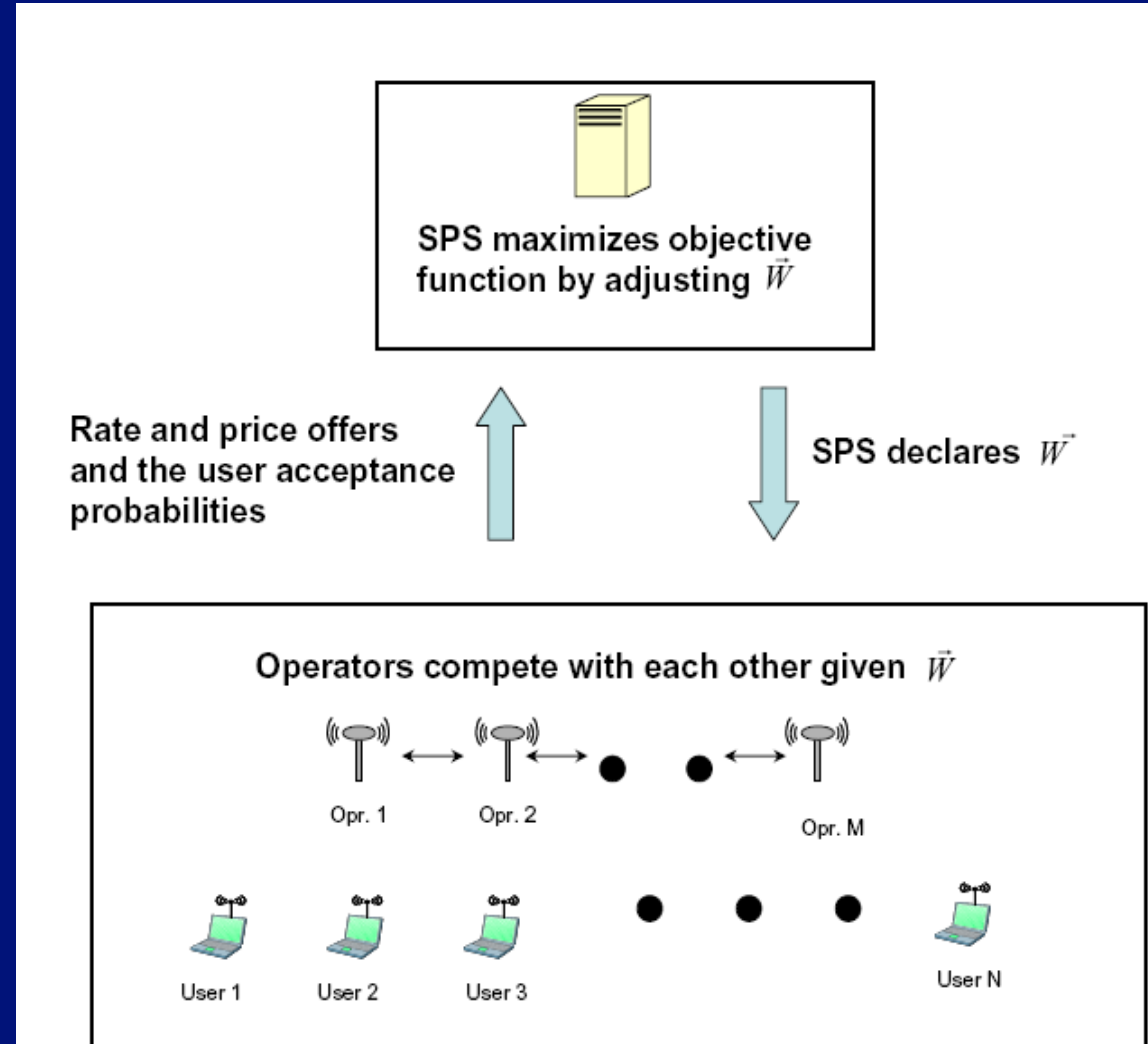
"Dynamic Property Rights Spectrum Access: Flexible Ownership Based Spectrum Management",  
IEEE DySPAN 2007.

- A central spectrum policy server (SPS) is proposed to coordinate spectrum demands of multiple CR operators.
- Operators dynamically compete for customers as well as portions of available spectrum



# Spectrum Policy Server (SPS) Scheme: Concept

- Operators work in mixed-common/property-rights regime under the regulation of an SPS
- Operators compete for both spectrum & potential customers
- Operators in return pay the SPS for the usage of spectrum





# SPS Scheme: Concept

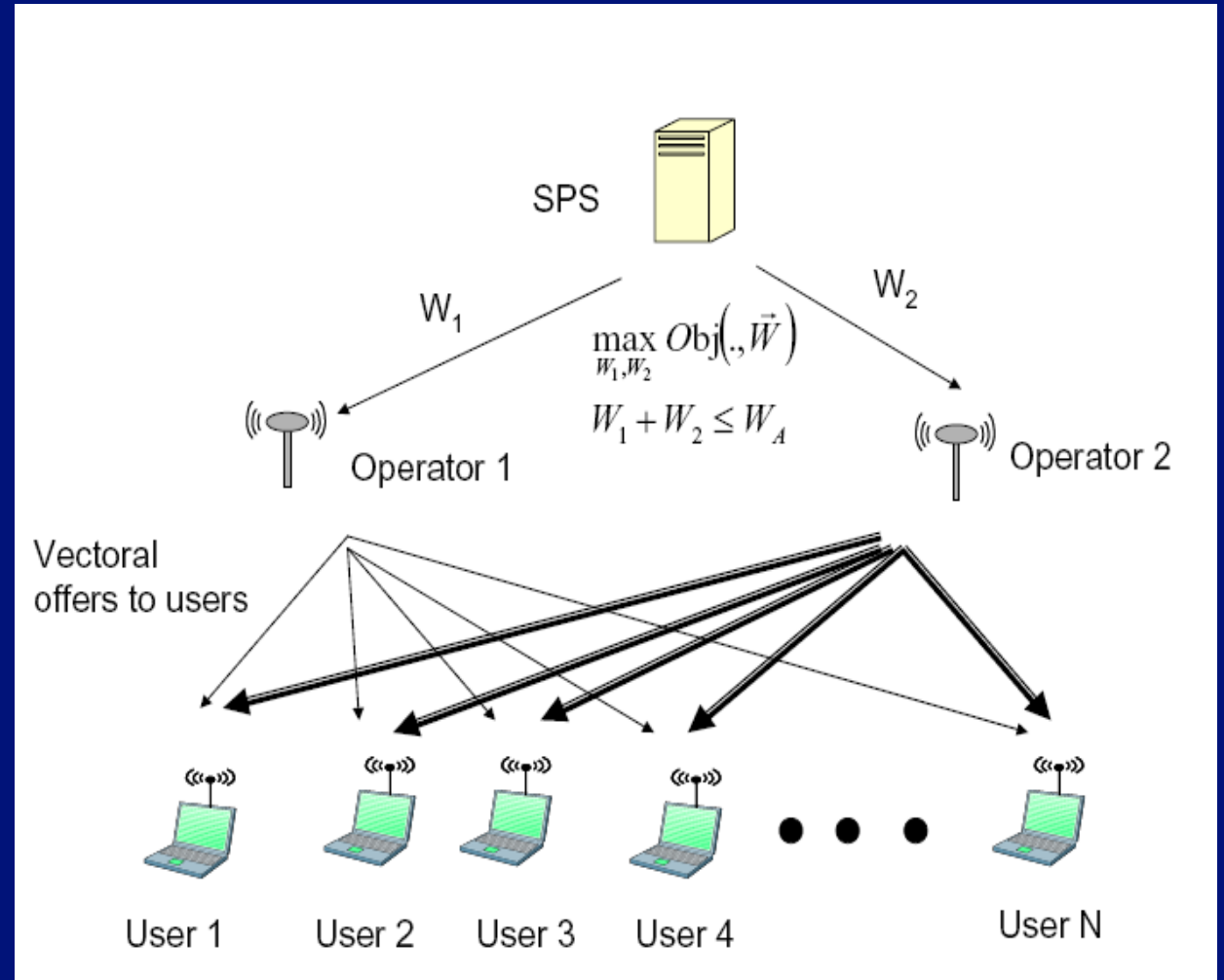
- Operators try to attract customers through “demand responsive pricing”
- Operators offer different spectral efficiency  
 $r$  [bps/Hz], rate  $R$  [bps] at total price of  $P$  [\$]
- Customer responses to each offer through an acceptance probability  $A(R, P)$
- $A(R, P)$  reflects the willingness of customers to buy the offered service
- Various operators compete to ensure that the users accept their offer with the highest probability



# SPS Scheme: Model

## Assumptions:

- A limited region is under the control of a local SPS
- Two operators, Operator 1 and Operator 2, provide services to a user
- SPS keeps track of the vacant spectrums  $W_A$





# SPS Scheme: Model

The model can be explained more elaborately breaking it into sub-models:

- Users' Acceptance sub-model
- Operator Profit sub-model
- Operator Interaction sub-model



# SPS Scheme: Model

## Users Acceptance Sub-Model

■ Acceptance probability  $A(u, P)$  is a function of  $u$  &  $P$

\*  $u$  = Utility

\*  $P$  = Price

$A$  is an increasing function dependent on  $u$ .

$A$  is a decreasing function dependent on  $P$ .

$$\frac{\partial A}{\partial u} \geq 0, \quad \frac{\partial A}{\partial P} \leq 0$$

$$\forall P > 0, \quad \lim_{u \rightarrow 0} A(u, P) = 0$$

$$\lim_{u \rightarrow \infty} A(u, P) = 1$$

$$\forall u > 0, \quad \lim_{P \rightarrow 0} A(u, P) = 1$$

$$\lim_{P \rightarrow \infty} A(u, P) = 0$$



# SPS Scheme: Model

## Users Acceptance Sub-model

$$A(u, P) = 1 - e^{-Cu^\mu P^{-\epsilon}}$$

$$u(R) = \frac{(R/K)^\xi}{1 + (R/K)^\xi}$$

- $\mu$  is the utility sensitivity of the user
- $\epsilon$  is the price sensitivity
- $C$  is a constant
- $u$  is parameterized as a function of offered rate  $R$  only
- Role of transmit power utility is ignored
- $K$  and  $\xi$  are constants

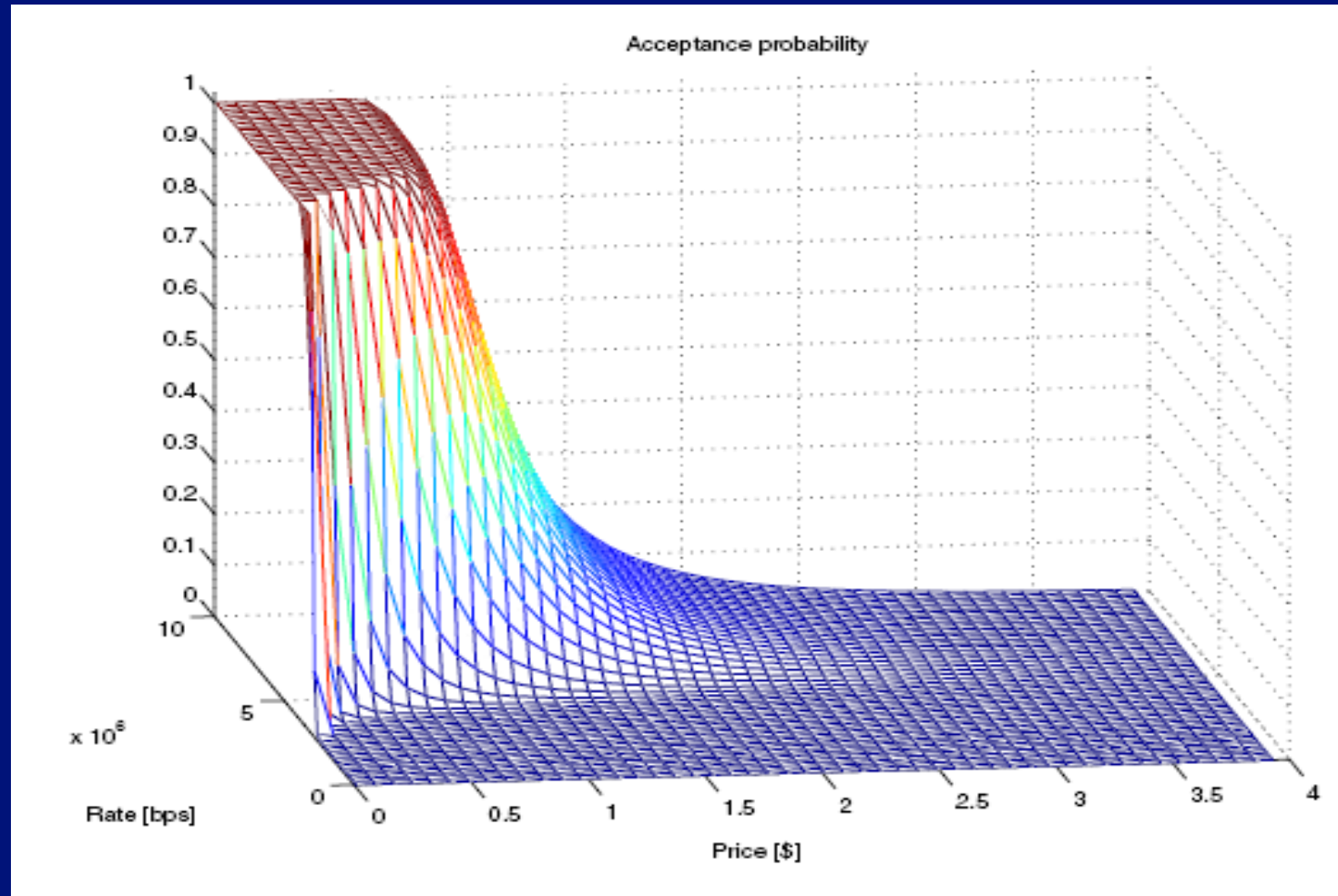




# SPS Scheme: Model

## Users Acceptance Sub-model

- It can be seen that the model chosen here obeys the Law of diminishing returns
- Utility function is plotted by choosing  $K=5 \times 10^6$ ,  
 $\zeta = 10$ ,  $C = 1$ ,  $\mu = 4$







# SPS Scheme: Model

## Operator Profit Sub-model

### ■ Assumption:

- Service is provided if revenue generated is high enough to compensate for the costs incurred
- Operators are able to provide spectral efficiencies of  $r_i$  [bps/Hz] to a specified user
- $r_i$  depends on the following parameters:
  - Technology used by the operator
  - Density of the base stations belonging to the operator
  - Location of the user



# SPS Scheme: Model

## Operator Profit Sub-Model

- Required bandwidth is  $W_i(R_i)$  to offer rate  $R_i$
- $W_i$  inversely proportional to the spectrum efficiency  $r_i$

$$W_i(R_i) = R_i/r_i$$



# Operator Profit Model: Operator's Perspective

$Q_i(R_i, P_i) = P_i - F_i - V_i R_i / r_i$       the profit for operator  $i$

$\bar{Q}_i(R_i, P_i) = A(R_i, P_i) Q_i(R_i, P_i)$       the expected profit for operator  $i$

$P_i$  : the offered price by operator  $i$

$R_i$  : the offered rate by operator  $i$

$F_i$  [\$] : the fixed cost incurred by operator  $i$

$V_i$  [\$ / Hz] : the price per unit bandwidth that the SPS charges operator  $i$

$r_i$  : spectral efficiency of operator  $i$  ( bps/Hz)

$A$  : Acceptance Probability

## NOTE:

- For fixed  $r_i$ , the acceptance probability  $A(R_i, P_i)$  is increasing in  $R_i$  and decreasing in  $P_i$
- Relation is opposite for profit  $Q_i(R_i, P_i)$



# SPS Scheme: Model

## Operator Interaction Sub-Model

- Operators are competing for resources and the user preferences
  - This can be represented as a game  $G = [N, \{S_i\}, \beta_i]$ 
    - $N = \{1, 2\}$  is the index set of the players (operators)
    - $S_i$  = strategy space available to Operator  $i$
    - $\beta_i$  = expected profit associated with the operator



# SPS Scheme: Model

## Operator Interaction Sub-model

Strategy space  $S_i$  for Operator  $i$  consists of all  $(R, P)$  pairs which satisfy the bandwidth constraint

$$S_i = \{ \forall (R, P) \mid (F_i + V_i R_i / r_i) \leq P \leq P_{\max}; \quad 0 \leq R \leq W_A r_i \}$$

$W_A$ : Available Bandwidth



# SPS Scheme: Model

## Operator Interaction Sub-model

The resulting expected profit  $\beta_i$  of operator  $i$   
(given the strategy of the opponent operator  $j$ )

$$\beta_i = \begin{cases} 0 & \text{if } A(R_i, P_i) < A(R_j, P_j) \\ 0.5\bar{Q}_i(R_i, P_i) & \text{if } A(R_i, P_i) = A(R_j, P_j) \\ \bar{Q}_i(R_i, P_i) & \text{if } A(R_i, P_i) > A(R_j, P_j) \end{cases}$$



# Operator Interaction Sub-Model as a Non-Cooperative Game

- Two offers  $(R_1, P_1)$  and  $(R_2, P_2)$  are made by Operators 1 and 2
- The offer for which  $A(R, P)$  is lower is ignored by the user and the other offer is accepted with the associated acceptance probability
- If  $A(R_1, P_1) = A(R_2, P_2)$  each offer is equally likely to be accepted



# SPS Scheme: Model

## Operator Interaction Sub-model

■ Non-cooperative operator game is given by

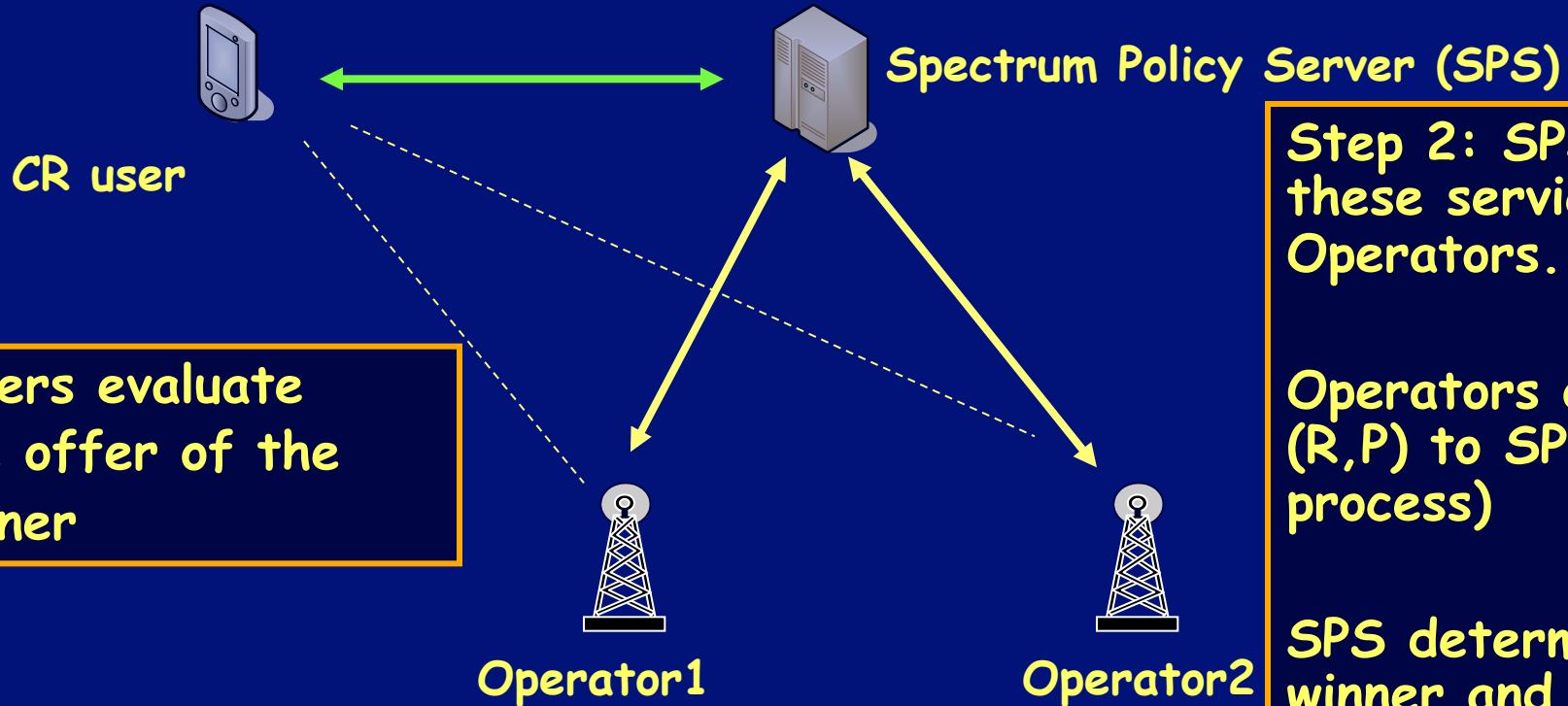
$$\max_{(R_i, P_i) \in \mathcal{S}_i} \beta_i(R_i, P_i, R_j, P_j) \quad \text{for } i, j \in \{1, 2\}$$





# SPS Scheme: Mediator for Iterative Bidding

Step 1: Users send specific (e.g., location) information and service requests to the SPS



Step 3: Users evaluate the offer of the winner

Step 2: SPS announces these service requests to Operators.

Operators offer their (R,P) to SPS. (Bidding process)

SPS determines the winner and announces it to the operators & users



# SPS Scheme: Mediator for Iterative Bidding

- Operators make an offer in each iteration
- Operators make the offer such that the related  $A(R,P)$  is greater than its opponent's offer while maximizing the expected profit
- Iterative bidding is initialized by allowing the operators to choose their service offers without consideration of the opponent strategy



# SPS Scheme: Mediator for Iterative Bidding

- Iteration process is terminated when a zero value for expected profit is declared by at least one operator
- Achieves a Nash equilibrium of the operator game
- Opportunity to offer service to the user is then given to the operator that wins
- Winning operator uses its most recent bid  $(R_{\text{winner}}, P_{\text{winner}})$ , as a service offering to the user



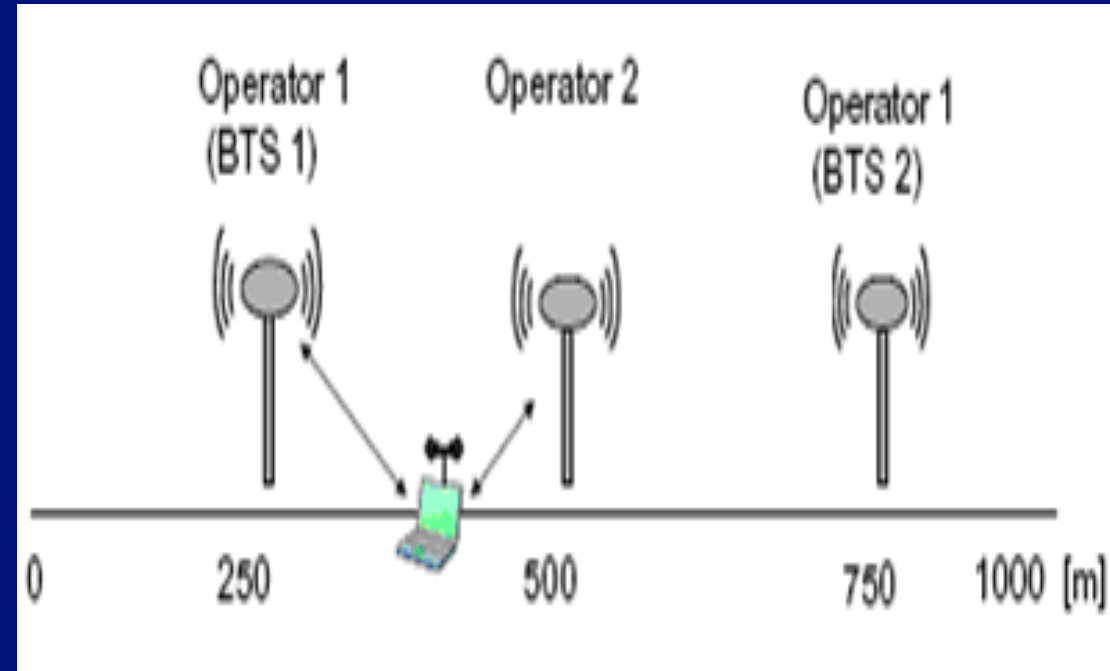
# SPS: Numerical Results for Single User System

O. Ileri, D. Samardzija, N. Mandayam,

"Dynamic Property Rights Spectrum Access: Flexible Ownership Based Spectrum Management",  
IEEE DySPAN 2007.

## Assumptions:

- Both operators use the same technology
- Both have different infrastructure density
- The associated fixed cost for Operator 1 will be twice the fixed cost of Operator 2, i.e.,  $F_1 = 2F_2$
- SPS will be charging both operators at the same variable cost rate  $V$  [\$/Hz].





# SPS: Numerical Results for Single User System

Spectral efficiency for Base Station  $k$  and user mobile terminal is

$$r_k = \log_2 \left[ 1 + \frac{P_s}{N_o} \left( \frac{d_k}{L/4} \right)^{-2} \right]$$

where  $P_s$  is the signal power

$N_o$  is the AWGN variance

$d_k$  is the distance between the BS  $k$  and user, and

$L$  is the total length of the linear region

Path loss coefficient is  $-2$ .

- Operator 1 always selects a BS that provides higher spectral efficiency to serve the user



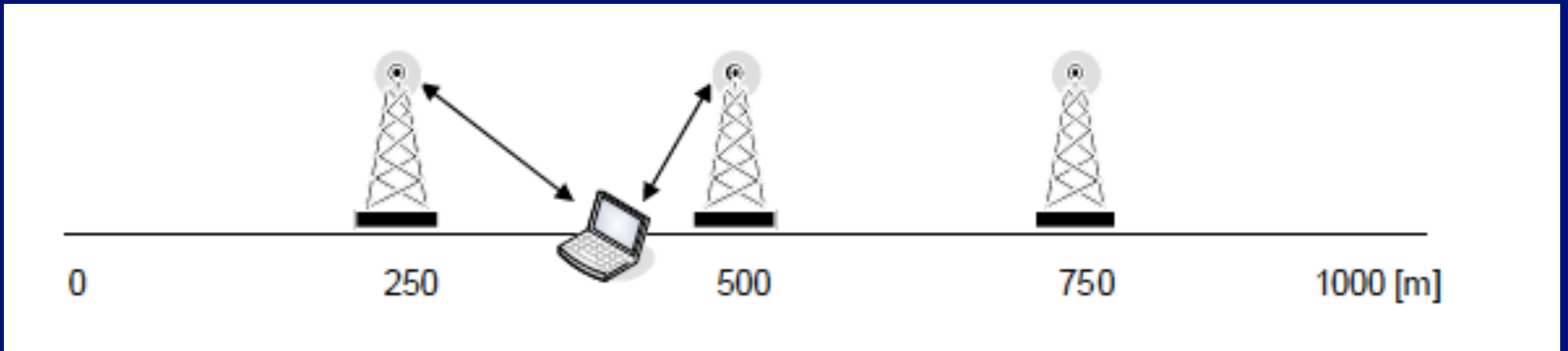
# SPS: Numerical Results for Single User System

- Cost structure is characterized by the ratio  $\eta = V/F$ 
  - $V =$  variable cost
  - $F =$  fixed cost per base station  $F = F_1/2 = F_2$  [\$]
- Lower values of  $\eta$  correspond to the spectrum being less expensive than the infrastructure
- Furthermore the absolute values for  $F$  and  $V$  are selected such that  $F + V \cdot W_A = 2\$$ , while  $F_1 = 2F$  and  $F_2 = F$

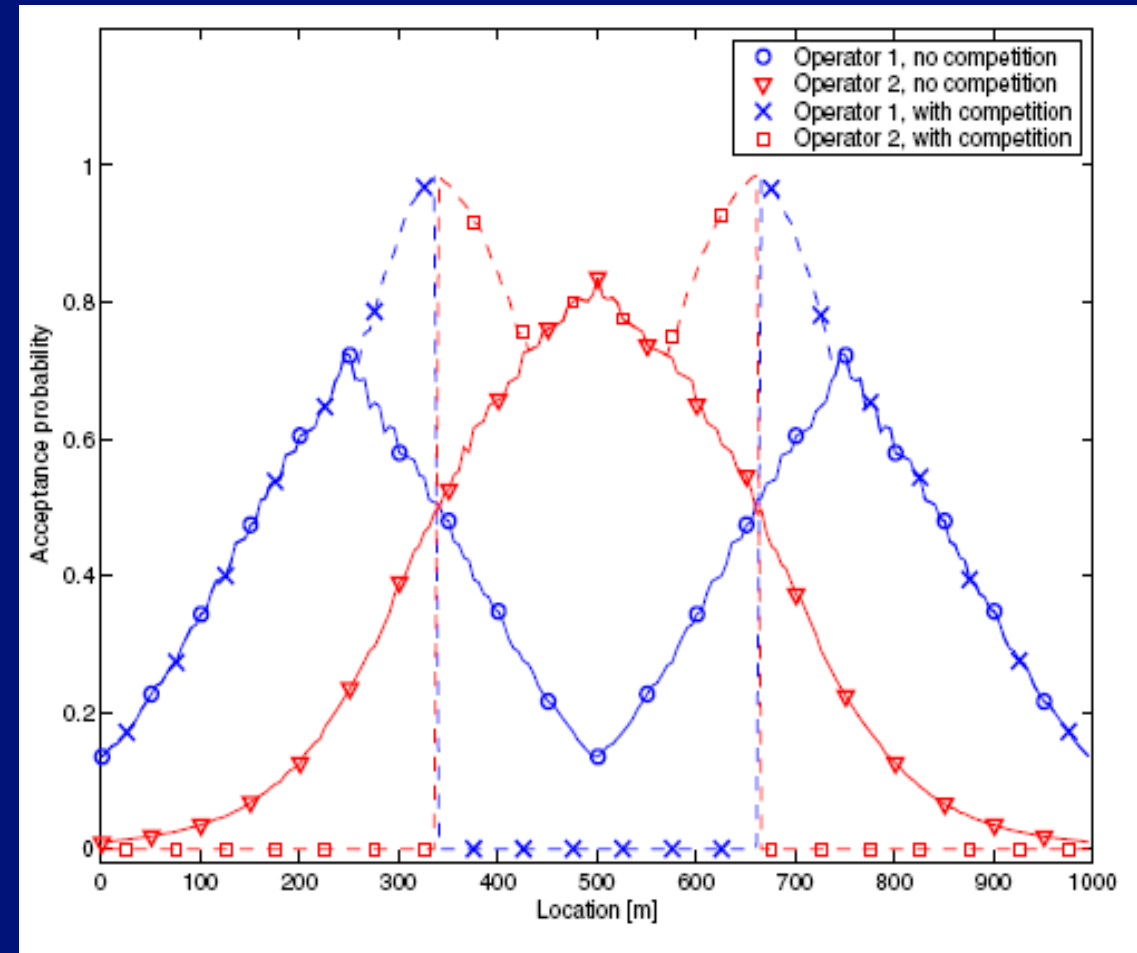
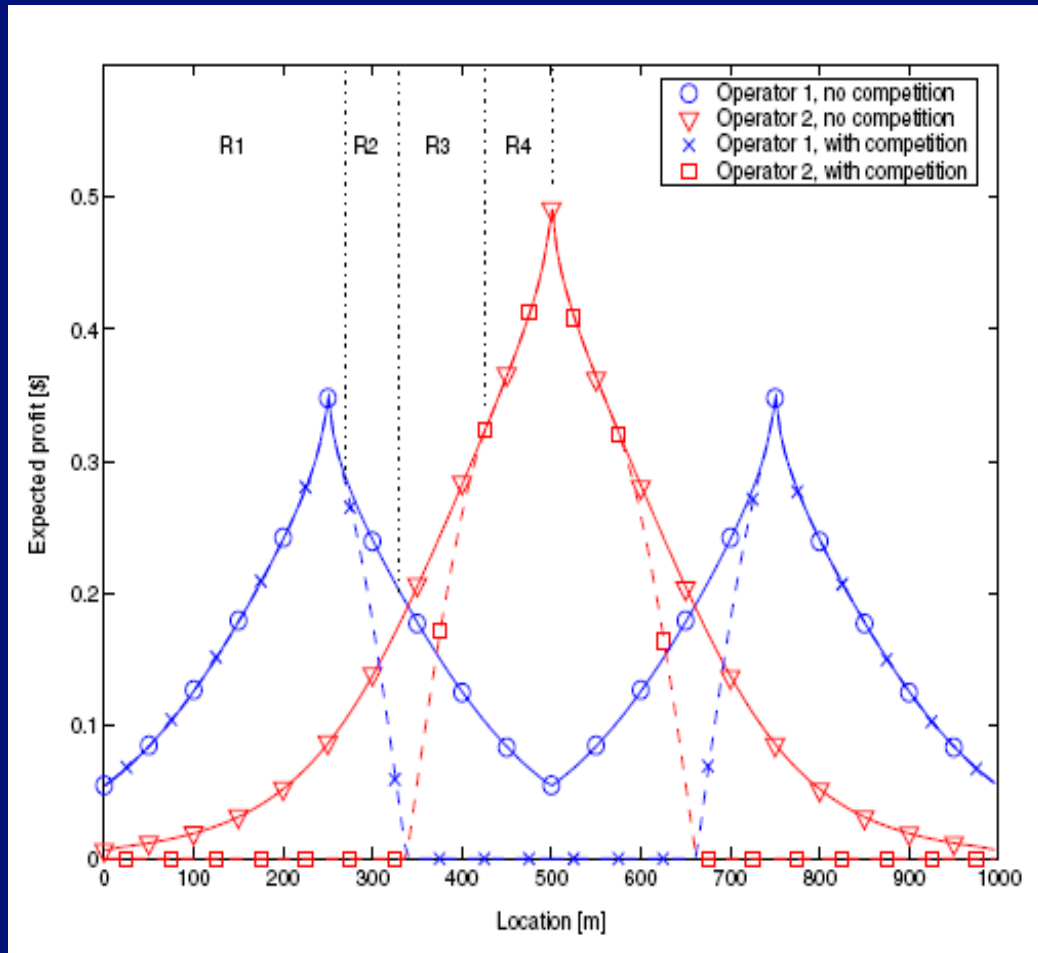


# EXAMPLE

## Single user and 2 Operators



# SPS: Numerical Results for Single User System

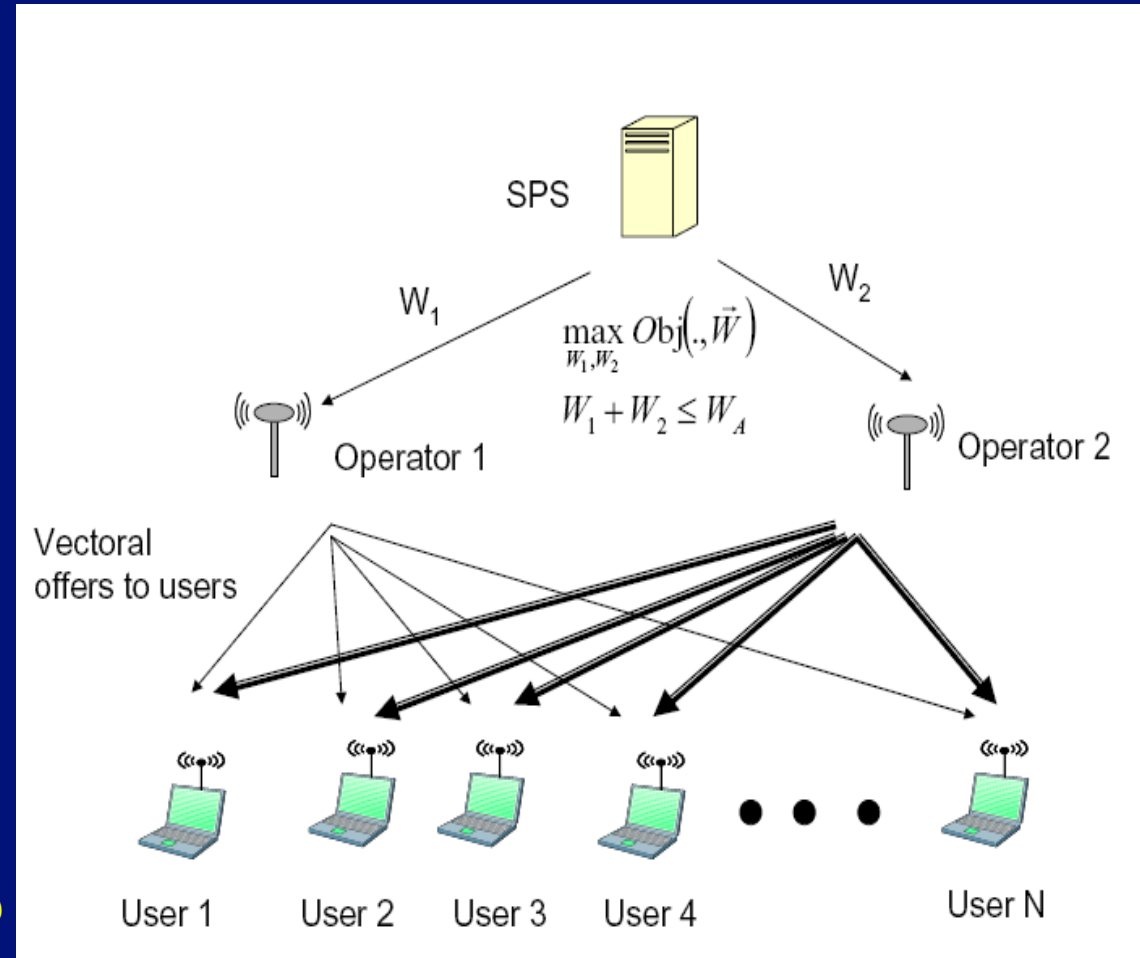






# SPS Scheme: Multi-User System

- Two operators are competing for spectrum and  $N > 1$  users
- Operators compete for each user individually
- Sessions corresponding to the user-operator pairs are held in non-overlapping spectrum portions thus leading to interference free transmission
- Total available bandwidth  $W_A$  is partitioned among these sessions by the SPS
- Total available bandwidth  $W_A$  is sufficient to support all the winning offers
- Operators, if not assisted by SPS, can not keep track of the winning bids





# SPS Scheme: Multi-User System

## Resource Allocation Model

- SPS maximizes its expected revenue  $R_{SPS}(\cdot, W)$

$$W = [W_1 W_2 \dots W_N]^T \quad (\text{BW allocation vector})$$

- Operators competing for user  $n$  must not make offers that require BWs greater than  $W_n$
- SPS maximizes its expected revenue subject to the constraint that the total allocated BW does not exceed the total available BW  $W_A$



# SPS Scheme: Multi-User System

## Resource Allocation Model

- SPS optimization problem is expressed by

$$\max_W R_{SPS}(\cdot, W) \quad s.t. \quad \sum_{n=1}^N W_n \leq W_A$$

- Expected revenue  $R_{SPS}$  is defined as the sum of the expected BW utilizations of the users scaled by the variable cost per BW  $V$  [\$/Hz]
- Function of the BW allocation vector  $W$  as well as the user locations and cost parameters, i.e.,



# SPS Scheme: Multi-User System Resource Allocation Model

$$R_{SPS}(r_1, r_2, F_1, F_2, V, W) = \sum_{n=1}^N V A_n^f(., W) W_n^f(., W)$$

- $r_1, r_2$  are the N dimensional spectral efficiency vectors for Operators 1 and 2.
- Each element of the vector denotes the service spectral efficiency the operators enjoy while providing service to a specific user
- $F_1$  and  $F_2$  denote the fixed costs of Operators 1 and 2
- $A_n^f$  and  $W_n^f$  refer to the winning bid acceptance probability and bandwidth usage achieved
- $W_n^f$  depends on the winning rate offer and the winning operator's spectral efficiency

$$W_n^f = R_{\text{winner}} / r_{\text{winner}}$$



# Conclusions

- Operator bidding scheme achieves high throughput leading to higher revenue for the operators, as well as a lower price for the users according to their requirements.
- This work opens a new perspective by incorporating competition for users as well as the spectrum in CR networks.



# Spectrum Allocation and Service Pricing through Auctions

S. Sengupta, M. Chatterjee, and S. Ganguly,

"An Economic Framework for Spectrum Allocation and Service Pricing with Competitive Wireless Service Providers"

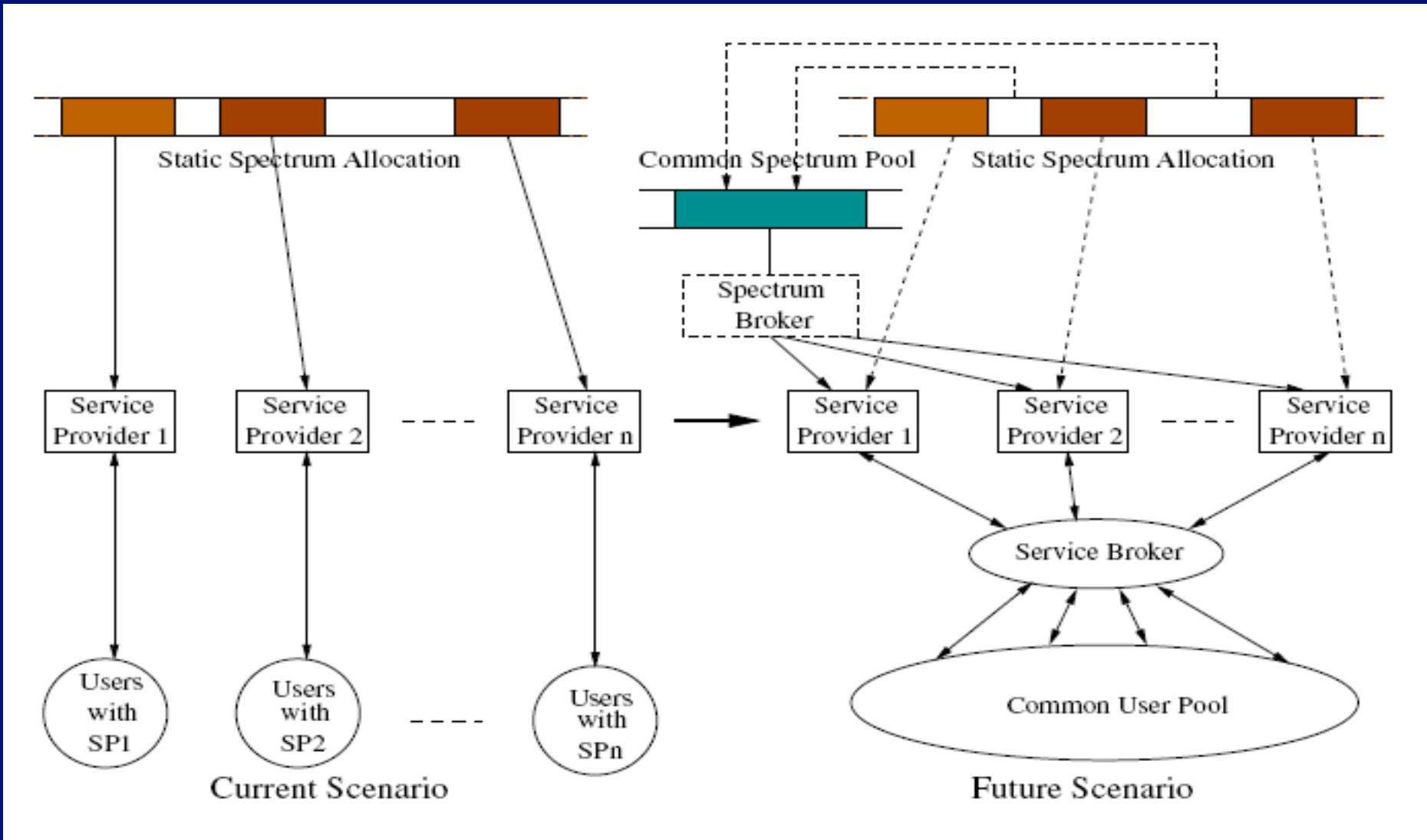
Proc. IEEE DySPAN 2007.

- Two-tier trading system based on a winner determining sealed-bid knapsack auction mechanism for

- Dynamic Spectrum allocation to WSPs based on their bids

DYNAMIC PRICING STRATEGY based on GAME THEORY to capture the interaction of users (their COIs) with the WSPs BOTH try to max their respective utilities.

# "Allocation through Auction": Concept





# "Allocation through Auction": Concept

## ■ Two major units:

### - Spectrum Broker:

Responsible to allocate spectrum to providers from common pool, whenever they require

### - Service Broker:

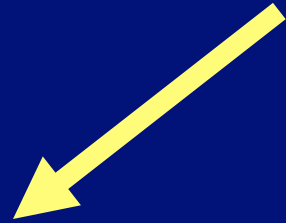
Technically known as Mobile Virtual Network Operators.

It acts as an interface between providers and the users.





# "Spectrum Allocation through Auction"



Asynchronous Allocation



Synchronous Allocation



# Spectrum Allocation through Auction

Asynchronous Allocation	Synchronous Allocation**
Whenever service provider requires a band, it makes a request to a spectrum broker	Spectrum allocation or de-allocation is done at a fixed time and is repeated after a fixed interval of time
If available broker assigns a chunk of spectrum; Leased period depends on the requirement of the provider	Lease time is a discrete unit short span of intervals
Provider is able to cater users and when a request comes in	Provider has to wait until next interval for spectrum allocation takes place
Provider takes the spectrum on lease for exactly the same duration for which it needs the channel	Provider may require to keep the channel at most for one more interval than it is required
Allocation scheme is not good from the brokers point of view	Spectrum broker is able to compare all the requests to maximize revenue



# "Spectrum Allocation through Auction"

## Alternative Classification

### Pricing Strategies

Depending on the total demand of spectrum from service providers;  
If total demand of spectrum does not exceed the spectrum available in CAB



Service Provider  
Dominant Strategy

Spectrum Broker  
Dominant Strategy

- Providers advertise the price they are willing to pay

- Broker advertises the price per unit spectrum; service providers respond by deciding on the amount of spectrum they can acquire



# "Spectrum Allocation through Auction"

- If the total demand of spectrum exceeds the total spectrum available in the CAB (often the case) →

one of the strategies for spectrum broker is to put up the spectrum for bids and decide on the allocation based on the bids →

**ADOPT AN AUCTION MODEL!!!**



# "Spectrum Allocation through Auction"

## ■ Assumption:

- All bands of the total available spectrum are homogenous

## ■ Spectrum broker is the seller and determination of winners heavily depends on the strategy adopted by broker

## ■ Three important issues regarding the design of the auction

- How to maximize revenue generated from bidders?
- How to entice bidders by increasing their probability of winning?
- How to prevent collusion among providers?



# WHAT IS COLLUSION ?

- An agreement between two or more parties, sometimes illegal and therefore secretive, to limit open **competition by deceiving, misleading, or defrauding others** of their legal rights, or to obtain an objective forbidden by law typically by defrauding or gaining an unfair advantage.
- It is an agreement among firms or individuals to divide a market, set prices, limit production or limit opportunities.
- It can involve "**wage fixing, kickbacks, or misrepresenting** the independence of the relationship between the colluding parties".
- In legal terms, all acts affected by collusion are considered **void**.



# COLLUSION IS ILLEGAL !!

- Collusion is largely illegal in the US, Canada and most of the EU due to competition/antitrust law, but implicit collusion in the form of price leadership and tacit understandings still takes place.
- Examples in the USA:
  - Market division and price-fixing among manufacturers of heavy electrical equipment in the 1960s, including General Electric.
  - An attempt by Major League Baseball owners to restrict players' salaries in the mid-1980s.
  - Price fixing within food manufacturers providing cafeteria food to schools and the military in 1993.
  - Market division and output determination of livestock feed additive, called lysine, by companies in the US, Japan and South Korea in 1996.





# COLLUSION - BARRIERS

- **Number of Firms:** As the number of firms in an industry increases, it is more difficult to successfully organize, collude and communicate.
- **Cost and Demand Differences between Firms:** If costs vary significantly between firms, it may be impossible to establish a price at which to fix output.
- **Cheating:** There is considerable incentive to cheat on collusion agreements; although lowering prices might trigger price wars, in the short term the defecting firm may gain considerably. This phenomenon is frequently referred to as "chiseling".
- **Potential Entry:** New firms may enter the industry, establishing a new baseline price and eliminating collusion (though anti-dumping laws and tariffs can prevent foreign companies entering the market).
- **Economic Recession:** An increase in average total cost or a decrease in revenue provides incentive to compete with rival firms in order to secure a larger market share and increased demand.





# "Allocation through Auction" Scheme: Formulation of Auction Rules

- This is analogous to "Knapsack problem"
  - **Knapsack problem:**  
Goal is to fill a sack of finite capacity such that its value is maximized
- Sack represents the finite spectrum band in the CAB which is to be allocated to WSPs such that the revenue generated from the WSPs becomes maximum
- "Winner Determining Sealed Bid Knapsack Auction" is proposed
  - Sealed bid because provider does not have knowledge of other bidders' price and quantity



# "Allocation through Auction" Scheme: Formulation of Auction Rules

Consider  $L$  WSPs (bidders) who compete for total spectrum  $W$

ALL SERVICE PROVIDERS SUBMIT THEIR DEMANDS at the same time in a sealed bid manner since sealed bid auction performs well.

Each WSP has knowledge about its own bidding quantity and bidding price but have no idea about other's quantity and price.



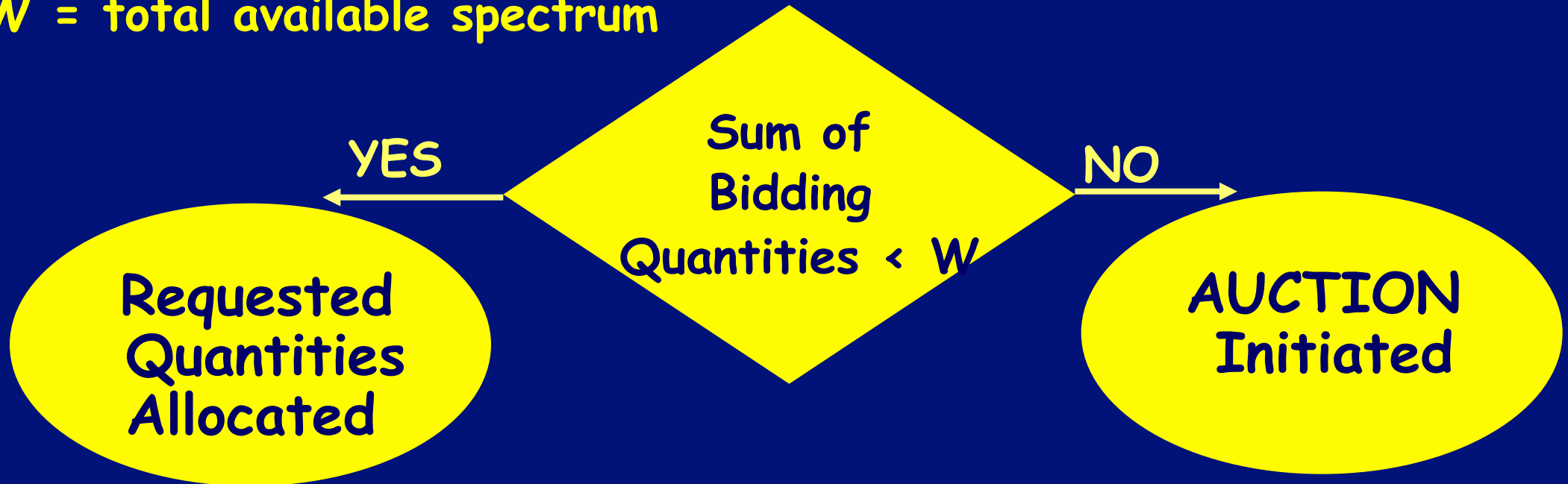
# "Allocation through Auction" Scheme: AUCTION PROCEDURE

- Service provider  $i$ 's strategy is denoted by:

$$q_i = \{w_i, x_i\}$$

where  $w_i$  is the spectrum requested and  $x_i$  is the price the service provider is willing to pay

- $W$  = total available spectrum





# "Allocation through Auction" Scheme

## Goal:

To solve winner-determination problem such that the spectrum broker maximizes revenue (total price) by choosing a bundle of bidders ( $q_i$ ) subject to the condition that the total spectrum allocated does not exceed  $W$ , i.e.,

$$* \text{maximize } \sum x_i \text{ such that } \sum w_i \leq W \text{ over } i=1, \dots, L$$



# "Allocation through Auction":

**Bidder Strategies** for first and second price bidding schemes  
under knapsack problem

- In first price auction, bidders with the winning bids pay their winning bids
- In second price, bidders with the winning bids do not pay their winning bid but pay the second highest bid.



# "Allocation through Auction": Bidder Strategies

- Each bidder submits its demand tuple  $q_i$
- Optimal Allocation is done by considering all  $q_i$  and the result is denoted by  $M$
- $M$  is a set of all winning demand tuples (subject to the condition\* above)
- Aggregate bid can be computed by  $\sum x_i$  (summing all the bids from bidder) (for  $i \in M$ )
- Suppose particular bidder  $j$  was allocated spectrum and belongs to  $M$



# "Allocation through Auction": Bidder Strategies

- Then the aggregate bid generated from optimal allocation  $M$  minus the bid of bidder  $j$  is  $\rightarrow \sum x_i$  where  $i \neq j, i \in M, j \in M$
- Now consider that bidder  $j$  does not exist in the original bid list and auction is among  $L-1$  bidders.
- Now let the optimal allocation denoted by  $M^*$   
 $\rightarrow \sum x_i$  where  $i \neq j, i \in M^*, j \notin M^*$
- Therefore, the minimum winning bid of bidder  $j$  must be at least greater than  
 $X_j = \sum x_i \text{ (for } M^*) - \sum x_i \text{ (for } M)$



# "Allocation through Auction": Bidder Strategies

- If  $x_j > X_j$ , the bidder  $j$ 's request is granted
- If  $x_j < X_j$ , the bidder  $j$ 's request is rejected
- If  $x_j = X_j$  the bidder  $j$  is indifferent between winning and losing

## REMARK:

Former eq. gives the winning bid for  $j$  but it may happen that  $j$  may not afford the price to pay.

There exists a price threshold (bidder's reservation price) beyond which a bidder is simply unwilling to pay.





# "Allocation through Auction" Bidder's Price Reservation

- Bidder Reservation Price is the max price that a bidder is willing to pay
  - This depends on the revenue which a bidder can generate from it
  - Let  $R$  be the total revenue generated
  - $R_{\text{static}}$  goes towards fixed cost
  - $R_{\text{dynamic}} = R - R_{\text{static}}$ , is the reservation price (max price) that the provider can afford for the extra spectrum from CAB



# "Allocation through Auction": Bidder's Price Reservation

- Two price bidding schemes under knapsack model

- **First Price Auction:**

- Winning bidder pays with the bid it offered

- **Second Price Auction:**

- Winning bidder does not pay with the bid it offered but with the second highest bid



# "Allocation through Auction": Bidder's Price Reservation

## First Price Auction:

- Price paid = bid price
- Reservation price is the above bidding threshold
- Expected payoff obtained by  $j^{\text{th}}$  bidder is  $E_j$

$$E_j = r_j - x_j$$

where  $r_j$  = reservation price,  $x_j$  = bid price

- To increase payoff  $E_j > 0$
- Condition: To win the bid  $x_j > X_j$



# "Allocation through Auction": Bidder's Price Reservation

## Proposed Second Price\* Auction:

- Dominant strategy is to bid at reservation price
- Bidder  $j$  will win if bid is at least  $X_j$
- Then according to policy,  $j$  needs to pay the threshold price than the second highest price as in classical case, i.e.,  $X_j$
- The max expected Payoff for  $j$ -th bidder is  $E_j = r_j - X_j$   
i.e., bidding any other price (higher or lower) than its reservation price  $r_j$  will not increase the payoff.
- Result  $\rightarrow$   $j$ -th bidder's true bid is its reservation price  $r_j$



# "Allocation through Auction": Service Provisioning using Game Theory Model

- "Always greedy and profit seeking" model exists between WSPs and users
- User selects service providers depending on profit they obtain for the price they pay
- **Conflict model:**
  - Users are the potential service buyers
  - User compares offers in terms of QoS and price
  - User does not have any information about other users as the price is dynamic
  - So, every user has its own strategy
  - Hence, we assume every user is selfish
  - Problem is modeled as a non-cooperative game
- **Assumptions:**
  - User device can connect to any provider
  - WSPs are selected on session by session basis

(See the paper for the Game Theory Model details...)



# "Allocation through Auction": Game Theory Model

## ■ Decision model:

- The decision problem is to select the best service provider for a session request
- How to select best service?
  - Based on QoS provided
  - Quality of service depends on traffic load and the pricing strategies
- Best service provider must be selected by performing **cost benefit analysis**
- But an equilibrium can exist where no user would like to change the strategy unilaterally (**Nash Equilibrium**)
- Provider is unaware of other providers' strategy
- So, provider needs to optimize price such that it is able to make profit even at a low price



# "Allocation through Auction": Simulation Results

- **For Spectrum Auctioning**

- **Factors considered for demonstrating performance of Knapsack Auction:**

- Revenue generated by spectrum broker
- Total spectrum usage
- Probability of winning for bidders

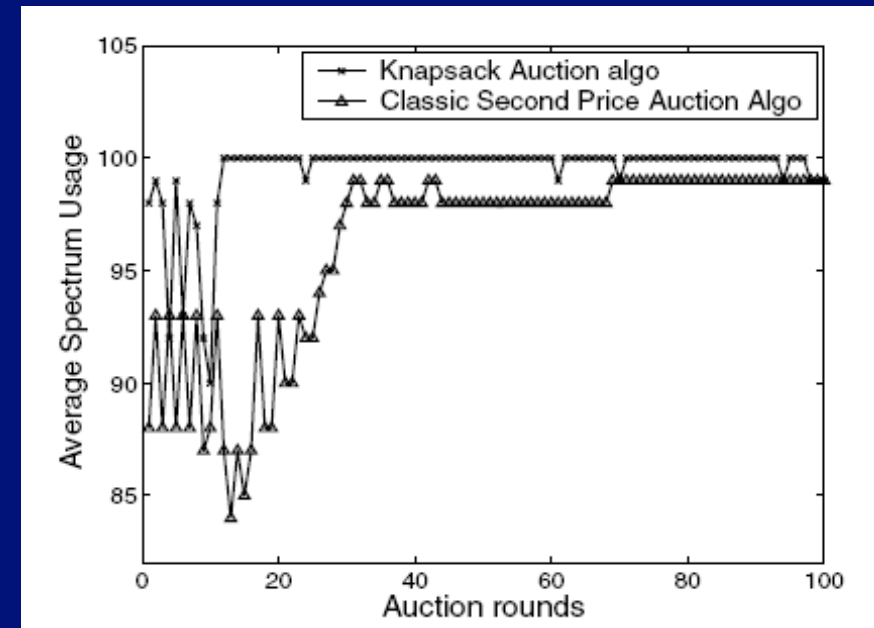
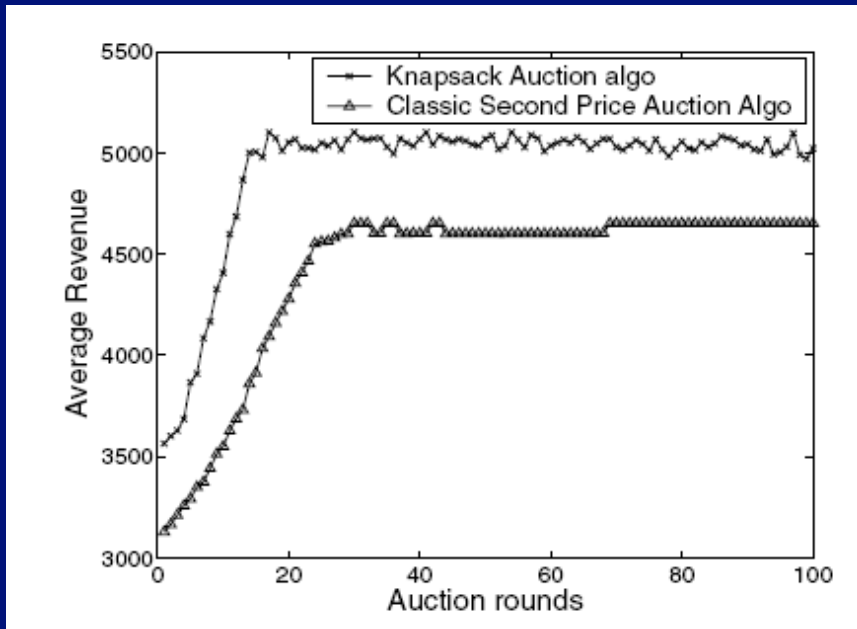
- **For simulation second price sealed bid mechanism is considered**

- **Assumptions:**

- All bidders are present for all auction rounds
- Bidders take feedback from previous rounds & generate tuple for next round



# "Allocation through Auction": Simulation Results



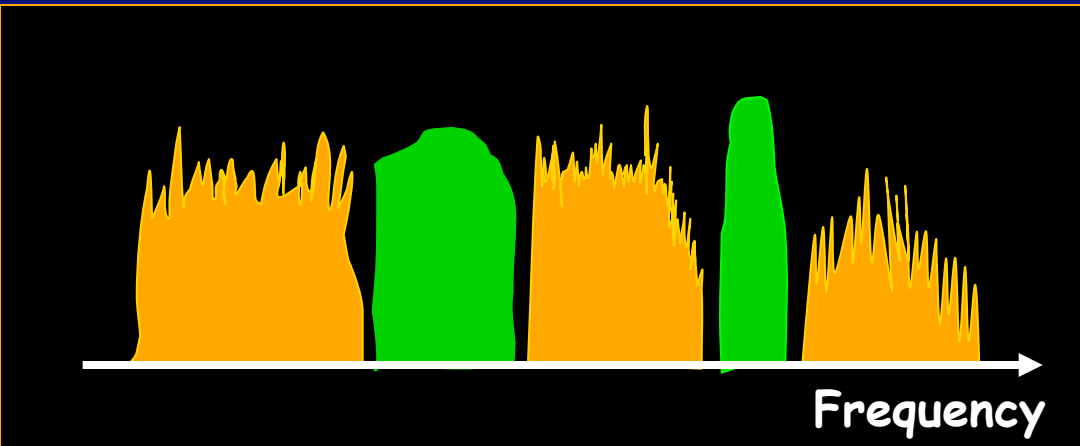
- For simulation assumed quantities:
  - CAB = 100 units; 10 bidders
  - Max & Min Bid 11 & 50 respectively
  - Min bid per unit of spectrum = 25 units



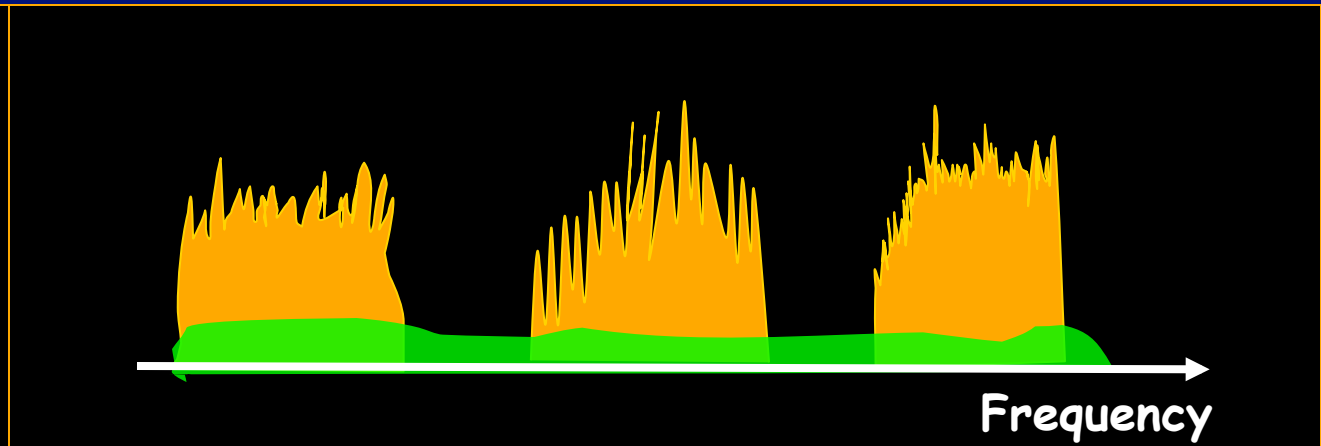


# Classification of Spectrum Sharing based on Spectrum Access Techniques

 Primary user       CR user



Overlay Spectrum Sharing



Underlay Spectrum Sharing



# Overlay Spectrum Sharing

- A CR user accesses the primary network using a portion of the spectrum that has not been occupied by PUs.
- As a result, interference to the primary system is minimized.



# Underlay Spectrum Sharing

- Exploits spread spectrum techniques
- Once a spectrum allocation map has been acquired, a CR user begins transmission such that its transmit power at a certain portion of the spectrum is regarded as noise by the primary users.  
(Interference temperature idea)
- Requires sophisticated spread spectrum techniques and can utilize increased BW compared to overlay techniques.



# Comparison of Underlay and Overlay Approaches

R. Menon, R. M. Buehrer, J. H. Reed,

"Based Comparison of Underlay and Overlay Spectrum Sharing Techniques Outage Probability,"

Proc. IEEE DySPAN, pp. 101-109, Nov. 2005.

Based on the influence of the CR network on the primary network in terms of outage probability

(probability that the primary network will experience interference from the CR network)

→ three spectrum sharing techniques are considered.



# Comparison of Underlay and Overlay Approaches

## ■ METHOD 1: Spreading Based Underlay

requires CR users to spread their transmit power over the full spectrum such as CDMA or UWB.



# Comparison of Underlay and Overlay Approaches

- **METHOD 2: Interference Avoidance Overlay**  
requires CR users to choose a frequency band to transmit such that the interference at a PU is minimized.



# Comparison of Underlay and Overlay Approaches

## ■ METHOD 3: Hybrid Technique (Spreading based Underlay with Interference Avoidance)

A CR user spreads its transmission over the entire spectrum where a PU is transmitting.



# Comparison of Underlay and Overlay Approaches

## ■ Perfect system knowledge

- **Overlay scheme** outperforms the **underlay scheme** in terms of outage probability.
- **Underlay scheme with interference avoidance** guarantees smaller outage probability than the pure interference avoidance.





# Comparison of Underlay and Overlay Approaches

## ■ Limited System Knowledge (more realistic)

- **The overlay schemes** result in poor performance due to imperfections at spectrum sensing.
- **Underlay with interference avoidance** → the interference caused to the PU is minimized.
- Another important result is that a higher number of CR users can be accommodated by the hybrid scheme than the pure interference avoidance scheme.



# Spectrum Sharing Challenges:

## Spectrum Unit

J. Zhao, H. Zheng, and G.-H. Yang,

"Distributed Coordination in Dynamic Spectrum Allocation Networks,"

Proc. IEEE DySPAN, pp. 259-268, Nov. 2005.

- All spectrum sharing techniques consider "a channel" as the basic spectrum unit for operation.
- Many algorithms and methods have been proposed to select the suitable *channel* for efficient operation in CR networks.



# Spectrum Sharing Challenges:

## Spectrum Unit

However, in some work, the channel is vaguely defined as

- \* Orthogonal non-interfering,
- \* TDMA, FDMA, CDMA, or
- \* A combination of them, or
- \* A physical channel as in IEEE 802.11, or
- \* A logical channel associated with a spectrum region or a radio technology”.

→ Definition of a channel as a spectrum unit for spectrum sharing is crucial in further developing algorithms.