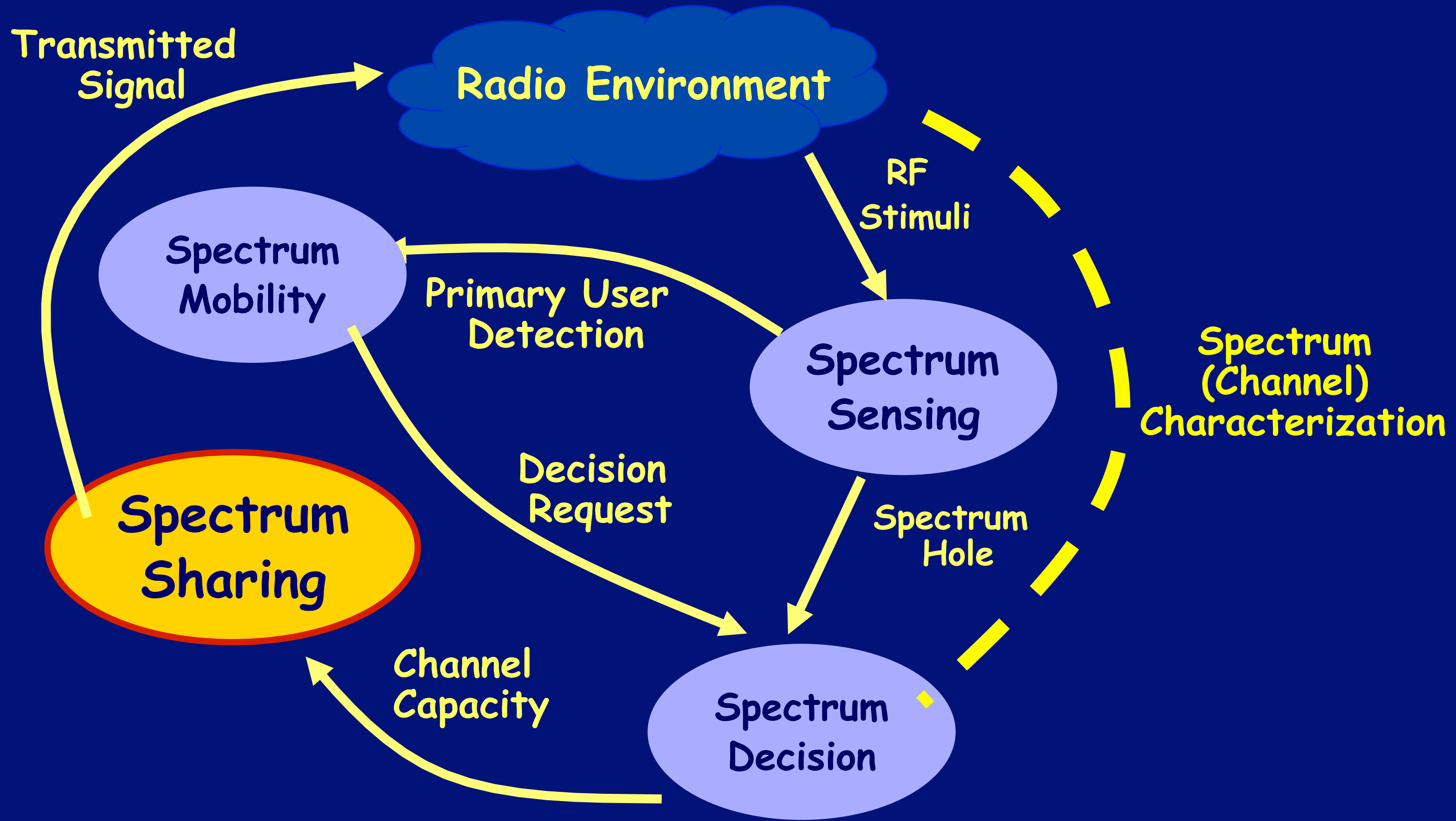
CHAPTER 7.
SPECTRUM SHARING

Spectrum Sharing



Spectrum Sharing

- Spectrum Sharing → similar to MAC Problems
  - Multiple CR users try to access the spectrum
  - Access must be coordinated
    (to prevent collisions in overlapping portions of the spectrum)
- Uniqueness
  - Coexistence with licensed (primary) users
  - Wide range of available spectrum

SPECTRUM SHARING CLASSIFICATION

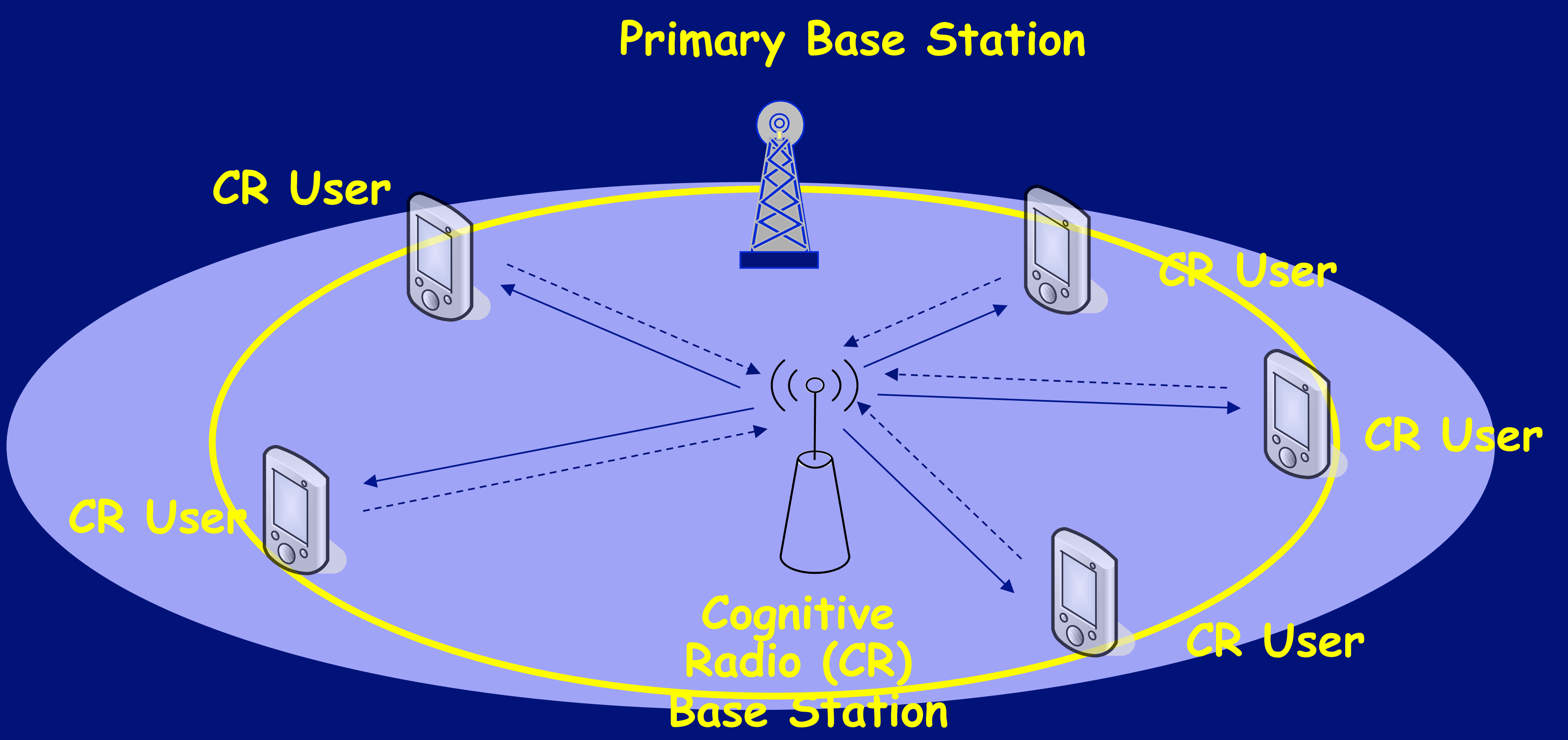
Intra-Network SS

- Centralized (Infrastruct.)
- Distributed (Ad hoc)
  - Cooperative
  - Non-cooperative

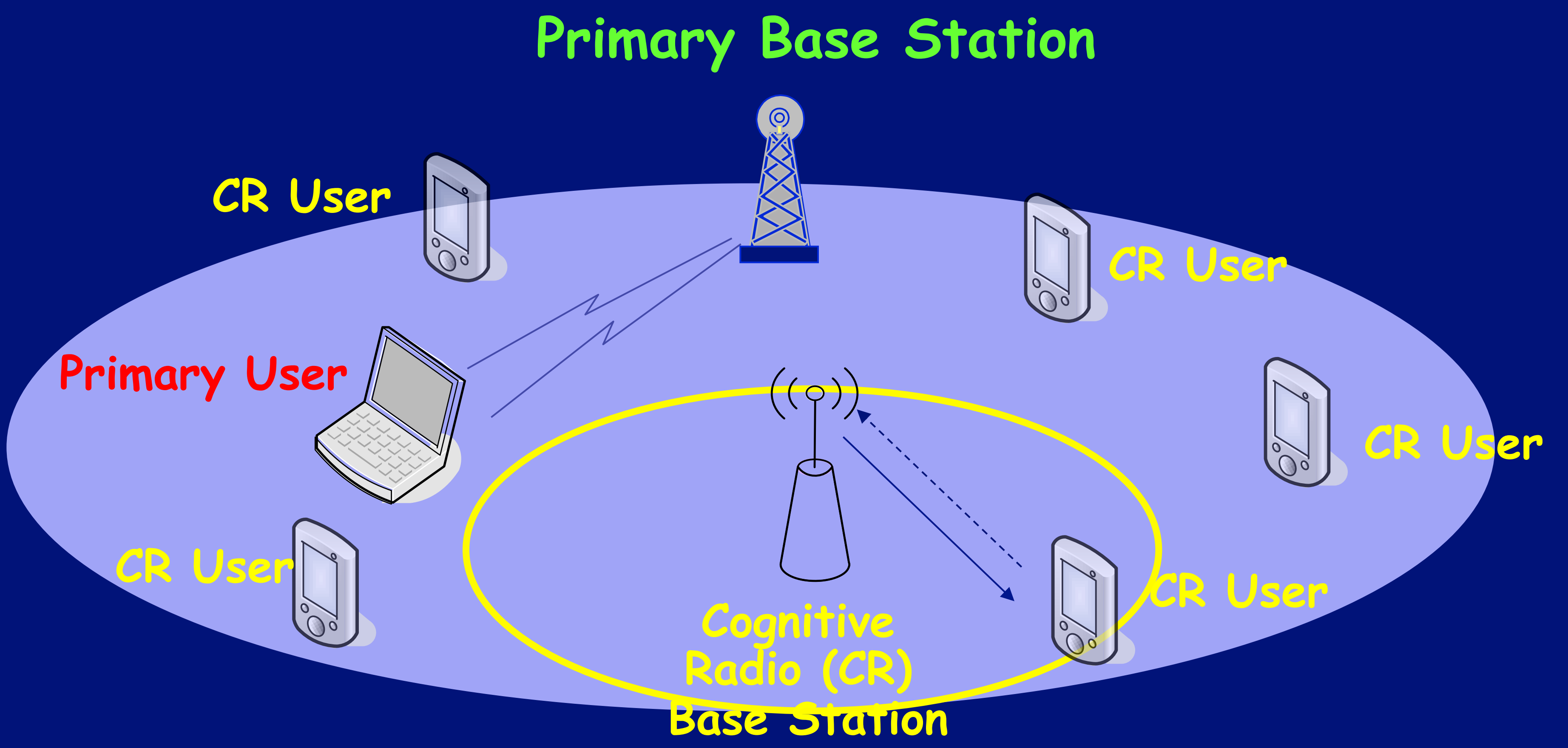
Inter-Network SS

* Centralized
* Distributed

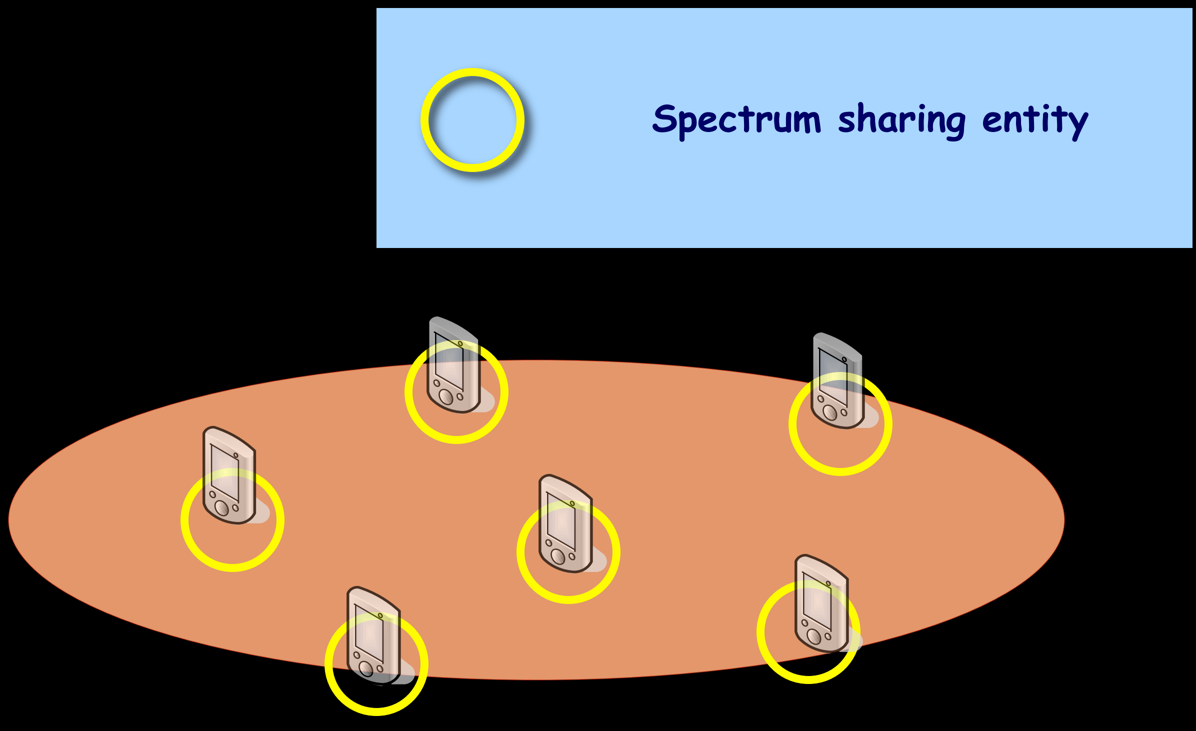
Intra-Network Centralized Spectrum Sharing



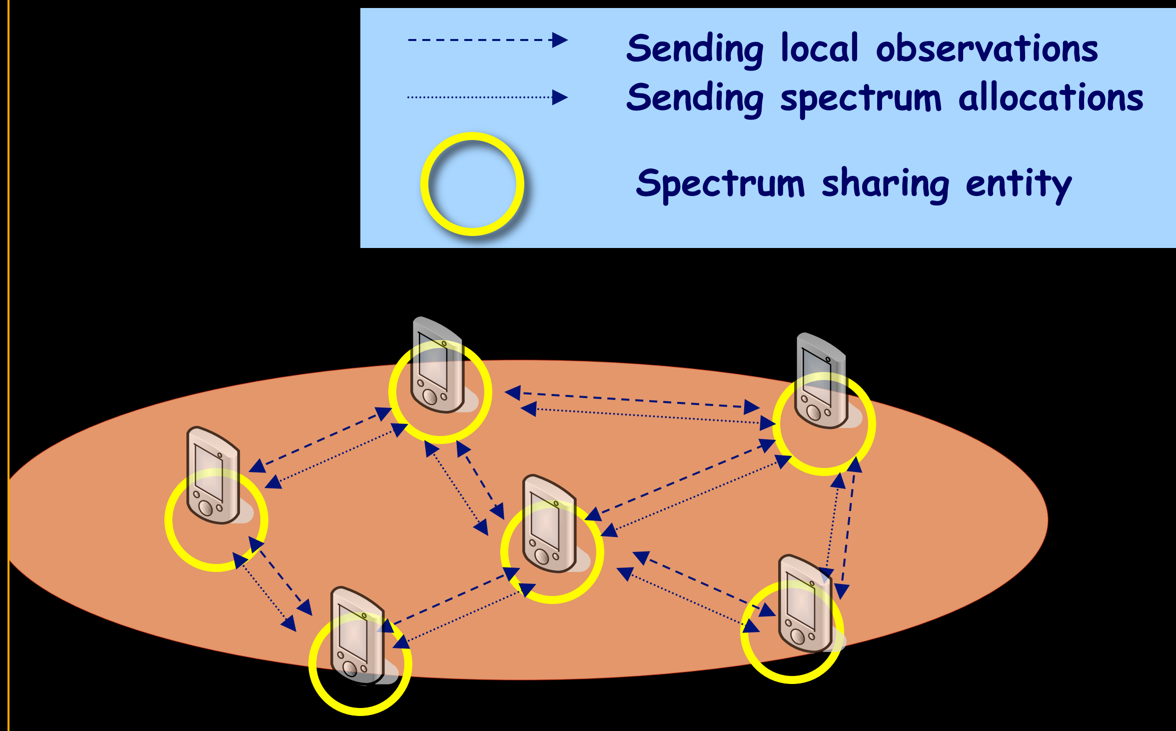
Intra-Network Centralized Spectrum Sharing



Intra-Network (Ad Hoc Network) Spectrum Sharing



Distributed Spectrum Sharing
(Non-Cooperative)

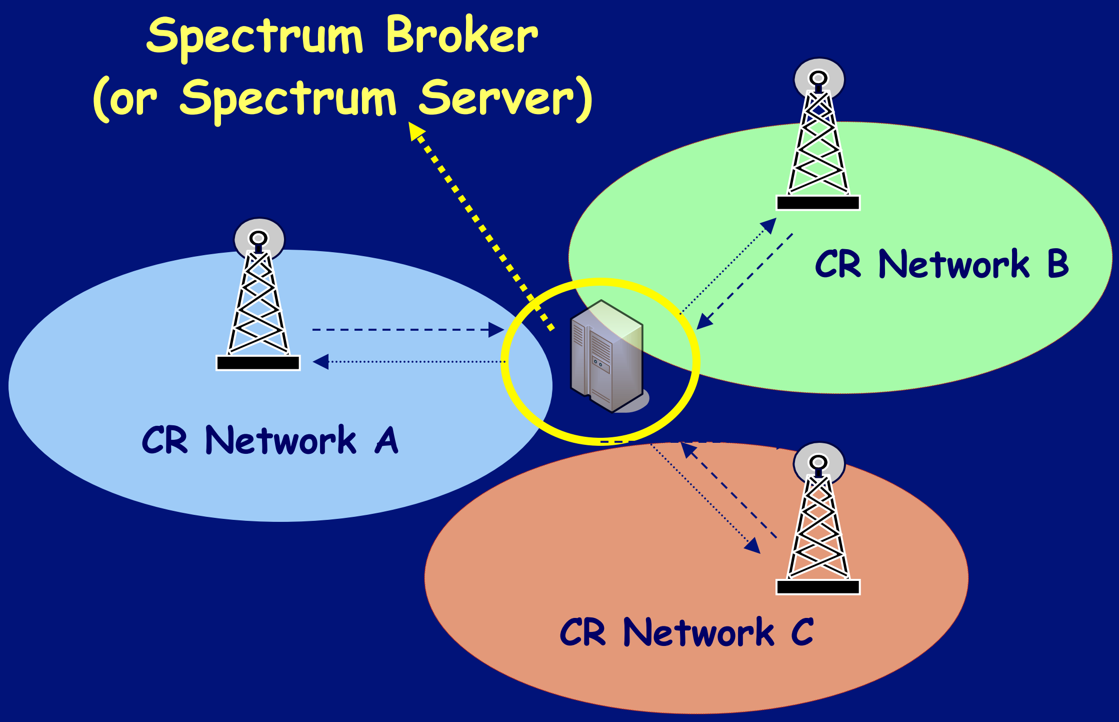


Distributed Spectrum Sharing
(Cooperative)

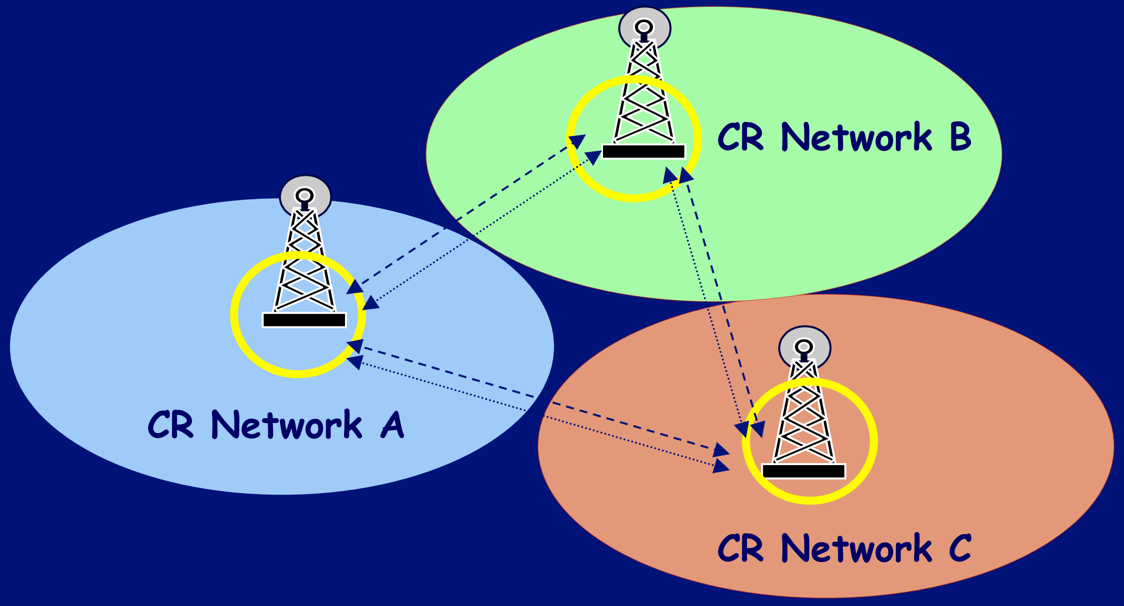
Intra-Network Spectrum Sharing

- Spectrum sharing inside a CR network ←→ same as MACs
- Focuses on "spectrum allocation" between CR users
  - Coordinates multiple accesses among CR users in order to prevent their collision in overlapping portions of the spectrum
- Also CR users need to access the available spectrum without causing interference to the PUs.

Inter-Network Spectrum Sharing



Centralized Spectrum Sharing



Distributed Spectrum Sharing

Inter-Network Spectrum Sharing

- Multiple systems are deployed in overlapping locations and spectrum bands

- Spectrum sharing among these systems is an important research topic in CR networks

EXAMPLE: Inter-Network Spectrum Sharing for CR Ad Hoc Networks

A CR Ad Hoc Network co-exists with a WiFi and a Bluetooth.

So far only the interference issues in the ISM band have been investigated in the literature ‼

(No centralized control) ‼

Evolution of Game Theory Perspective

- CR users make intelligent decisions on spectrum usage and communication parameters based on the sensed spectrum dynamics and other users' decisions.

- To analyze intelligent behaviors of CR Users,
  ➔ GAME THEORETICAL PERSPECTIVE

GAME THEORY

- Mathematical models and techniques developed in economics to
  * analyze interactive decision processes
  * predict the outcomes of interactions, &
  * identify optimal strategies

- Game theory techniques were adopted to solve many protocol design issues (e.g., resource allocation, power control, cooperation enforcement) in wireless networks

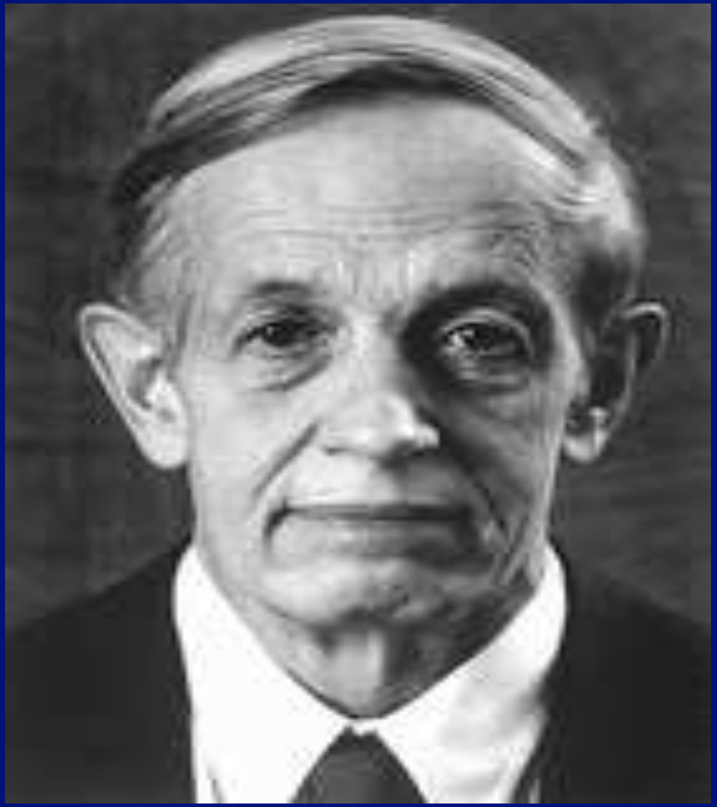
- Fundamental component of game theory is the notion of a game.

Game Theory – Short History

- John von Neumann (1903-1957)
- "Theory of Games and Economic Behavior" with Oskar Morgenstern
- This book established game theory as a field

Game Theory – Short History

- John F. Nash, Jr. (1928-)
- One of the contributions is the introduction of the equilibrium notion now known as Nash equilibrium
- 1994 Nobel prize winner in economics with the game theorists John Harsanyi and Reinhard Selten



Game Theory

- **Definition**
  - A collection of mathematical models and techniques for the analysis of interactive decision processes
  - Provides strategic interactions among users
  - Enables the choice of optimal behavior when costs and benefits of each option depend upon the choices of other users.

Why Game Theory for Spectrum Sharing?

- Excellent match to the spectrum sharing in CR networks
  - Provides a well-defined model to describe conflict and cooperation among intelligent rational decision makers

Why Game Theory for Spectrum Sharing in CR Networks?

- CR users have a common interest to have the spectrum resources as much as possible.
  - However, CR users have competing interests to maximize their own share of the spectrum resources, i.e., the activity of one CR user can impact the activities of the others
  - Also CR user's rational decisions require anticipating rivals' responses

Why Game Theory?

- Provides an efficient distributed spectrum sharing scheme.
- Provides the well-defined equilibrium criteria for the spectrum sharing problem to measure the optimality in various network scenarios.

What is a GAME??

All games involve three features:

- Rules
- Strategies
- Payoffs

GAME THEORY

- A game is described by
  - * A set of rational players
  - * Strategies associated with the players, and
  - * The payoffs for the players.

- A rational player has his own interest, and therefore, will act by choosing an available strategy to achieve his interest.

- A player is assumed to be able to evaluate exactly or probabilistically the outcome or payoff (usually measured by the utility) of the game which depends not only on his action but also on other players' actions.

Game Theory: Basic Components

- **Game**: A model of interactive decision process
- **Player**: A decision making entity
- **Actions (Strategies)**: Adaptations available to the player.
- **Outcomes (Payoffs)**: Outputs determined by the actions and the particular system in which the players are operating
- **Preference**: A decision maker objective
  (To capture the preference relation in a more compact way; we employ utility functions (payoff functions) where each player assigns a real number to each outcome)

Game Theory: Recap

Output (outcomes) of the process (game) is the function of the inputs (actions) from several different decision makers (players)

who may have potentially conflicting objectives (preferences) with regards to the outcome of the process.

Non-cooperative Game Theory

- Rational players having conflicting interests
  - e.g. scheduling in wireless networks
- Defined by
  - Set of players
  - Set of strategies for each player
- Players engage in the game while being selfish
  - Each player wishes to maximize his payoff or 'utility'
- Solution: the Nash equilibrium
  - No user can unilaterally improve his payoff
  - Can be inefficient

Normal Form Games (Strategic Form Games)

- Synchronous Single Shot Play:
  All players make their decisions simultaneously and take only a single decision without knowing the actions of the other

- Specified by 3-tuple $\Gamma = \langle N, A, \{u_j\} \rangle$

  - A set of players $N = \{1,2,\ldots,N\}$

  - Action Space A; formed from the Cartesian product of each players' strategy set $A = A_1 \times A_2 \times \ldots \times A_N$

  - PAYOFFS: A set of utility functions $\{u_j\}$
    such that each player $j \in N$ has its own utility function, $u_j : A \to R$
    (R is a set of real numbers)

Normal Form Games (Strategic Form Games)

- Example: Paper (P) – Rock (R) - Scissors (S) Game
  - N = {P1, P2}
  - A = {(P,P), (P,R), (P,S), …, (S,S)}
  - $\{u_j\}$ = {-1, 0, 1}   (-1: loss, 0: tie, 1: win)

P1 \ P2	P	R	S
P	(0,0)	(1,-1)	(-1,1)
R	(-1,1)	(0,0)	(1,-1)
S	(1,-1)	(-1,1)	(0,0)

No Nash Equilibrium !

Details: http://www.youtube.com/watch?v=sQVbeAEorsc

Nash Equilibrium (NE)

DEFINITION:
A set of actions (strategies) where no player has anything to gain by changing only his/her own strategy unilaterally.

NEs correspond to the steady-states of the game and are then predicted as the most probable outcomes of the game.

I.o.w.
When each player is taking the *best* action given best actions taken by other players !

Under the Nash Equilibrium, no players want to deviate

Nash Equilibrium (NE)

If each player has chosen a strategy and no player can benefit by changing his/her own strategy while other players keep theirs unchanged,

then the current set of strategy choices and the corresponding payoffs constitute a NE.

Nash Equilibrium (NE)

SIMPLY:

You and I are in NE if I make the best decision I can, taking into account your decision, and you make the best decision you can, taking into account my decision.

Likewise, many players are in NE if each one is making the best decision he can, taking into account the decisions of the others.

Pareto Optimality

DEFINITION:

A set of actions if some (all) players must/may be hurt in order to improve the payoff of other players

Pareto Optimality is used to measure the efficiency of game outcomes.

- A set of actions which is a NE need not to be Pareto optimal.
- A set of actions which is Pareto optimal need not to be a NE.

- Generally, it is desirable for a NE to be Pareto optimal.

NE vs Pareto

- Pareto Optimal:
  Check every column and row (diagonals) and find that all cases are
  WIN/LOSS or LOSS/WIN or LOSS/LOSS.

  If one case is not met, then it is not Pareto Optimal.

- NE:
  Check the states from each players' perspective and see there is any improvement.

  If no improvement, then STUCK → NE.

Pareto Optimality

- **Example Games**

Legend: Pareto Optimal (blue ellipse); NE (orange ellipse); NE + PO (striped ellipse)

Pareto Optimal ✓

Consider this case

Player 1 \ Player 2	a_2	b_2	
a_1	1,1	-5,5	✓
b_1	5,-5	-1,-1	
	✓	✓	

L/W (1,1 → -5,5); W/L (1,1 → 5,-5); L/L (1,1 → -1,-1)

Player 1 \ Player 2	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	3, 3

Pareto Optimality

- Example Games

Pareto Optimal ✓

Pareto Optimal

NE

NE + PO

Player 2

Player 1

	a_2	b_2
a_1 ✓	1,1	-5,5
b_1	5,-5	-1,-1
	✓	✓

W/L W/L W/L

Player 2

Player 1

	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	3, 3

Pareto Optimality

- **Example Games**

Pareto Optimal ✓

◯	Pareto Optimal
◯	NE
◯	NE + PO

Player 2

Player 1	a_2	b_2
a_1	1,1 ✓	-5,5 ✓
b_1	5,-5	-1,-1 ✓

L/W L/W L/W

Player 2

Player 1	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	3, 3

Pareto Optimality

- **Example Games**

Pareto Optimal

NE

NE + PO

Not Pareto Optimal

Player 2

Player 1	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	-1,-1

W/W L/W W/L ✓ ✓

Player 2

Player 1	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	3, 3

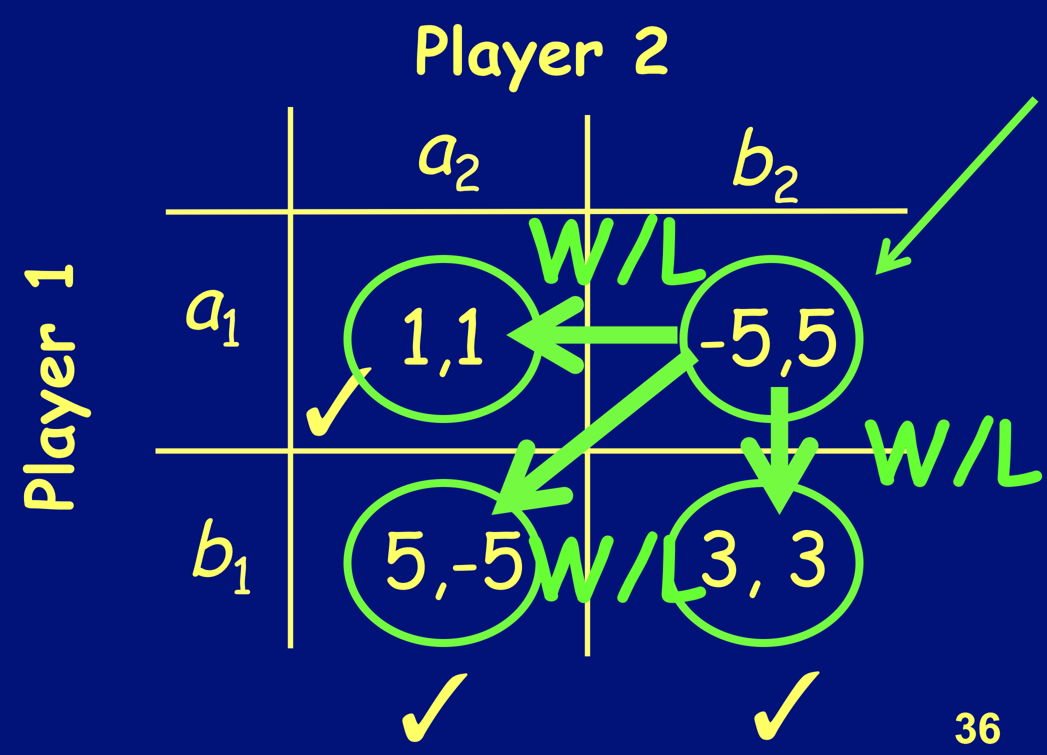
Pareto Optimality

- **Example Games**

Pareto Optimal — Pareto Optimal; NE; NE + PO

Player 2 (columns), **Player 1** (rows)

	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	-1,-1



Pareto Optimality

- **Example Games**

Pareto Optimal | NE | NE + PO

Pareto Optimal

Player 2 (columns), Player 1 (rows)

	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	-1,-1

Player 2 (columns), Player 1 (rows)

	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	3, 3

L/L

L/W

W/L

Pareto Optimality

- **Example Games**

Pareto Optimal

Legend: Pareto Optimal · NE · NE + PO

Player 2 (columns), Player 1 (rows)

	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	-1,-1

Player 2 (columns), Player 1 (rows)

	a_2	b_2
a_1	✓ 1,1	-5,5 ✓
b_1	5,-5	3, 3 ✓

L/W L/W L/W

Pareto Optimality

- **Example Games**

Legend: Pareto Optimal (blue oval); NE (orange oval); NE + PO (blue/orange striped oval)

Not Pareto Optimal

Player 2 (columns), **Player 1** (rows)

	a_2	b_2
a_1	1,1 (Pareto Optimal)	-5,5 (Pareto Optimal)
b_1	5,-5 (Pareto Optimal)	-1,-1

Player 2 (columns), **Player 1** (rows)

	a_2	b_2
a_1	1,1	-5,5 ✓
b_1	5,-5	3, 3

Annotations from 1,1: L/W → -5,5; W/L → 5,-5; W/W → 3, 3; ✓ (below the a_2 column)

Pareto Optimality

- Example Games

Pareto Optimal

NE

NE + PO

Player 2

Player 1	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	-1,-1

Player 2

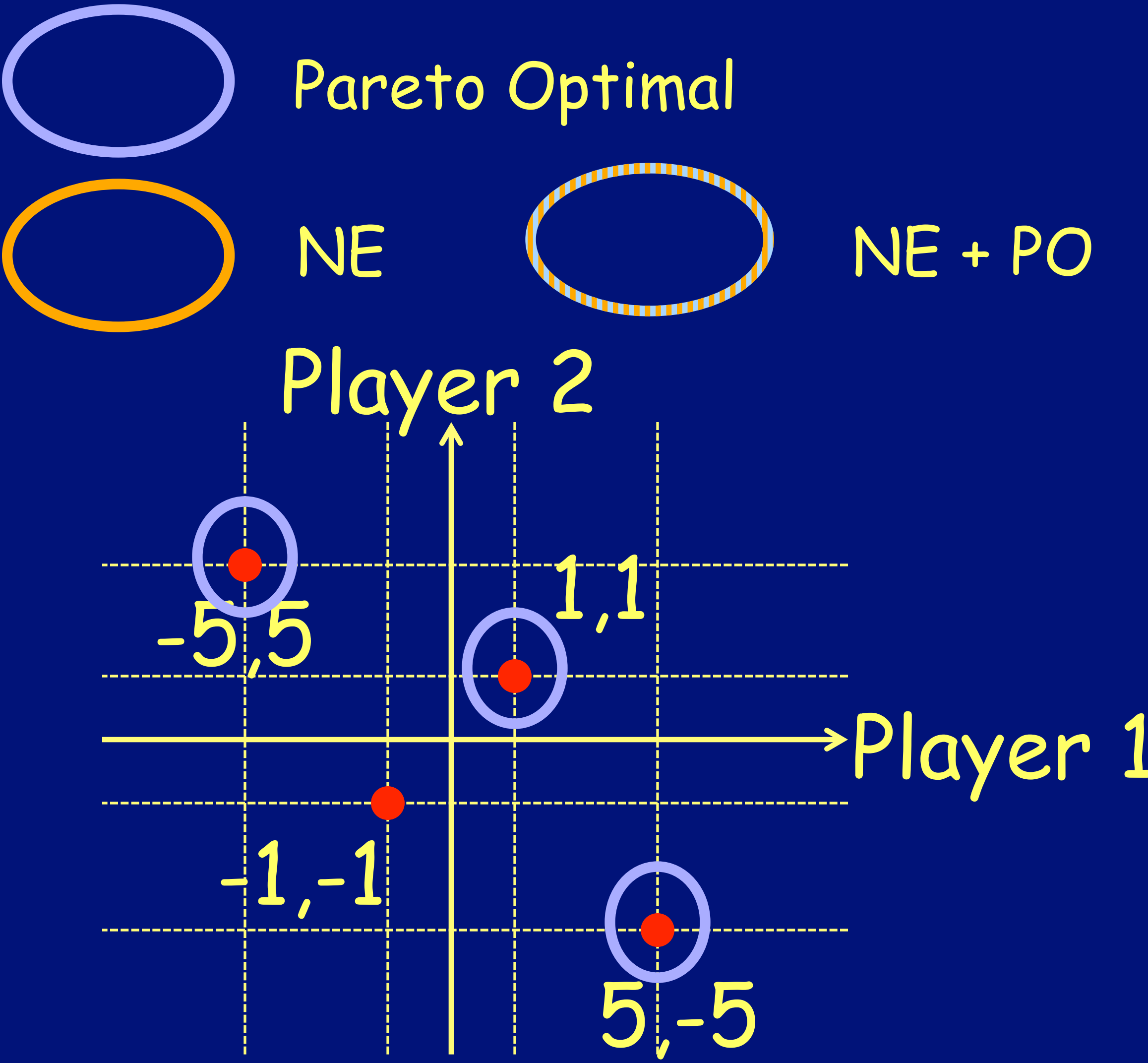
Player 1	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	3, 3

Pareto Optimality – Graphical Method

- **Example Games**

Player 2

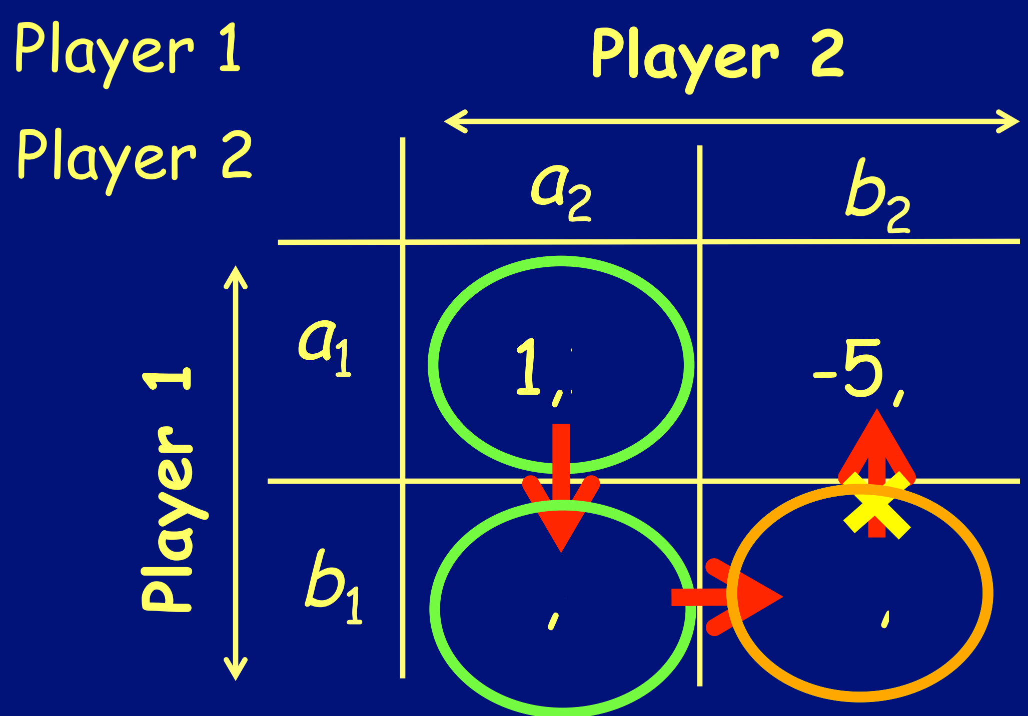
Player 1	a_2	b_2
a_1	1,1	-5,5
b_1	5,-5	-1,-1

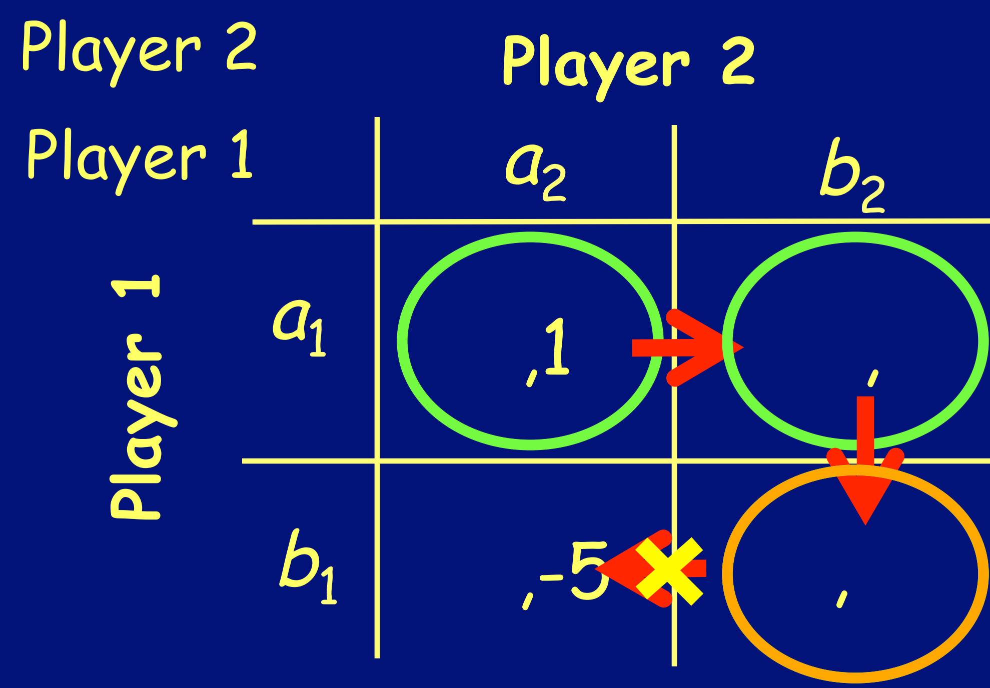


Nash Equilibrium vs Pareto Optimality

- **Example Games** — NE

Start from any state and examine the rule from a single player's perspective at a time



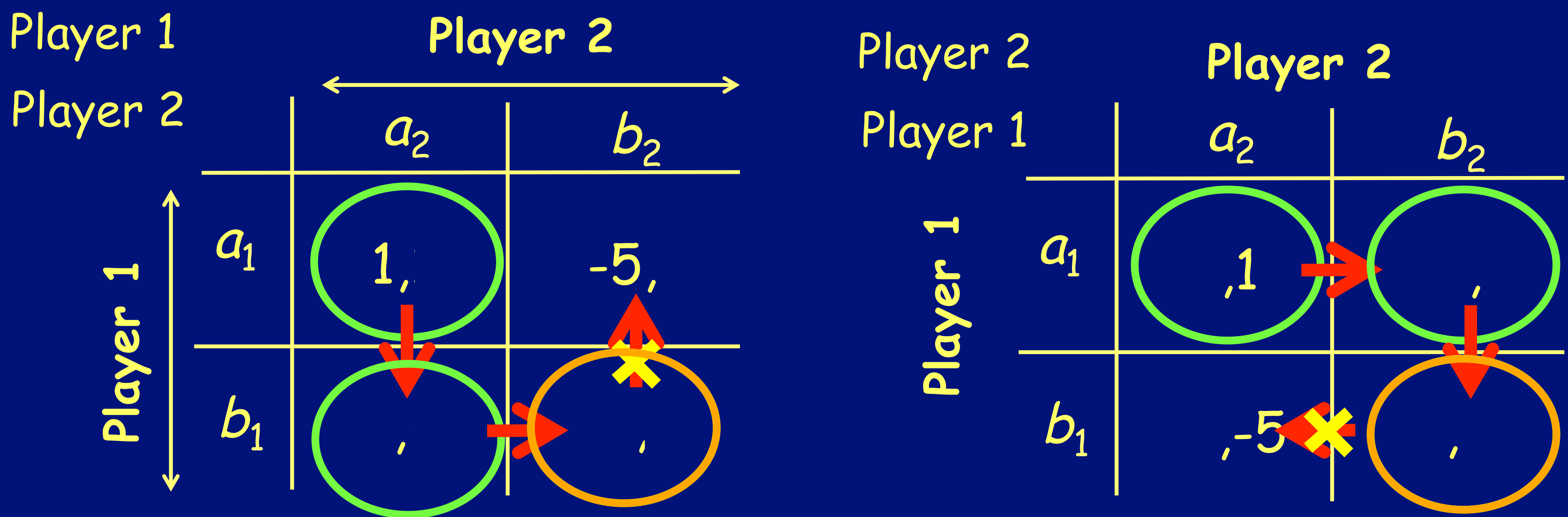


Nash Equilibrium vs Pareto Optimality

- **Example Games**

NE

Start from any state and examine the rule from a single player's perspective at a time



Nash Equilibrium vs Pareto Optimality

- **Example Games**

Legend: Pareto Optimal (blue ellipse); NE (orange ellipse); NE + PO (blue/orange striped ellipse)

Player 2 (columns), Player 1 (rows):

	a_2	b_2
a_1	1,1 (Pareto Optimal)	-5,5 (Pareto Optimal)
b_1	5,-5 (Pareto Optimal)	-1,-1 (NE)

Player 2 (columns), Player 1 (rows):

	a_2	b_2
a_1	1,1	-5,5 (Pareto Optimal)
b_1	5,-5 (Pareto Optimal)	3, 3 (NE + PO)

EXAMPLE WITH THREE STRATEGIES

RULE: If the 1st P1 payoff # is the max of the column of the cell and if the 2nd P2 # is the max of the row of the cell, then the cell represents a Nash equilibrium.

NE: (B,A), (A,B), (C,C)

(B,A) → 40 is the max of the 1st column;
25 is the max of the 2nd row.
(A,B) → 25 is the max of the 2nd column;
40 is the max of the 1st row
(Same with C,C)

Find the max of a column and check if the 2nd member of the pair is the max of the row.

	Option A	Option B	Option C
Option A	(0,0)	(25,40)	(5,10)
Option B	(40,25)	(0,0)	(5,15)
Option C	(10,5)	(15,5)	(10,10)

* If these conditions are met, the cell represents a NE.

* Check all columns this way to find all NE cells

* An NxN matrix may have between 0 and NxN Nash equilibria.

NE Identification

- Direct Application of Definition (Greedy Search)
  - Exhaustively evaluate all action tuples in light of the definition of NE to find out which ones are NE.

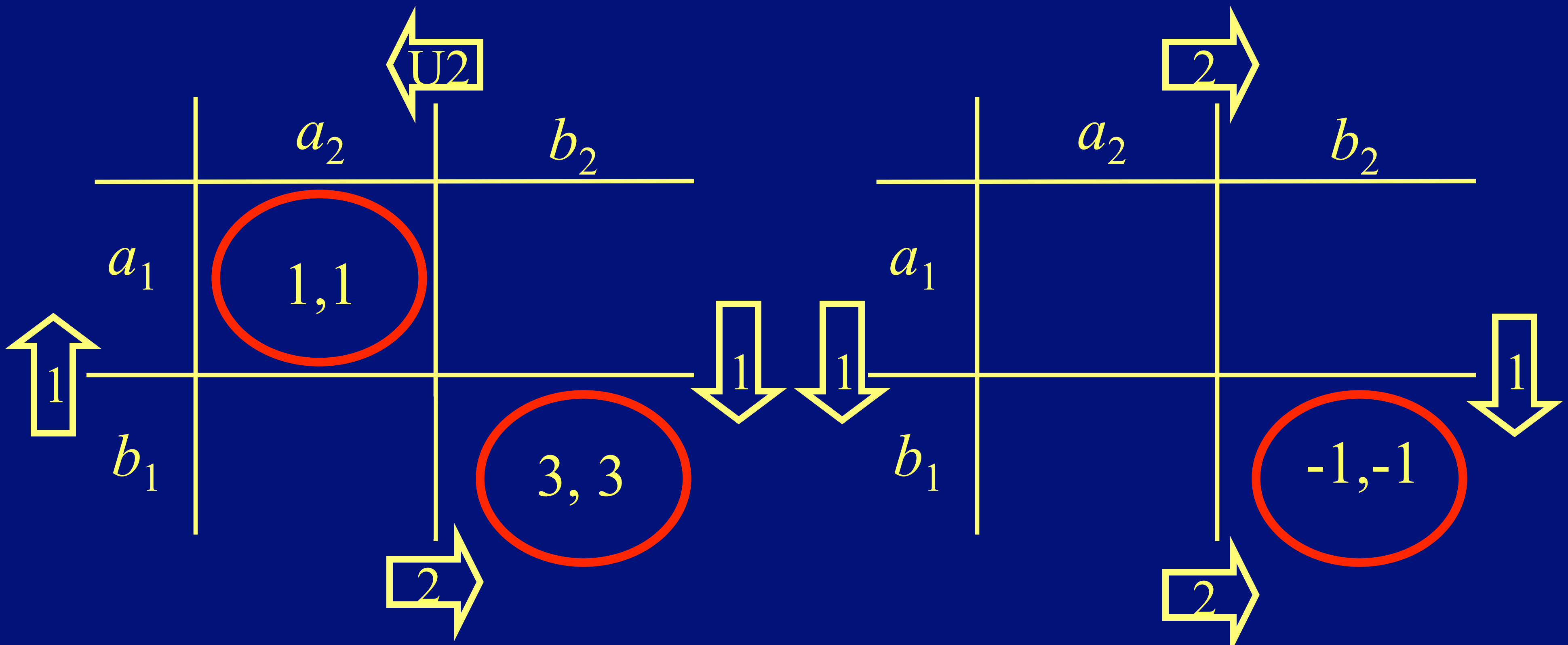
NE Identification: Improvement Deviations

- An improvement deviation is a unilateral deviation from one action tuple to another which shows greater utility function.

- All points which have no improvement deviations must be a NE.

- Why not follow improvement deviations until a NE is reached, or a loop is found.

NE Identification: Improvement Deviations (Example)



NE Identification: IEDS (Iterative Elimination of Dominated Strategies)

- Sometimes a player's actions are not preferable, no matter what the other players do.

- These actions would thus never rationally be played and can be eliminated from consideration in any NE action vector.

NE Identification: IEDS Example

Iteration 1.

Note the following

$$u_1(C,D) > u_1(D,D)$$
$$u_1(C,C) > u_1(D,C)$$

So *D* is dominated by *C* for player 1. So we remove *D* for player 1 from the game.

Iteration 2.

Note the following

$$u_2(C,C) > u_2(C,D)$$

So in the remaining game *D* is dominated by *C* for player 2. So we remove *D* for player 2 from the game.

User 2

User 1		C
C		-5, -5

As there is only one action tuple left (thus no deviation is possible, nor is a profitable deviation), it must be a NE

The Prisoners' Dilemma

- Art and Bob been caught stealing a car: sentence is 2 years in jail.
- DA wants to convict them of a big bank robbery: sentence is 10 years in jail.
- DA has no evidence and to get the conviction, he makes the prisoners play a game.

PRISONER'S DILEMMA: RULES

- Players cannot communicate with one another.
  - If both confess to the larger crime, each will receive a sentence of 3 years for both crimes.
  - If one confesses and the accomplice does not, the one who confesses will receive a sentence of 1 year, while the accomplice receives a 10-year sentence.
  - If neither confesses, both receive a 2-year sentence.

PRISONER'S DILEMMA: STRATEGIES

- The strategies of a game are all the possible outcomes of each player.

- The strategies in the prisoners' dilemma are:

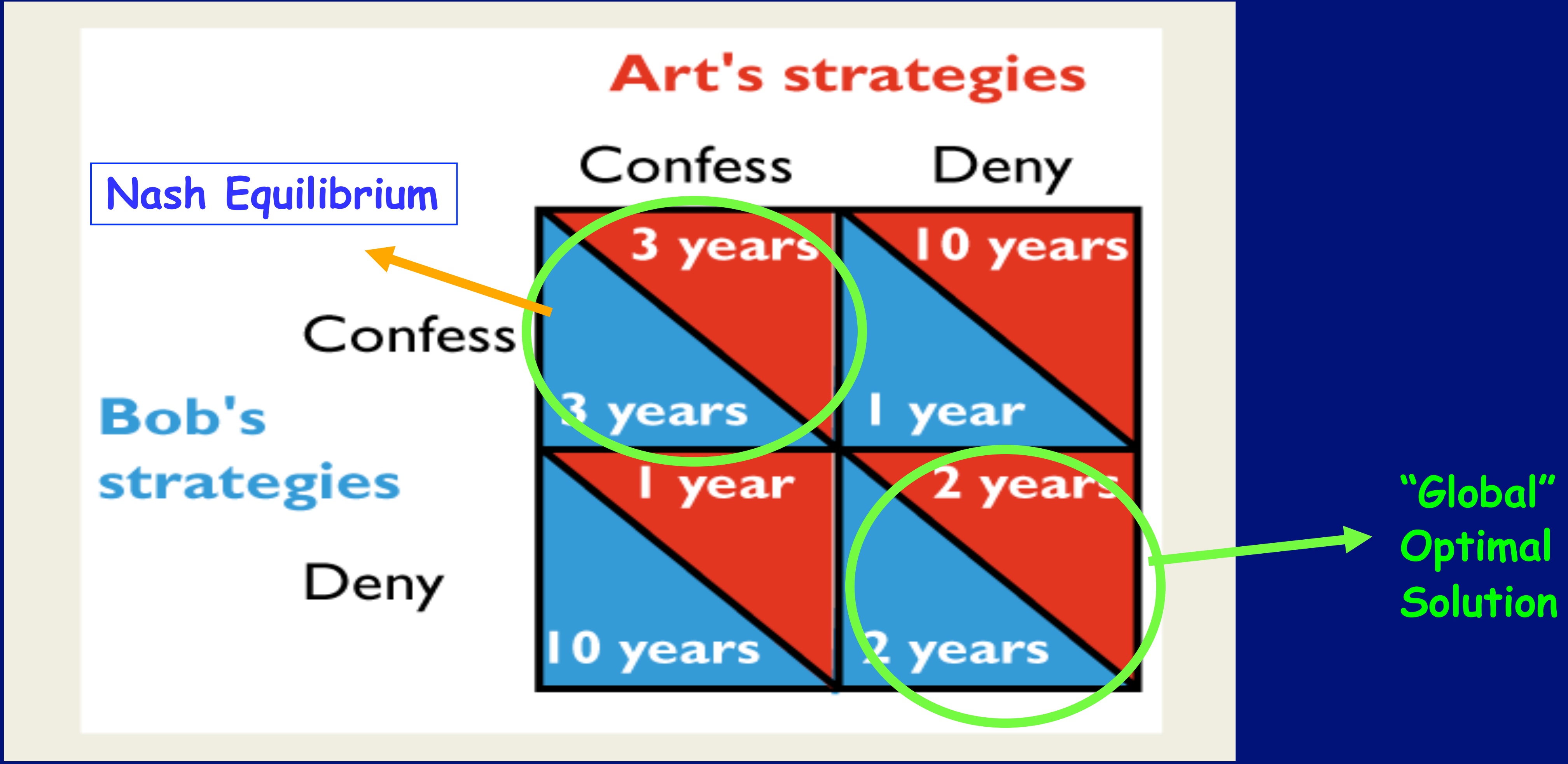
  - Confess to the bank robbery
  - Deny the bank robbery

PRISONER'S DILEMMA: PAYOFFS

- Four outcomes:
  - Both confess
  - Both deny
  - Art confesses and Bob denies
  - Bob confesses and Art denies

- A payoff matrix is a table that shows the payoffs for every possible action by each player given every possible action by the other player.

PRISONER'S DILEMMA: PAYOFF MATRIX



In the non-cooperative game, Nash Equilibrium cannot always provide an optimal solution.

PRISONER'S DILEMMA

Equilibrium

- Occurs when each player takes the best possible action given the action of the other player.

Nash equilibrium

- An equilibrium in which each player takes the best possible action given the action of the other player.

PRISONER'S DILEMMA

- The Nash equilibrium for Art and Bob is to confess.
- Not the Best Outcome
- The equilibrium of the prisoners' dilemma is not the best outcome.

Example: Prisoner's Dilemma

The action profile (Confess, Confess) is the only NE.

To show that a pair of actions is not a Nash equilibrium, it is enough to show that one player wishes to deviate
(an equilibrium is immune to any unilateral deviation).

In general, at the Nash equilibrium, the action for a player is optimal if other players choose their Nash equilibrium actions, but some other action is optimal if the other players choose non-equilibrium actions.

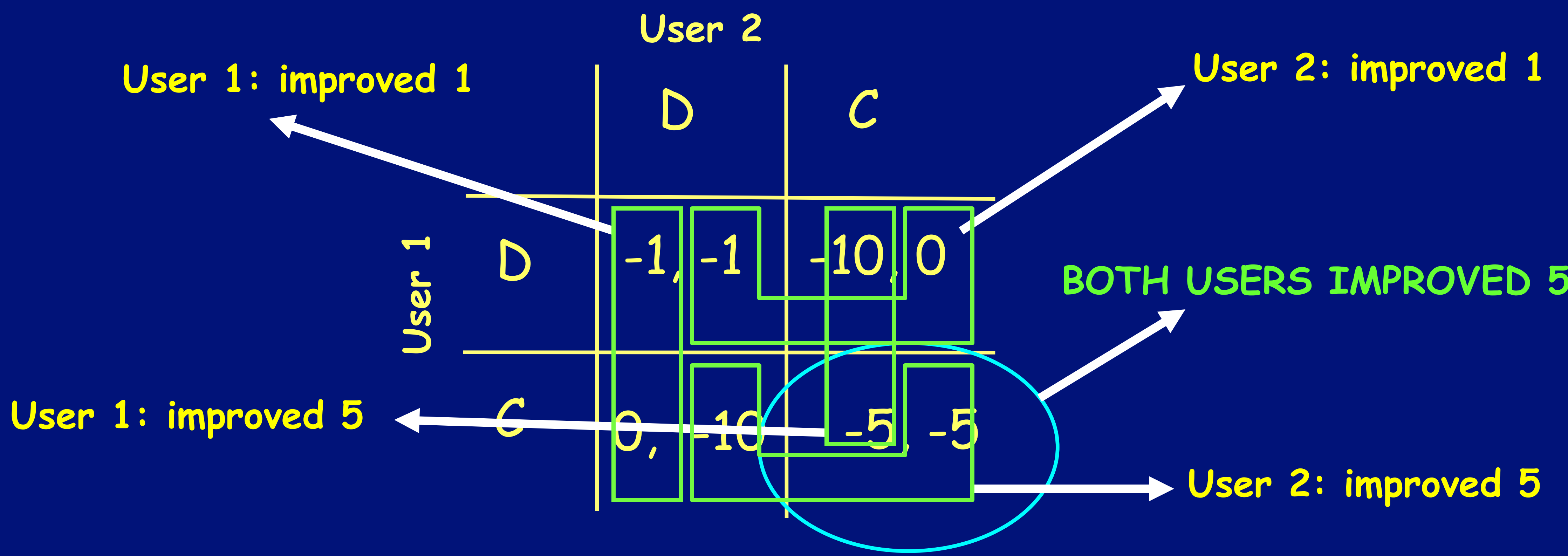
NE Identification: Best Response Analysis

- If a sequence of improvement deviations exists
  where a single player deviates without other players deviating,

  Why not immediately skip ahead to the action tuple that yields the largest improvement

  Best response → thus eliminating the intermediate steps.

NE Identification: Best Response Analysis Example



Best Response Function

For any given actions of the players other than i, the best actions of player i which yield the highest payoff for player i, denoted by, $B_i\ (a_{-i})$

B_i = Best response function of player i.

Mathematically:

$B_i\ (a_{-i}) = \{a_i \text{ in } A_i: u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}) \text{ for all } a_i' \text{ in } A_i\}$,

i.e., any action in $B_i\ (a_{-i})$ is at least as good for player i as every other action of player i when the other players' actions is given by a_{-i}.

Best Responses in Prisoner's Dilemma

- BR of Bob to each action of Art:
  - Art chooses C → BR of Bob is C (i.e., (C,C))
  - Art chooses D → BR of Bob is C (i.e., (C,D))
- BR of Art to each action of Bob:
  - Bob chooses C → BR of Art is C (i.e., (C,C))
  - Bob chooses D → BR os Art is C (i.e., (D,C))

  RULE: If Art picks a strategy, look columns for Bob
  If Bob picks a strategy, look for rows for Art
- The game has one NE: (C,C)

		Art	
		Confess	Deny
Bob	Confess	(-5*,-5*)	(3,-10)
	Deny	(-10,0)	(-2,-2)

Example for Best Responses

- Find the best response of P1 to each action of P2.
- If P2 chooses L, then P1's best response is M
  (2 is the highest payoff for P1 in this column)
- Indicate the best response by attaching a star to P1's payoff to (M,L).
- If P2 chooses C, then P1's best response is T, indicated by the star attached to P1's payoff to (T,C).
- And if P2 chooses R, then both T and B are best responses for P1; both are indicated by stars.

	L	C	R
T	(1,2*)	(2*,1)	(1*,0)
M	(2*,1*)	(0,1*)	(0,0)
B	(0,1)	(0,0)	(1*,2*)

Example for Best Responses

- Second find the best response of P2 to each action of P1
  (for each row find the highest payoff of P2)
- Best responses are indicated by stars to P2's payoffs.
- Find the boxes in which both players' payoff are starred.
- Such box is a NE:
  Star in P1's payoff means that P1's action is a best response to P2's action, and
  Star on P2's payoff means that P2's action a best response to P1 action.
- Conclude that the game has 2 Nash equilibria:
  (M,L) and (B,R).

	L	C	R
T	(1,2*)	(2*,1)	(1*,0)
M	(2*,1*)	(0,1*)	(0,0)
B	(0,1)	(0,0)	(1*,2*)

Multi-Player Games Example: Competitive Advantage of 3 Firms

Each firm has the choice between staying put and adopting the new technology

If no firm adopts the new technology then there is no competitive advantage and the payoff vector is (0,0,0)

If exactly one firm adopts the new technology then the firm gets the competitive advantage a, while each firm at a competitive disadvantage looses {-a/2} →(-a/2,-a/2,a) for Firm 3;

Thus, only if Firm 1 adopts the new technology, then the payoff vector is (a,-a/2, -a/2); i.e., Firm 1 takes market share from both Firm 2 & 3

(-a/2,a,-a/2) for Firm 2

Multi-Player Games Example: Competitive Advantage of 3 Firms

If exactly two firms adopt the new technology, then these two firms split the competitive advantage, each gaining $a/2$, and the firm at a disadvantage looses a

Finally if all firms adopt the new technology there is no competitive advantage and the payoff vector is $(0,0,0)$.

Multi-Player Games Example: Competitive Advantage of 3 Firms

FIRM 3: ADOPT

Firm 2 (columns) / Firm 1 (rows)

	Adopt	Stay put
Adopt	(0,0,0)	(a/2,-a,a/2)
Stay Put	(-a,a/2,a/2)	(-a/2,-a/2,a)

FIRM 3: STAY PUT

Firm 2 (columns) / Firm 1 (rows)

	Adopt	Stay put
Adopt	(a/2,a/2,-a)	(a,-a/2,-a/2)
Stay put	(-a/2,a,-a/2)	(0,0,0)

Multi-Player Games Example: Competitive Advantage of 3 Firms

Each firm has a dominant strategy which is to adapt the new technology

The unique equilibrium occurs when all 3 firms play the pure strategy: i.e., adopt the new technology

This is precisely what happened with 2 firms

No firm can be left behind in the race to adopt the new technology

This is as just true for n players as it is for 2 or 3.

How to Model CR Networks using Game Theory?

- **Player** → CR Users (and Primary Users)
- **Action (Strategy)**
  - CR Networks:
    - Which licensed channels will be used by the CR users?
    - Which transmission parameters (transmission power, time duration) to use for CR users? or
    - The price they agree to pay for leasing certain channels from the primary networks.

How to model CR networks using Game Theory?

Action (Strategy)

- Primary Networks:
  - Which unused spectrum they will lease?
  - How much they will charge CR users for using their spectrum resources, etc. ?

How to Model CR Networks using Game Theory?

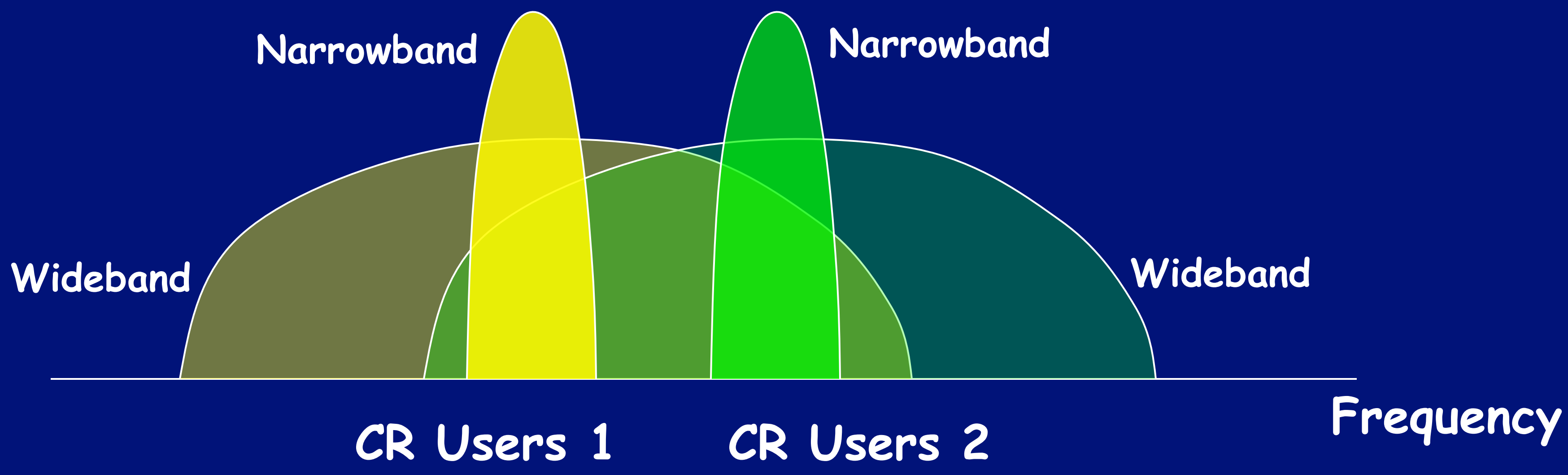
- **Outcome (Payoff)** → Network State (SNR, BW, etc)
- **Utility Functions** → Target QoS parameters (Throughput, Delay, BER, Cost, etc.)

Example Models

- **Player:** Two CR Users
- **Action:**

  Select either a low-power narrowband waveform N, or a higher power wideband waveform W
- **Outcome:** Network States (SNR, BW)
- **Utility Function:** Throughput
- **Preference:** To maximize throughput

Example Models



CR users 2

CR user 1		Narrowband	Wideband
	Narrowband	(9.6,9.6)	(3.2, 21)
	Wideband	(21,3.2)	(7,7)

Nash Equilibrium: (7,7)

(kbps)

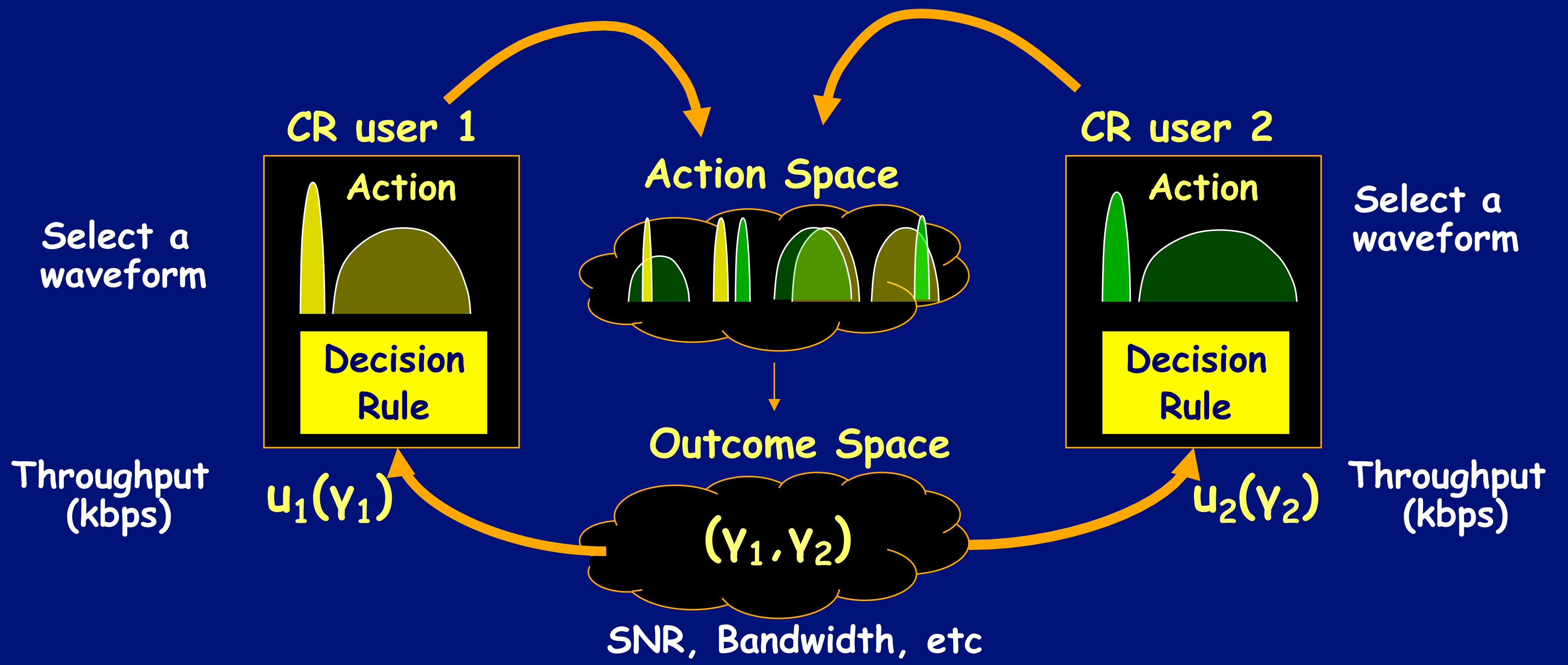
Repeated Game (Extensive Form Game)

- Specified by 4-tuple $\Gamma = \langle N, A, \{u_j\}, \{d_j\} \rangle$,

d_j: Decision rule

  - To adapt repeatedly with synchronous timing
  - Especially well-suited for wireless networks where users incorporate punishment and reward strategies

Repeated Game (Extensive Form Game)♪



Other Game Models for CR Networks

- **Myopic Repeated Game**
  - Specified by 5-tuple $\Gamma = \langle N, A, \{u_j\}, \{d_j\}, T_j \rangle$,

    T_j: decision timing

  - Adapts to the most recent state of networks under a variety of different decision timings

Other Game Models for CR Networks (Normal Form Game)

Mixed (Probabilistic) Strategy Game

- Specified by 3-tuple $\Gamma = \langle N, \Delta(A), \{U_j\} \rangle$,
- $\{U_j\}$: Expected utility of user j
- Mixed strategy for user: the probability of each action for user j
- $\Delta(A)$: all possible mixed strategy tuples
- Models scenarios where users can probabilistically play different waveforms

Other Game Models for CR Networks

- **Potential Game**
  - A normal form game $\Gamma = \langle N, A, \{u_j\} \rangle$

    which has the property that there exists a function known as *the potential function,*

    $$V : A \rightarrow R$$

    that reflects the change in value accrued by every unilaterally deviating player.

Other Game Models for CR Networks

- Exact Potential Game

If there exists a function, $V : A \rightarrow R$,
known as an exact potential function, that satisfies

$$\frac{\partial u_i(a)}{\partial a_i} = \frac{\partial V(a)}{\partial a_i} \quad or \quad \frac{\partial^2 u_i(a)}{\partial a_i \partial a_j} = \frac{\partial^2 u_j(a)}{\partial a_j \partial a_i}, \quad \forall i, j \in N, a \in A$$

Other Game Models for CR Networks

- **Supermodular Game**
  - A normal form game, $\Gamma = \langle N, A, \{u_j\} \rangle$, if all players' utility functions satisfy

$$\frac{\partial^2 u_i(a)}{\partial a_i \, \partial a_j} \geq 0, \quad \forall i \neq j \in N, a \in A$$

SPECTRUM SHARING CLASSIFICATION

Intra-Network SS

- Centralized (Infrastruct. based)
- Distributed (Ad hoc - based)
  - Cooperative
  - Non-cooperative

Inter-Network SS

* Centralized
* Distributed

Intranetwork Spectrum Sharing: Centralized

1. Auction Based Spectrum Sharing Game

   J. Huang, R. Berry, and M. L. Honig,

   ACM Monet Journal, 2006

Intranetwork Spectrum Sharing: Distributed – Cooperative

1. Local Bargaining
   Cao/Zheng, IEEE SECON 2005.

2. Interference Compensation Based Spectrum Sharing
   J. Huang, R. A. Berry, M. L. Honig,
   "Spectrum Sharing with Distributed Interference Compensation,"
   Proc. IEEE DySPAN, Nov. 2005.

Intranetwork Spectrum Sharing: Distributed – Non-Cooperative

1. Device Centric Approach:
   H. Zheng and L. Cao,
   "Device-centric Spectrum Management,"
   Proc. IEEE DySPAN, Nov. 2005.

2. Belief Assisted Pricing
   J. Zhu and Ray Li,
   Proc. of IEEE SECON, 2006.

Centralized Intranetwork Spectrum Sharing

- A centralized node (e.g., CR base station) controls the spectrum allocation and access procedures.

- Each CR user forwards their measurements about the spectrum allocation to the BS which then constructs a spectrum allocation map.

Background on Auction Theory

- Highest bidder gets the good and pays the bid
- **Elements of auction:**
  - * Good: resource
  - * Auctioneer (Manager): representing seller of the good
  - * Bidders (Users): buyers of the good
- **Rules of auction:**
  - * Bids: what the bidders submit to the auctioneer
  - * Allocation: how auctioneer allocates the good to the bidders
  - * Payments: how the bidders pay the auctioneer
- **Types of Auctions**
  - * Indivisible Auction
  - * Divisible Auction: suitable for communication resource allocation

AUCTION THEORY IN SPECTRUM SHARING

- Auction is a process of buying and selling goods or services through a bidding process.
- Goods or services are sold to the winning bidders.
- Auction is applied when the price of the goods and services is undetermined and it varies with demand

Auction Based Spectrum Sharing Game

J. Huang, R. Berry, and M. L. Honig,
"Auction-based Spectrum Sharing,"
ACM/Springer Mobile Networks and Apps., 2006.

Auction mechanisms for Spectrum Sharing subject to interference temperature at a measurement point.

Two Auctions to allocate the received power:

1. Weighted max-min fair SINR Allocation
   Users are charged for received SINR combined with logarithmic utilities

2. Auction for Power → maximizes total utility when BW is large enough and receivers are co-located.

One-Dimensional Auctions with Pricing (Power-based / SINR-based)

Two Different Payments

- SINR Auction: User n pays $C_n(\pi) = \pi \, SINR_n$

  * User-centric payment

  * Proportional to user's achieved QoS (SINR)

  * Leads to fair allocation

- Power Auction: User n pays $C_n(\pi) = \pi \, p_n \, h_n$

  * Network-centric payment

  * Proportional to the allocated resource (power)

  * Leads to efficient allocation

Requirement for Efficient and Fair Spectrum Sharing

- Interference Temperature Constraint P

- Condition for satisfying the above constraint:

  Total received power at a specified measurement point must satisfy

$$\sum_{i=1}^{M} p_i h_{i0} \le P$$

where p_i – User i's transmitted power

h_{i0} – Channel gain from User i's tansmitter to measurement point

System Model

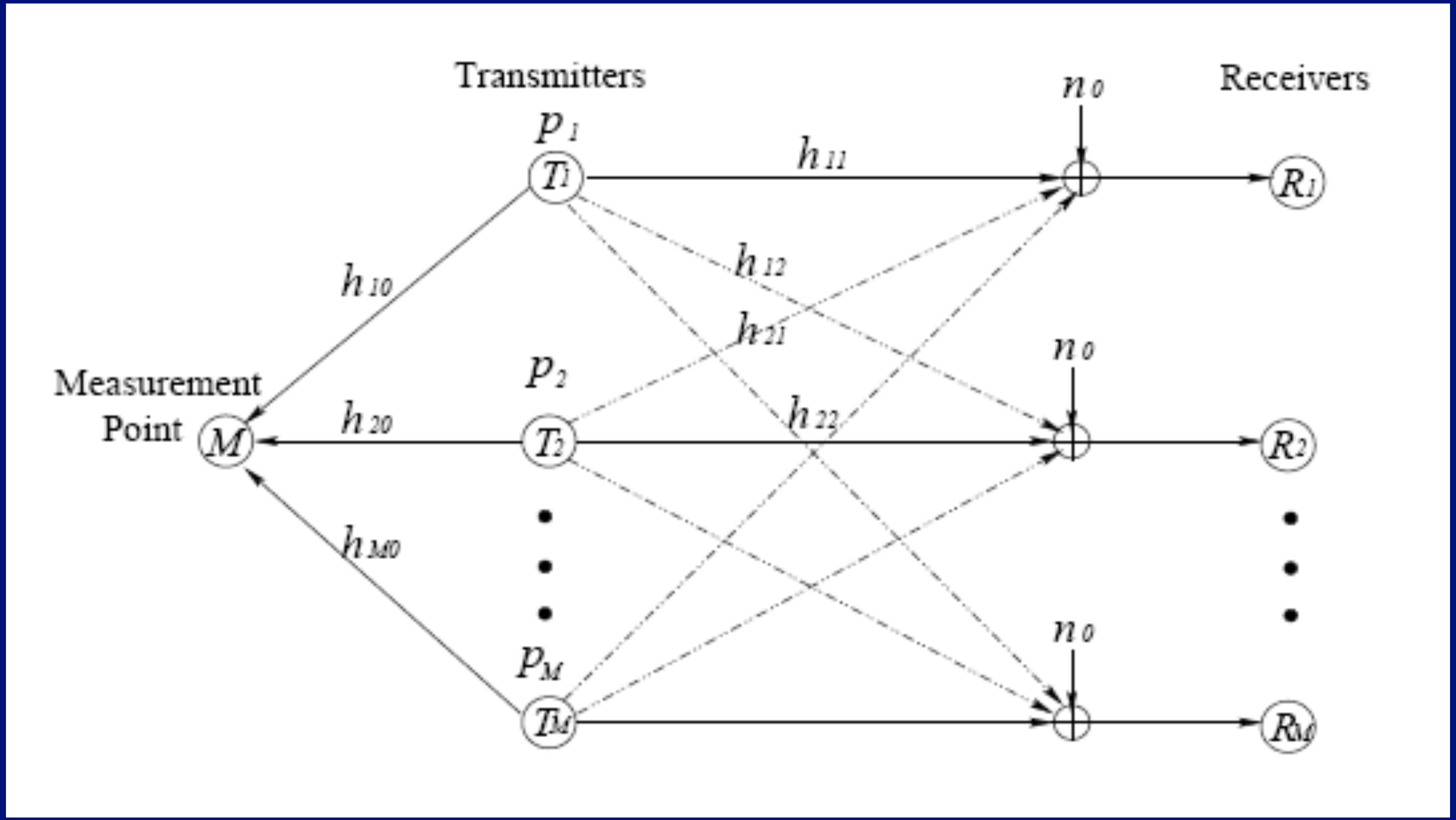


Figure: System Model for M Transmitters-Receiver pairs

Notations:

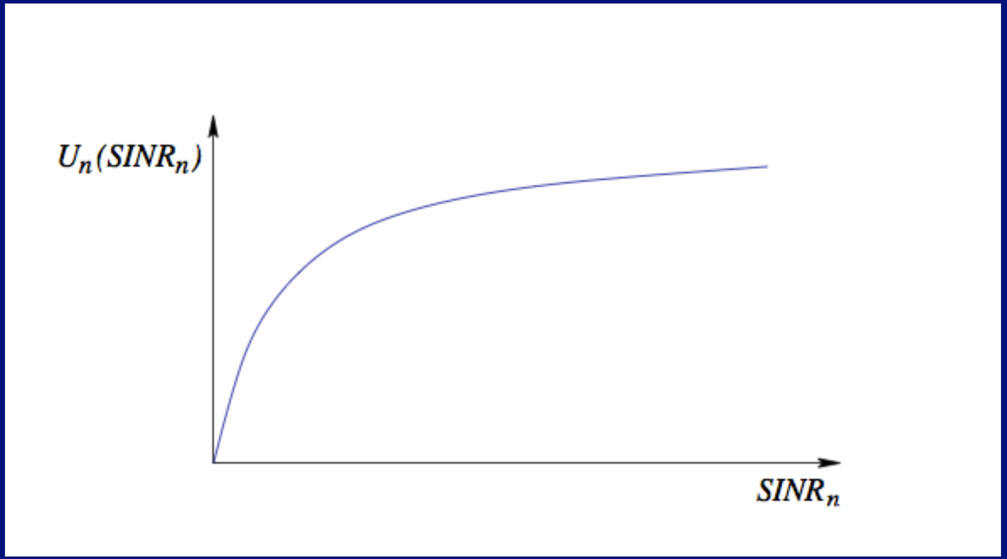
p_i – User i's Tx power

h_{ij} – channel gain from user i's transmitter to user j's Receiver

h_{i0} – channel gain from user i's Tx to measurement point

System Model

- Spectrum covers a Bandwidth B
- This spectrum is to be shared among M spread spectrum users.
- User i's evaluation of the spectrum is characterized by a utility $U_i(\gamma_i)$, where γ_i = Received SINR at user i's receiver



System Model

- For each i, the received SINR at user i is:

$$\gamma_i = \frac{p_i h_{ii}}{n_0 + \frac{1}{B}\left(\sum_{j \neq i} p_j h_{ji}\right)}$$

where p_i is user i's transmission power

h_{ij} is the channel gain

n_0 = Background Noise Power (same for all users)

System Model

Lemma: A power allocation scheme is Pareto optimal if and only if the total received power constraint is tight, i.e.,

$$\sum_{i=1}^{M} p_i h_{i0} \leq P$$

thus, a power allocation is Pareto Optimal, if no user's utility is increased without decreasing another user's utility

System Model

A Special Case

If all receivers are co-located with the measurement point, then

$h_{i,j} = h_{i,0}$, for all $i,j \in \{1,2,\ldots,M\}$ $\quad p_i^r = p_i h_{i0}$

and in the Pareto optimal allocation for each user *i, SINR for each user i is:*

$$\gamma_i \equiv \gamma_i(p_i^r) = \frac{p_i^r}{n_0 + \frac{1}{B}(P - p_i^r)}$$

System Model

REMARK:

User i's utility $U_i(\gamma_i(p_i^r))$ under a Pareto optimal allocation does NOT depend on how the power is allocated among the interferers.

Network Objective I: Efficiency

Efficiency: **Maximize the total network Utility:**

Efficiency Problem

$$\begin{aligned} &\text{maximize} \sum_n U_n(\mathrm{SINR}_n(\mathbf{p})) \\ &\text{subject to} \sum_n p_n h_n \le P \\ &\text{variables } p_n \ge 0, \forall n \end{aligned}$$

Example: $U_n(\mathrm{SINR}_n) = \theta_n \log(1 + \mathrm{SINR}_n)$

Maximizing total weighted rate

Network Objective II: Fairness

Fairness: Fair share of resources, independent of location

$$\begin{aligned}
&\text{maximize } \mathrm{SINR}_1(\mathbf{p}) \\
&\text{subject to } U'_n(\mathrm{SINR}_n(\mathbf{p})) = U'_m(\mathrm{SINR}_m(\mathbf{p})), \forall m \neq n \\
&\qquad\qquad \sum_n p_n h_n \leq P \\
&\text{variables } p_n \geq 0, \forall n
\end{aligned}$$

Example: $U_n(\mathrm{SINR}_n) = \theta_n \log(\mathrm{SINR}_n)$

Weighted max-min fair

Challenges

- Non-convexity:
  * SINR and utility may not be concave in power
- Physically distributed:
  * Local Information: utility functions, channel gains
  * Selfish Objectives
- Performance Coupling:
  * Mutual Interference
  * Shared received power at measurement point
- Solution: Auction-based Resource Allocation Algorithm
  * Distributed in nature
  * Capture interactions between users

Evolution of Auction-Based Spectrum Sharing

Each user's utility is PRIVATE information

↓

Manager does NOT have a-priori knowledge of the information for each user

↓

A Mechanism is required for the purpose of power allocation without the manager having prior knowledge of channel gains, h_{ij}.

↓

AUCTION SCHEMES

AUCTION SCHEMES

- VCG AUCTION
- One-Dimensional Auctions with Pricing

VCG (Vickrey-Clarke-Groves) AUCTION

- A simple and rudimentary auction technique
- A weakly dominant strategy for users to bid truthfully
- How is it done ?

1. Users are asked to submit their utilities $U_i(\gamma_i)$
2. Manager computes the power allocation $p^* = (p_1^*, \ldots, p_M^*)$
3. Manager allocates power to users accordingly

$$U_{\max} = \sum_{j=1}^{M} U_j(\gamma_j(p^*))$$

Drawbacks of VCG Auction

- Excessive burden on the users due to several measurements →

  Computationally expensive for large *M*

- Stressful for the Manager due to (M+1) optimization problems which are non-convex due to interference

- Not suitable for on-line allocations.

AUCTION SCHEMES

VCG AUCTION

One-Dimensional Auction with Pricing

One-Dimensional Auction with Pricing

- **PRINCIPLE OF AUCTION:**

  In a shared auction, the user submits his demand curve &
  the auctioneer computes a market clearing price based on the set of demand curves

- **What is a Demand Curve?**

  The amount of goods/resources of a user desires as a function of the price.

One-Dimensional Auction with Pricing

- In Cognitive Radio Networks,
  CR users submit the demand curve in terms of Received Power or SINR
- Issues with Received Power as Demand Curve:
  it depends on the demands of the other users due to Interference
- Issues with SINR as Demand Curve:
  it is independent of other users, however, the market clearing price is NOT easy to find due to the received power constraint

One-Dimensional Auction with Pricing

- **SOLUTION:**
  A SIGNALLING SYSTEM (SINR-based or Power-based)

- **Role of the users in SS:**
  they submit one-dimensional bids (i.e., signals) representing their willingness to pay

- **Role of the manager (auctioneer) in SS:**
  it allocates the received power in proportion to the bids

One-Dimensional Auction with Pricing

- How does this auction scheme work?

The ultimate goal is to achieve Nash Equilibrium of the auction.

Assume:
All user's utilities and all channel gains are known to all users.

One-Dimensional Auction with Pricing: Algorithm

1. Manager announces two parameters:
   * A reserve bid $\beta >= 0$
   * A Price for power auction $\pi^p > 0$ or
   A Price for SINR auction $\pi^s > 0$

2. After observing these values, User $i \in \{1,\ldots,M\}$ submits a bid $b_i >= 0$

3. Manager keeps reserve power p_0 and allocates to each user i, a transmission power p_i, so that the received power at the measurement point is proportional to the bids, i.e.,

One-Dimensional Auction with Pricing: Algorithm

p_i=Assigned power to i

$$p_i h_{i0} = \frac{b_i}{\sum_{i=1}^{M} b_i + \beta} P$$

and

p_0=Reserved power

$$p_0 = \frac{\beta}{\sum_{i=1}^{M} b_i + \beta} P$$

Also, the resulting SINR for user *i* is

$$\gamma_i = \frac{p_i h_{ii}}{n_0 + \frac{1}{B}\left(\sum_{j \neq i} p_j h_{ji} + p_0 h_{0i}\right)}$$

h_{oi} → channel gain from the manager to user i's receiver. If $\sum_{i=1}^{M} b_i + \beta = 0$ then p_i=0. If h_{oi}=0 for all i ε {1,..,M} then the manager does not interfere with the users. In the co-located case we have h_{0i}=1 for all i.

One-Dimensional Auction with Pricing: Algorithm

4. User *i* pays $C_i = \pi^s \gamma_i$ or $C_i = \pi^p p_i h_{i0}$

- SINR Auction

$$C_i = \pi^s \gamma_i = \pi^s \frac{p_i h_{ii}}{n_0 + \frac{1}{B}\left(\sum_{j \neq i}^{M} p_j h_{ji} + p_0 h_{0i}\right)}$$

- Power Auction

$$C_i = \pi^p p_i h_{i0}$$

Nash Equilibrium of the Auctions

- A bidding profile is the vector containing the user's bids

$$\underline{b}=(b_1, \ldots, b_M)$$

- The bidding profile of user i's opponents is defined as

$$\underline{b}_{-i}=(b_1,\ldots, b_{i-1},b_{i+1},\ldots,b_M)$$

So that $\mathbf{b}=(b_i;b_{-i})$

- Each user i submits a bid b_i to maximize his Surplus Function

$$S_i(b_i;b_{-i})=U_i(\gamma_i(b_i;b_{-i}))-C_i$$

β and Π are omitted

Nash Equilibrium of the Auctions

Nash Equilibrium of the auction is associated with Bidding Profile b* s.t.

$$S_i(b_i^*;b_{-i}^*) \geq S_i(b_i^{'};b_{-i}^*) \quad \textit{for any } b_i^{'} \in [0,\infty) \quad \text{\& any user i}$$

$$b^* = (b_1^*, b_2^*, \cdots, b_M^*)$$

Define user i's best response given b_{-i} as the set

$$B_i(b_{-i}) = \left\{ \overline{b_i} \mid \overline{b_i} = \arg\max S_i(b_i;b_{-i}) \right\}$$

$$b_i \varepsilon [0,\infty)$$

i.e., the set of b_i's that maximize $S_i(b_i;b_{-i})$ given a fixed b_{-i}.

Nash Equilibrium of the Auctions

* NE bidding profile b* is a fixed point, i.e., no user has the incentive to deviate unilaterally.

* The existence and uniqueness of an NE depend on β and π^s (or π^P).

* Manager can influence the NE by choosing β and π^s (or π^P).

* Allows to reach Pareto optimal solutions instead of NE.

Tracing of the Auction Mechanism

- CONSTRAINTS:
  - P is the total power can be transmitted by CR users,
  - h_{i0} is the gain to the measurement (reference) point (BS)
  - Assumption of knowledge for all channel gains

$$\sum_{i=1}^{M} p_i h_{i0} \le P$$

- Manager announces to all CR nodes
  - A reserve bid, e.g., $\beta=5$
  - A price (based on SINR or power auction), e.g., $p_i=1$
  - Transmission power is computed at transmitter according to the allowed receiver power

$$p_i h_{i0} = \frac{b_i}{\sum_{i=1}^{M} b_i + \beta} P$$

$$\gamma_i = \frac{p_i \, h_{ii}}{n_0 + \frac{1}{B}\left(\sum_{j \neq i} p_j h_{ji} + p_0 h_{0i}\right)}$$

Example: Tracing of the Auction Mechanism

- Reserve bid $b_0 = 5$ P=20
- User bids $b_i = 3,5,2$ for i=1,2,3

$$p_1 h_{10} = \frac{3}{\sum_{i=1}^{M} b_i + 5} 20 = 4 \qquad p_2 h_{20} = \frac{5}{\sum_{i=1}^{3} b_i + 5} 20 = \frac{20}{3} \qquad p_3 h_{30} = \frac{2}{\sum_{i=1}^{3} b_i + 5} 20 = \frac{8}{3}$$

- For same channel gains h=0.5,
  - Allocation proportional to bid

$$p_1 = 8 \qquad p_2 = 40/3 \qquad p_3 = 16/3$$

Example: Tracing of the Auction Mechanism

- Note that manager decides on the transmitted power based on the receiver
  - It does not consider PU location in the network, it maybe closer than the CR receiver

- Cost is the multiplication of received power by price paid per received power

$$C_i = \pi^p p_i h_{i0} \qquad \pi^p = 3$$

$$C_1 = 12 \qquad C_2 = 20 \qquad C_3 = 8$$

Example: Tracing of the Auction Mechanism

- Auction is repeated until SURPLUS is maximized for each transmitter

$$S_i(b_i;b_{-i}) = U_i(\gamma_i(b_i;b_{-i})) - C_i$$

- Example: Utility Function $U_i = \ln(\gamma_i(b_i;b_{-i}))$,

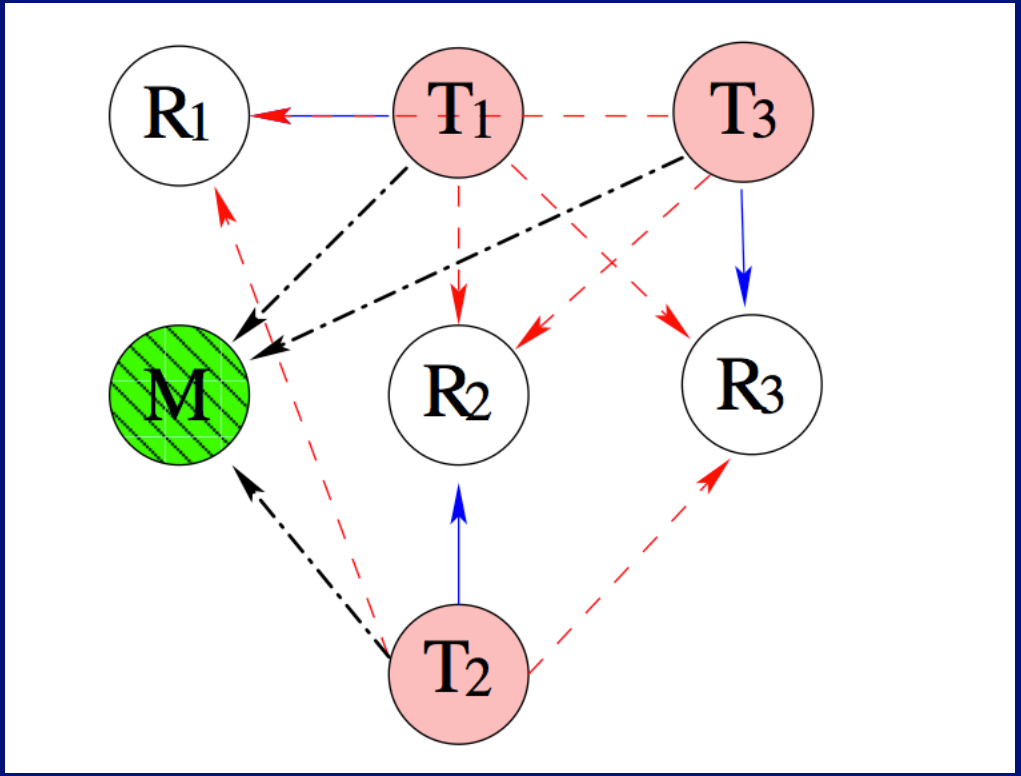
- For SNR calculation: $n_0 = 10^{-10}$ $B = 10^6$

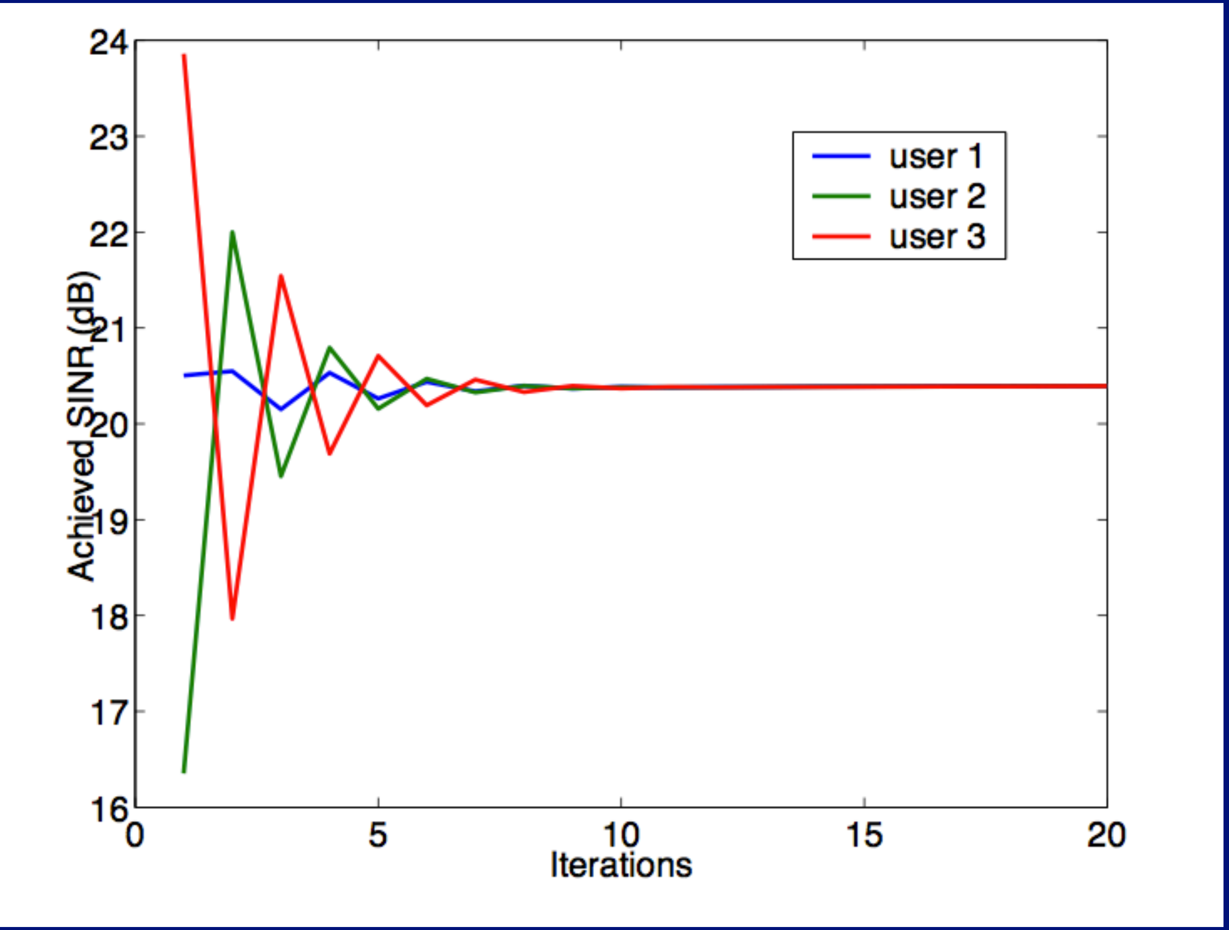
$$S_1 = \ln\frac{4}{10^{-10} + \frac{1}{10^6}15} - 12 = 0.4938$$

$$S_2 = \ln\frac{20/3}{10^{-10} + \frac{1}{10^6}15} - 20 = -4.2874$$

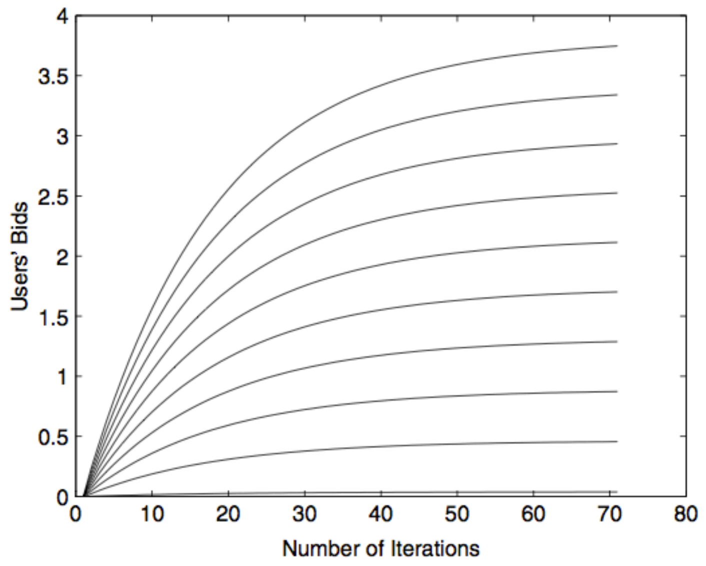
$$S_3 = \ln\frac{8/3}{10^{-10} + \frac{1}{10^6}15} - 8 = 6.7963$$

SINR AUCTION

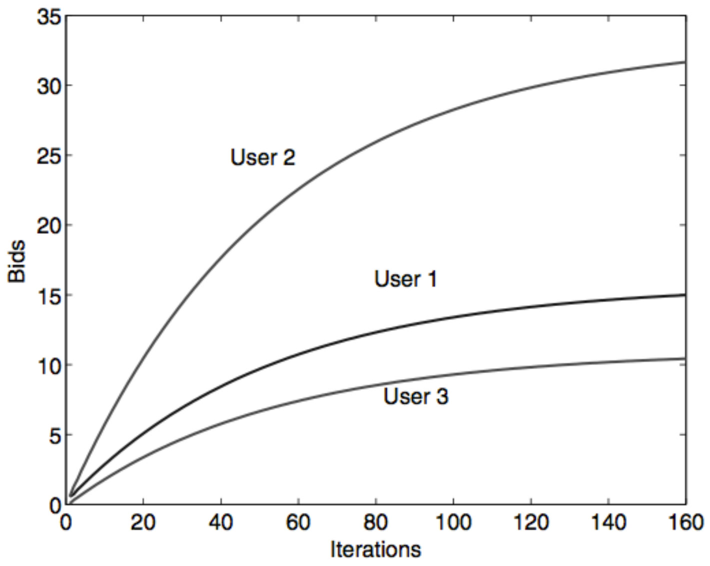




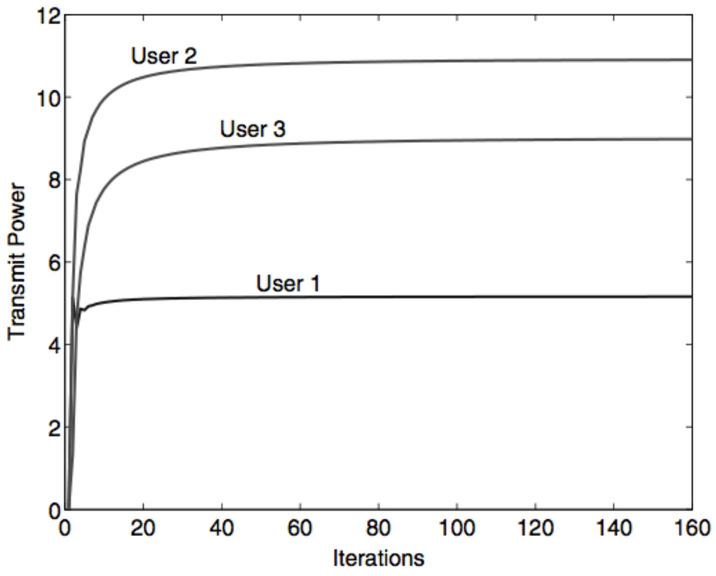
SINR AUCTION



Bids for each user



Convergence of transmit power in the three-user network



Convergence of transmit power in the three-user network

SPECTRUM SHARING CLASSIFICATION

Intra-Network SS

- Centralized (Infrastruct. based)
- Distributed (Ad hoc - based)
  - Cooperative
  - Non-cooperative

Inter-Network SS

* Centralized
* Distributed

Intra-Network Spectrum Sharing - Distributed & Cooperative

- Each CR user is responsible for the spectrum allocation and access is based on local policies.
- CR users exchange their information with other neighboring users for spectrum access
- Cooperative (or collaborative) solutions consider the effect of the CR user's communication on other users.
- I.o.w. the interference measurements of each CR user are shared among other CR users.

Local Bargaining - Motivating Factors

L. Cao, H. Zheng, "Distributed Spectrum Allocation via Local Bargaining,"
Proc. IEEE Sensor and Ad Hoc Communications and Networks (SECON), Sept. 2005.

- In a mobile (ad hoc) network, users are constantly moving and the network topology changes.
- Therefore, the network needs to completely re-compute spectrum assignments for all users after each change.

  → Centralized approach based on global optimization is infeasible

COOPERATIVE LOCAL BARGAINING

- A cooperative local bargaining (LB) scheme → for both spectrum utilization and fairness.
- Construct local groups according to a poverty line

  → ensures a minimum spectrum allocation to each user and

  hence focuses on fairness of users.

Problem Model and Utility Functions

- N={1,…,N} SUs competing for M={1,…, M} spectrum channels
- SUs select communication channels and adjust their transmit powers accordingly to avoid interference with PUs.
- Spectrum Access Problem → A Channel Allocation Problem,
  i.e., to obtain a conflict free channel assignment for each user that max. utility.

DEFINITIONS

1. **Channel Availability L(n)**

   Let $L(n) = \{1 \le m \le M \mid l_{m,n} = 1\}$ be the set of channels available at n.

   $l_{m,n} = 0$ → Channel m is occupied by PU.

2. **Interference Constraint C**

   Let $C = \{c_{n,k} | c_{n,k} \in \{0, 1\}\}_{N \times N}$, → NxN matrix,
   represents the interference constraints among users.

   If $c_{n,k}=1$, users n and k would interfere with each other if they use the same channel.

DEFINITIONS

3. Conflict Free Assignment A

$A = \{a_{m,n} | a_{m,n} \in \{0, 1\}\}_{M \times N}$ where $a_{m,n} = 1$

denotes that spectrum band m is assigned to user n, otherwise 0.

A satisfies all the constraints defined by C, i.e.,,

$$a_{m,n} + a_{m,k} \le 1, \text{ if } c_{n,k} = 1, \forall\ n, k < N, m < M.$$

4. User Dependent Channel Throughput B

Let $B = \{b_{m,n} > 0\}_{M \times N}$ describe the reward that a user gets by successfully acquiring a spectrum band m

$b_{m,n}$ represents the max BW/throughput that user n can acquire through using spectrum band m (assuming no interference from neighbors)

DEFINITIONS

5. **User Throughput of a Conflict Free Assignment**

Let $TP_A(n)$ represent the throughput that user n gets under assignment A, i.e.,

$$TP_A(n) = \sum_{m=1}^{M} (a_{m,n} . b_{m,n})$$

6. OBJECTIVE: Do optimal spectrum allocation (in terms of total user throughput) and maximize the total network utilization, i.e., A*=max argmax U(A) with the utility

$$U(A) = \sum_{n=1}^{N} \log(TP_A(n))$$

$$U(A) = \sum_{n=1}^{N} (TP_A(n))$$ → TOTAL USER THROUGHPUT

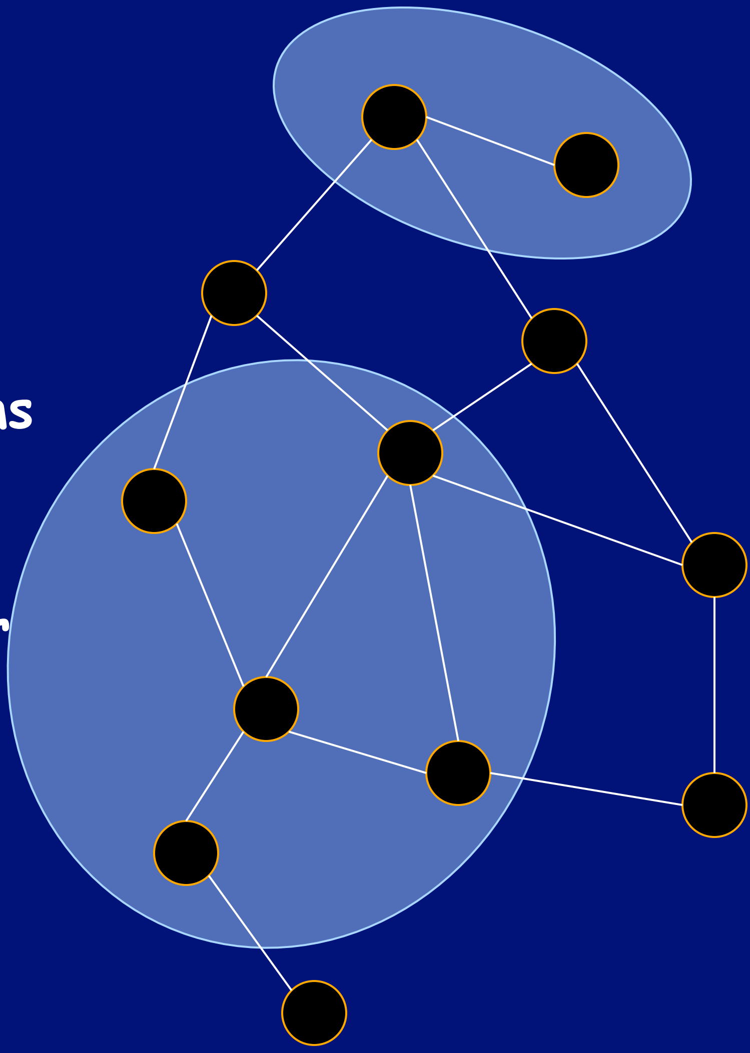
Local Bargaining – The Idea

- Devices self-organize themselves into bargaining groups

- Requester becomes group coordinator and performs bargaining computations

- Members of each group coordinate to adjust their spectrum usage

- Advantage:
  low cost, quick adaptation to network dynamics

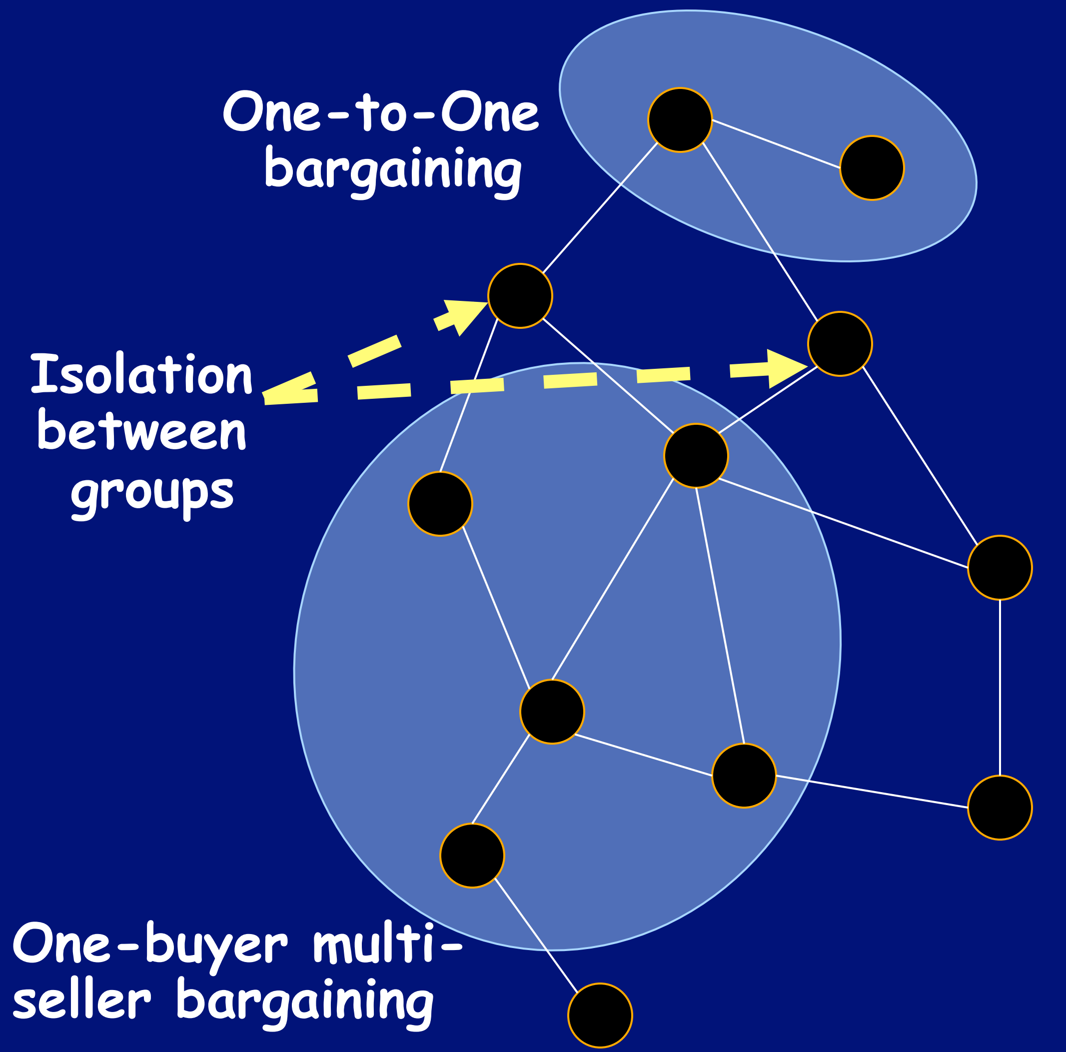


Local Bargaining – Constraints

Two Bargaining Strategies:

1. Limited Neighbor Bargaining
   1.1. One-to-one Bargaining
   1.2. One-buyer-multi-seller bargaining

2. Self Contained Group Bargaining
   2.1. Restricted Bargainable Channels
   2.2. Isolated Bargaining Groups



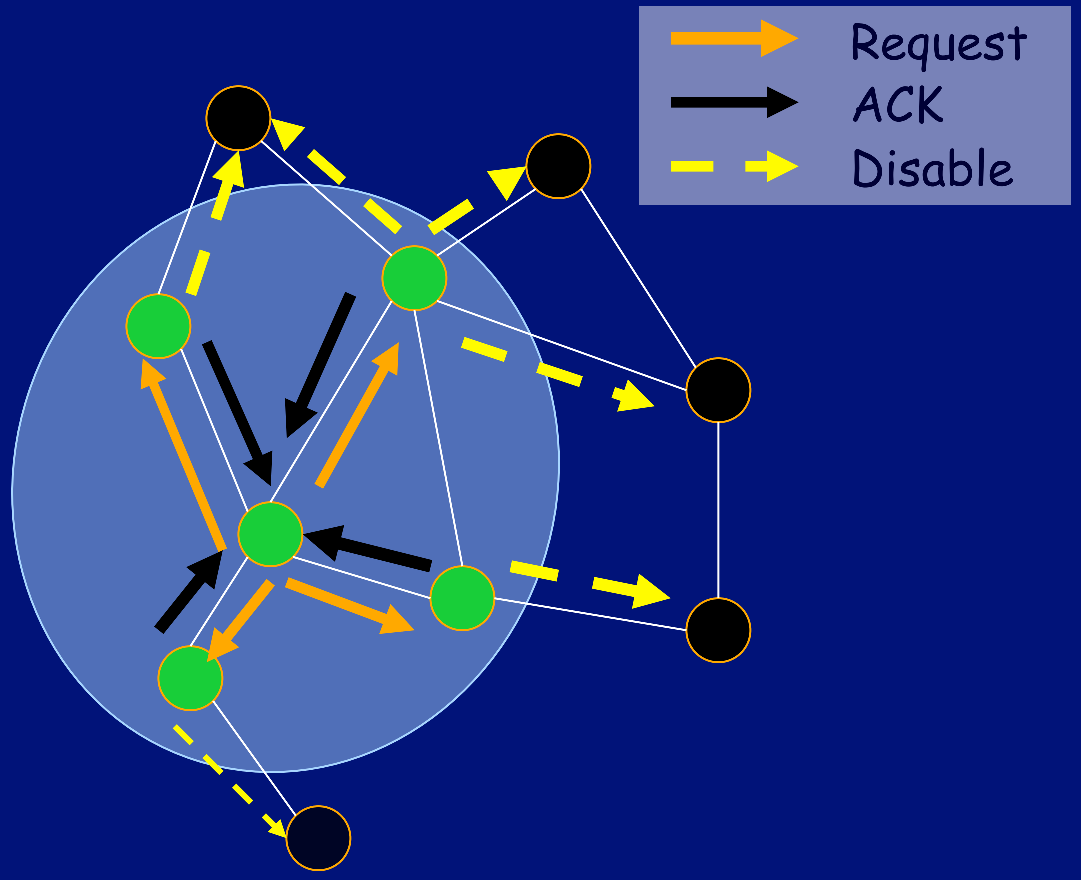
Bargaining Procedures

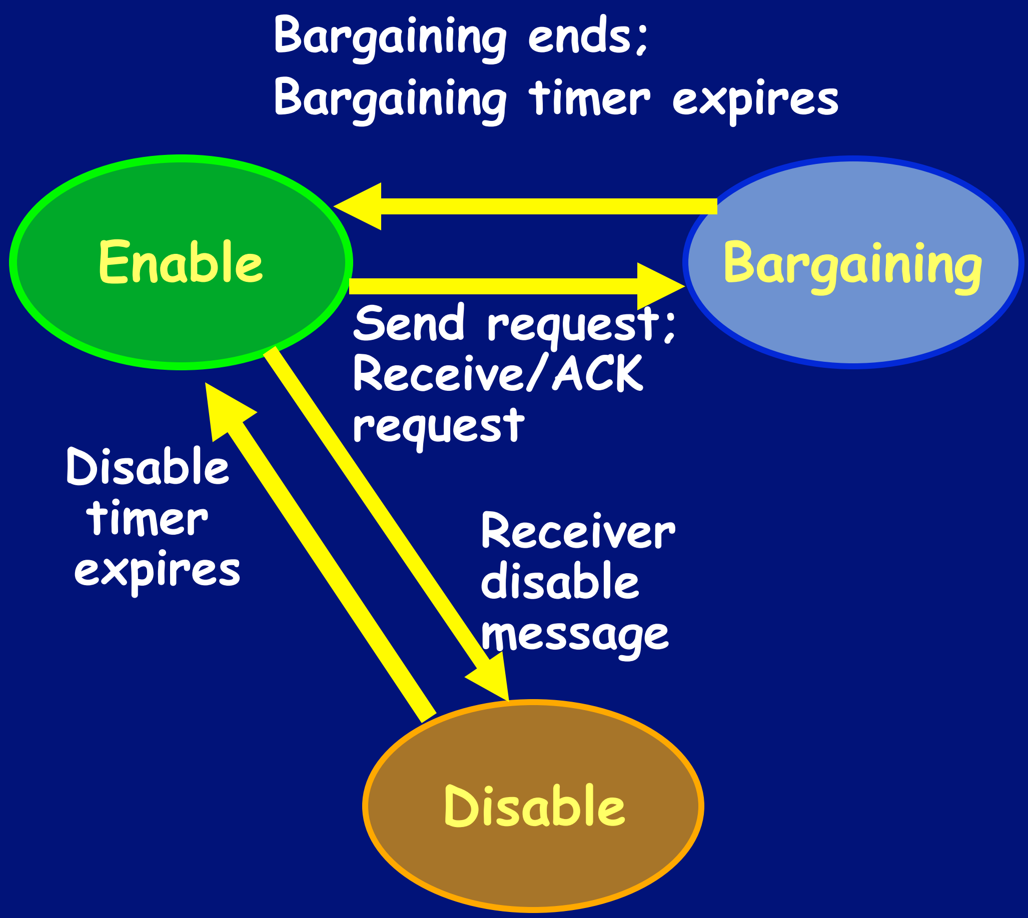
- Initialize bargaining request
- Acknowledge bargaining request
- Bargain group formation
- Bargaining
- Group dismissed

Each node has 3 states:

Bargaining Procedures

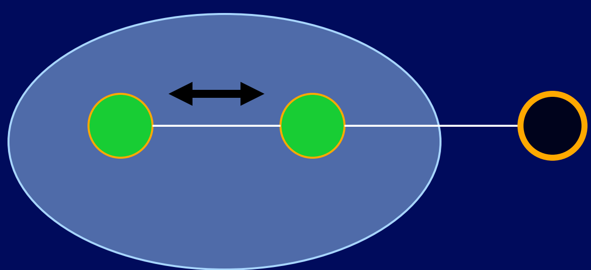
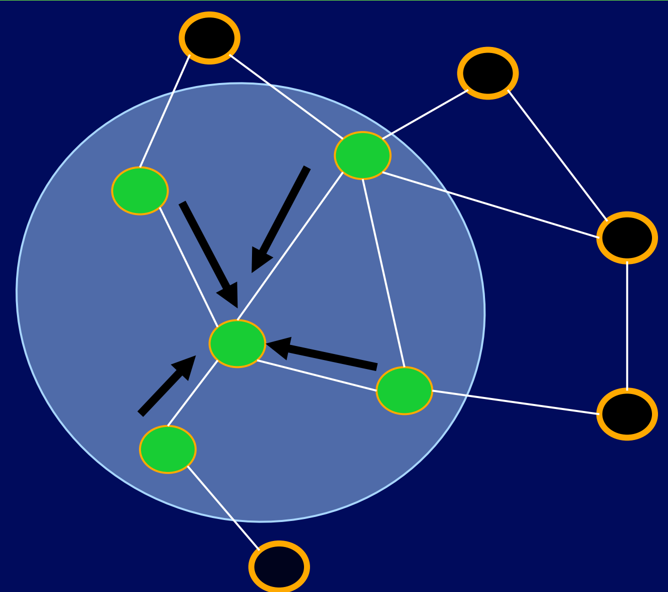
Only enabled nodes can perform bargaining.





Local Bargaining Schemes for Fairness

Global fairness utility increases if nodes with many assigned channels "give" some channels to nodes with few assigned channels.

1. One-to-One Bargaining	2. One-buyer-multi-seller bargaining (Feed Poverty Bargaining)
	

LOCAL BARGAINING SCHEMES

1. One-to-One Fairness Bargaining

- Allows two neighboring nodes n_1 and n_2 to exchange channels to improve system utility while complying with conflict constraints from other neighbors.

- Nash Bargaining Scheme (NBS): Optimization goal of local bargaining

For a current spectrum utilization $A_{M \times N}$ ONE-to-ONE FAIRNESS BARGAINING finds nodes n_1 and n_2 and their bargaining channel set $C_b(n_1,n_2)$ and modifies $A_{M \times N}$ to $A'_{M \times N}$ related to n_1, and n_2 and channels $C_b(n_1,n_2)$ s.t.

$$TP_{A'}(n_1) \cdot TP_{A'}(n_2) > TP_A(n_1) \cdot TP_A(n_2)$$

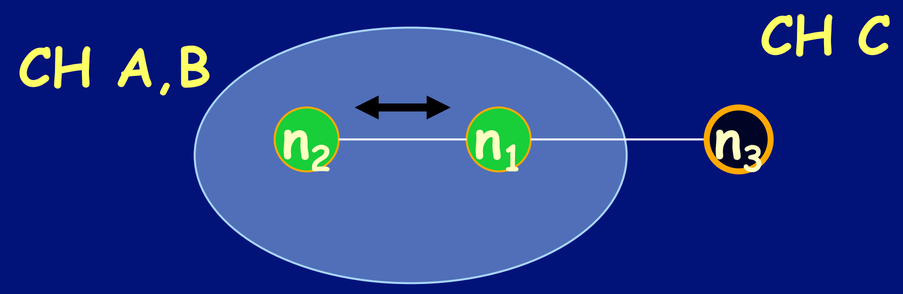
$TP_A(n)$ represent the throughput that user n gets under assignment A

EXAMPLE: LOCAL BARGAINING SCHEMES

1. One-to-One Fairness Bargaining

- n_2 and n_1 calculate the throughput of each user for A and A'
  - User Throughput: $TP_A(n) = \sum_{m=1}^{M} (a_{m,n} . b_{m,n})$

$$\begin{array}{c} \quad n_1 \quad n_2 \quad n_3 \\ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \text{Ch A} \\ \text{Ch B} \\ \text{Ch C} \end{array} \end{array} \qquad B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



$TP_A(n_1)=0+0+0=0$ $\qquad$ $TP_A(n_2)=1+1+0=2$

  - If n_2 gives Ch B to n_1, new channel assignment is denoted by A'

$$A' = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

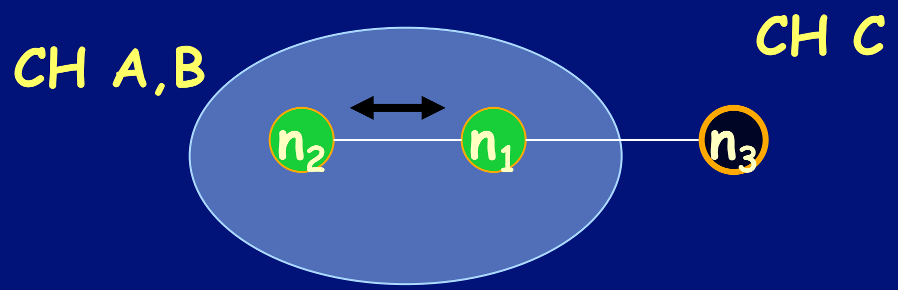
$TP_{A'}(n_1)=0+1+0=1$ $\qquad$ $TP_{A'}(n_2)=0+1+0=1$

EXAMPLE: LOCAL BARGAINING SCHEMES

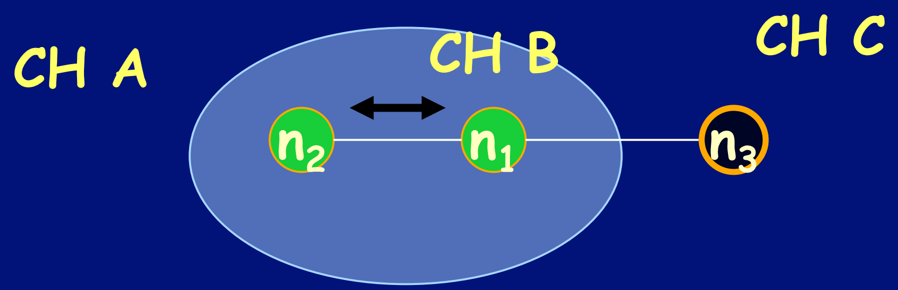
1. One-to-One Fairness Bargaining

- n_2 and n_1 compare the channel assignments A and A'

Initial assignment A



Assignment after bargaining A'



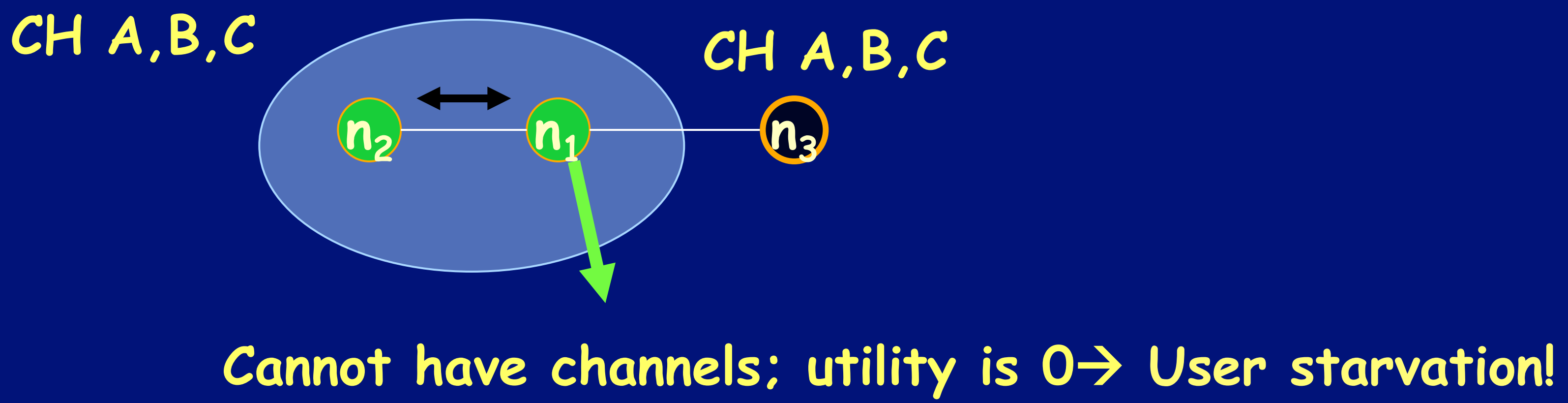
$$TP_A(n_1)=0 \;.\; TP_A(n_2)=2 \quad < \quad TP_{A'}(n_1)=1 \;.\; TP_{A'}(n_2)=1$$

After bargaining we increase Fairness

LOCAL BARGAINING SCHEMES

Problem of One to One Fairness Bargaining

- The effectiveness of one-to-one bargaining is constrained by the size of $C_b(n_1,n_2)$ → Cannot eliminate User Starvation



- $C_b(n_1,n_2)$: # of bargainable channels between n_1 and n_2)
- n_1 and n_2 cannot bargain due to constraint from n_3, i.e., $C_b(n_1,n_2) = \varnothing$
- n_1 and n_3 cannot bargain due to constraint from n_2, i.e., $C_b(n_1,n_3) = \varnothing$

→ FAIRNESS BARGAINING is not effective to eliminate USER STARVATION!

LOCAL BARGAINING SCHEMES

2. ONE BUYER MULTI SELLER: Feed Poverty Bargaining

- **Remedy for user starvation:**
  If n_2 and n_3 can give up channel A at the same time and feed it to n_0, we can remove the starvation at n_0 (example for one buyer multiseller bargaining)

- If a node (buyer) has very poor channel assignment, the neighboring nodes can collaborate together to feed it with some channels (also called a one-buyer-multi-seller bargaining).

- For an assignment $A_{M\times N}$, a feed poverty bargaining is to find some node n_1 and channel m_1, modify $A_{M\times N}$ to $A'_{M\times N}$, such that

$$A'_{m,n} = \begin{cases} 1 & : \; m = m_1 \text{ and } n = n_1 \\ 0 & : \; m = m_1 \text{ and } n \in Nbr(n_1) \\ A_{m,n} & : \; \text{otherwise} \end{cases}$$

$Nbr(n_1)$: A set of neighbors of node n_1

Fairness Bargaining with Feed Poverty (BF)

- Combine ONE-TO-ONE FAIRNESS BARGAINING and FEED POVERTY BARGAINING
- Each node who wants to improve its spectrum usage starts with negotiating One-to-One Fairness Bargaining with its neighbors.
- If there are no bargainable channels between itself and any of its neighbors (i.e., $|C_b| = \emptyset$,), that node (i.e., a starving node) can broadcast a Feed-Poverty request to its neighbors to initialize Feed Poverty Bargaining.
- The requestor sequentially selects multiple channels to maximize group utility.

Fairness Bargaining with Feed Poverty (BF)

- Overall, a channel assignment A is said to be BF-optimal if no further Fairness Bargaining with Feed Poverty can be performed on it.

- Poverty Line Guided Bargaining

  A node is entitled to request bargaining if its current throughput is below its poverty line

Fairness Bargaining with Feed Poverty (BF)

Minimum amount of spectrum a node is entitled to.

Poverty Line

$$TP(n) \geq \left\lfloor \frac{l_n}{d_n + 1} \right\rfloor$$

TP(n):
Spectrum usage of user n
(# of channels)

- l_n : # of available channels for user n
- d_n: # of the neighboring/ conflicting nodes for user n

Interference Compensation Based Spectrum Sharing

J. Huang, R. A. Berry, M. L. Honig, "Spectrum Sharing with Distributed Interference Compensation," Proc. IEEE DySPAN, Nov. 2005.

- **Interference Compensation:**
  - Each CR user senses the signal at a particular channel
  - Calculates how much interference will be created if it transmits on that channel.
  - If that limit is below a threshold then it sends on that channel. (Cooperative!!!)
- **Considers the problem of joint channel selection and power control**

SYSTEM MODEL

- Each user is represented by a transmitter-receiver node pair
- Single-hop and half-duplex transmissions
- Multiple channels available with fixed gains (slow fading)
- No centralized controller
- How to select channel and power in a distributed way with limited (scalable) information exchange ?

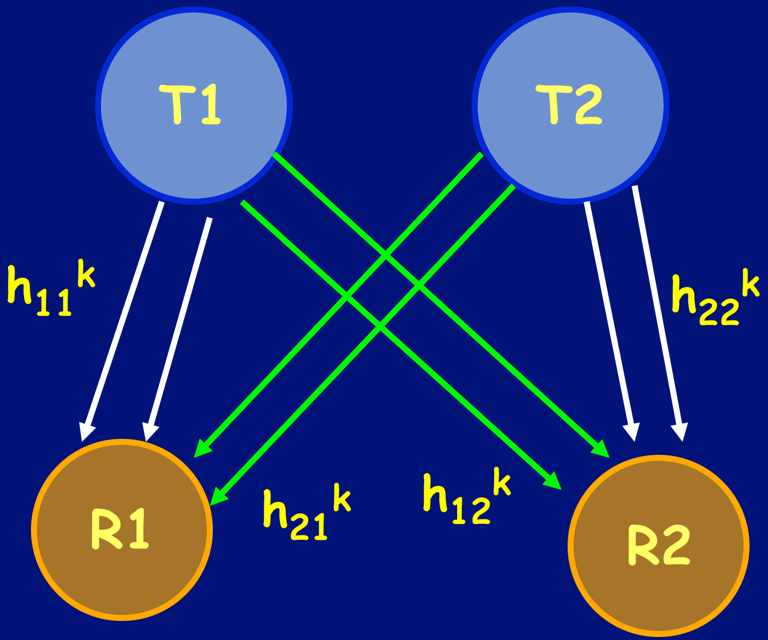
SYSTEM MODEL

- I transmitter-receiver pairs (users)
- K parallel channels for all users
- User i chooses to transmit in one channel, φ(i), with power $p_i^{\varphi(i)}$
  - Transmission power constraint

$$P_i^{\min} \leq p_i^{\phi(i)} \leq P_i^{\max}$$

  - Received SINR of user i in channel φ(i)

$$\gamma_i^{\phi(i)} = \frac{p_i^{\phi(i)} h_{ii}^{\phi(i)}}{n_o + \sum_{j \neq i} p_j^{\phi(i)} h_{ji}^{\phi(i)}}$$



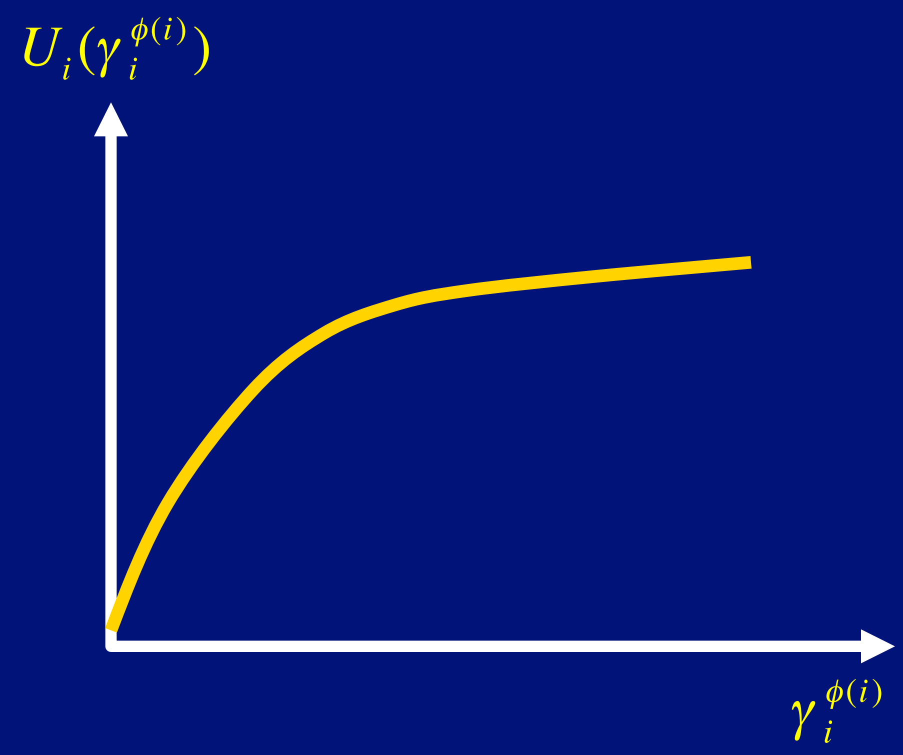
h_{ij}^k : gain between tx i and rx j for channel k

n_0 : background noise power

Utility Function

- User i 's QoS preference is given by utility $U_i(\gamma_i^{\varphi(i)})$
  - U_i is increasing and strictly concave in $\gamma_i^{\varphi(i)}$
  - Rate-adaptive applications with elastic demands.

- Network Performance = Total Network Utility



OBJECTIVE:
Total Utility Maximization Problem

- **Goal:**

  Select channel and allocate power in a distributed way to maximize total utility.

- **Challenges:**
  - Channel selection is a discrete (combinatorial) and possibly non-convex optimization problem → Difficult to solve
  - Power assignments across users are coupled due to mutual interference
  - Objective function may not be concave in power

- **Proposed HEURISTIC SOLUTION: SC-ADP ALGORITHM**

  Distributed cooperation by exchange of interference prices.

Single-Channel Asynchronous Distributed Pricing (SC-ADP) Algorithm

- Price Announcement: User i announces an interference price $\pi_i^{\varphi(i)}$ in the currently selected channel $\varphi(i)$

$$\pi_i^{\phi(i)} = \left| \frac{\partial U_i(\gamma_i^{\phi(i)})}{\partial(\sum_{j \neq i} p_j^{\phi(i)} h_{ji}^{\phi(i)})} \right|$$

Interference price reflects the marginal increase of user i's utility if its received interference (denominator) is decreased by one unit

Single-Channel Asynchronous Distributed Pricing (SC-ADP) Algorithm
CHANNEL SELECTION AND POWER UPDATE

- Based on the current interference prices and current level of interference,

User i chooses channel φ(i) and power $p_i^{\varphi(i)}$ to maximize its surplus

$$S_i = U_i(\gamma_i^{\phi(i)}(p_i^{\phi(i)})) - p_i^{\phi(i)} \sum_{j \neq i} \pi_j^{\phi(i)} h_{ij}^{\phi(i)})$$

↓ Utility gain at user i

↓ Utility loss at other users

Single-Channel Asynchronous Distributed Pricing (SC-ADP) Algorithm

- Repeat two steps asynchronously across users.

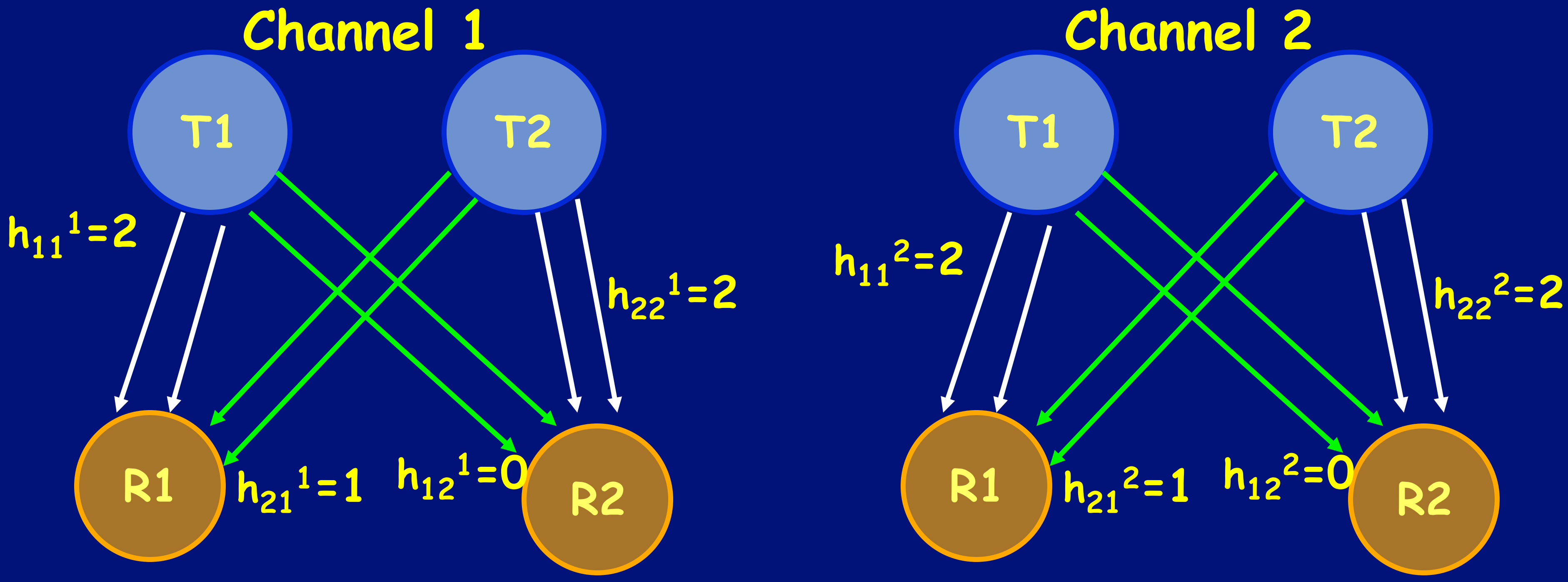
→

Announce the price $\pi_i^{\varphi(i)}$ and

measure local channel gains (h_{ij}^k for all j and k).

Single-Channel Asynchronous Distributed Pricing (SC-ADP) Algorithm

- Consider 2 users sharing 2 channels



- Assume that both user starts to use the same channel with max power: $p_1(\varphi(1))=p_2(\varphi(1))=1$
- User 1 computes its SINR and price where noise power $n_0=0.1$ using these formulas assuming a utility function $U(x)=\log(1+x)$

$$\gamma_i^{\phi(i)} = \frac{p_i^{\phi(i)} h_{ii}^{\phi(i)}}{n_o + \sum_{j \neq i} p_j^{\phi(i)} h_{ji}^{\phi(i)}}$$

$$\pi_i^{\phi(i)} = \left| \frac{\partial U_i(\gamma_i^{\phi(i)})}{\partial(\sum_{j \neq i} p_j^{\phi(i)} h_{ji}^{\phi(i)})} \right|$$

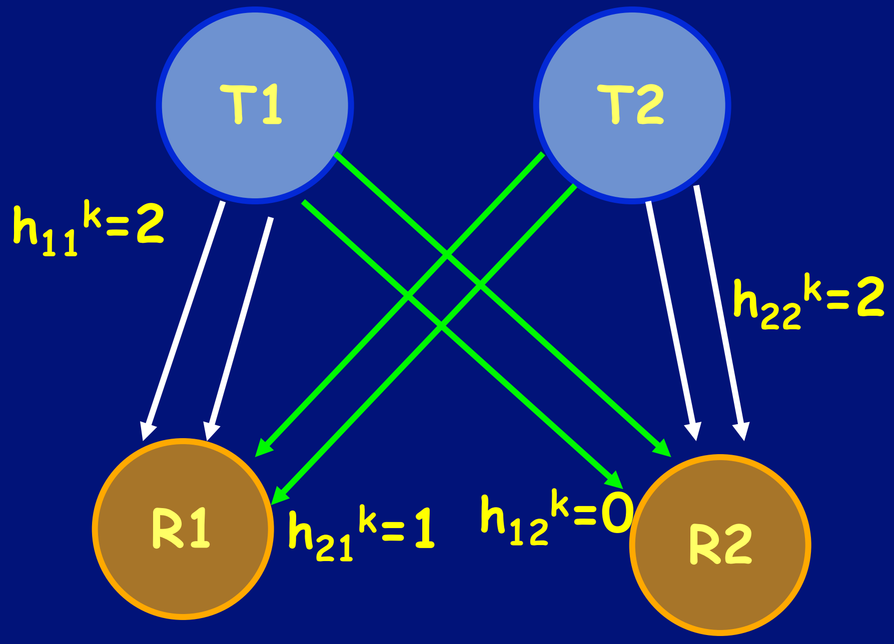
Single-Channel Asynchronous Distributed Pricing (SC-ADP) Algorithm- Example

- SINR of user 1:

$$\gamma_1 = \frac{1 \times 2}{0.1 + 1\times1} = 1.82$$

- Price of user 1 for using the channel is

$$\pi_1 = \frac{log(1.82+1) - log(12+1)}{1} = 0.69$$



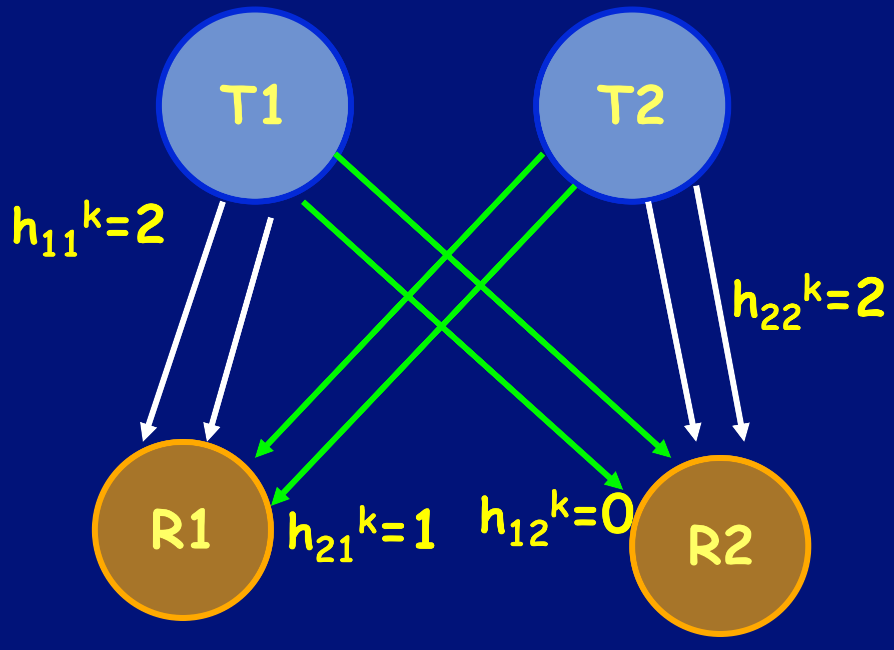
Single-Channel Asynchronous Distributed Pricing (SC-ADP) Algorithm- Example

- User 2 computes its SINR:

$$\gamma_2 = \frac{1 \times 2}{0.1 + 1\times1} = 1.82$$

- Price of user 2 for using the channel 1 is

$$\pi_2 = \frac{log(1.82 + 1) - log(0 + 1)}{1} = 0.69$$



Single-Channel Asynchronous Distributed Pricing (SC-ADP) Algorithm- Example

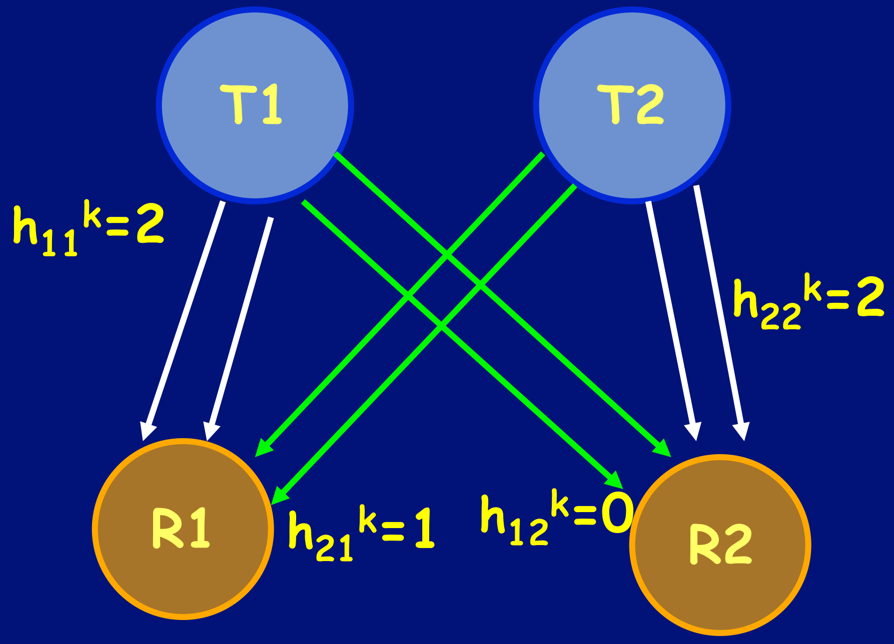
- The surplus function for user 1 is

$$S_i = U_i(\gamma_i^{\phi(i)}(p_i^{\phi(i)})) - p_i^{\phi(i)} \sum_{j \neq i} \pi_j^{\phi(i)} h_{ij}^{\phi(i)})$$

$$S_1 = log(1 + 1.82) - 1 \times 0.69 \times 1 = -0.24$$

- If user 1 drops the channel 1 and grabs the channel 2, its surplus function will become

$$S_1' = log(1 + 1.82) - 1 \times 0 \times 1 = 0.45$$



- Since $S_1' > S_1$, user 1 changes its channel from channel 1 to channel 2.
- Then it calculates its new SINR and new price and advertise the new price which are:

$$\gamma_1 = \frac{1 \times 2}{0.1 + 0 \times 1} = 12$$

$$\pi_1 = \frac{\log(12+1) - \log(12+1)}{1} = 0$$

- At this point, since both users are using different channels with maximum power, we reach the optimal point.

Performance Evaluation: SC-ADP

- SC-ADP Max Power:

  User i transmits with maximum power $P_{max}^{\varphi(i)}$ in the channel $\varphi(i)$ that maximizes surplus S_i.

Performance Evaluation: SC-ADP

- **Best SINR:**

  User i transmits with maximum power in the channel that yields the highest SINR:

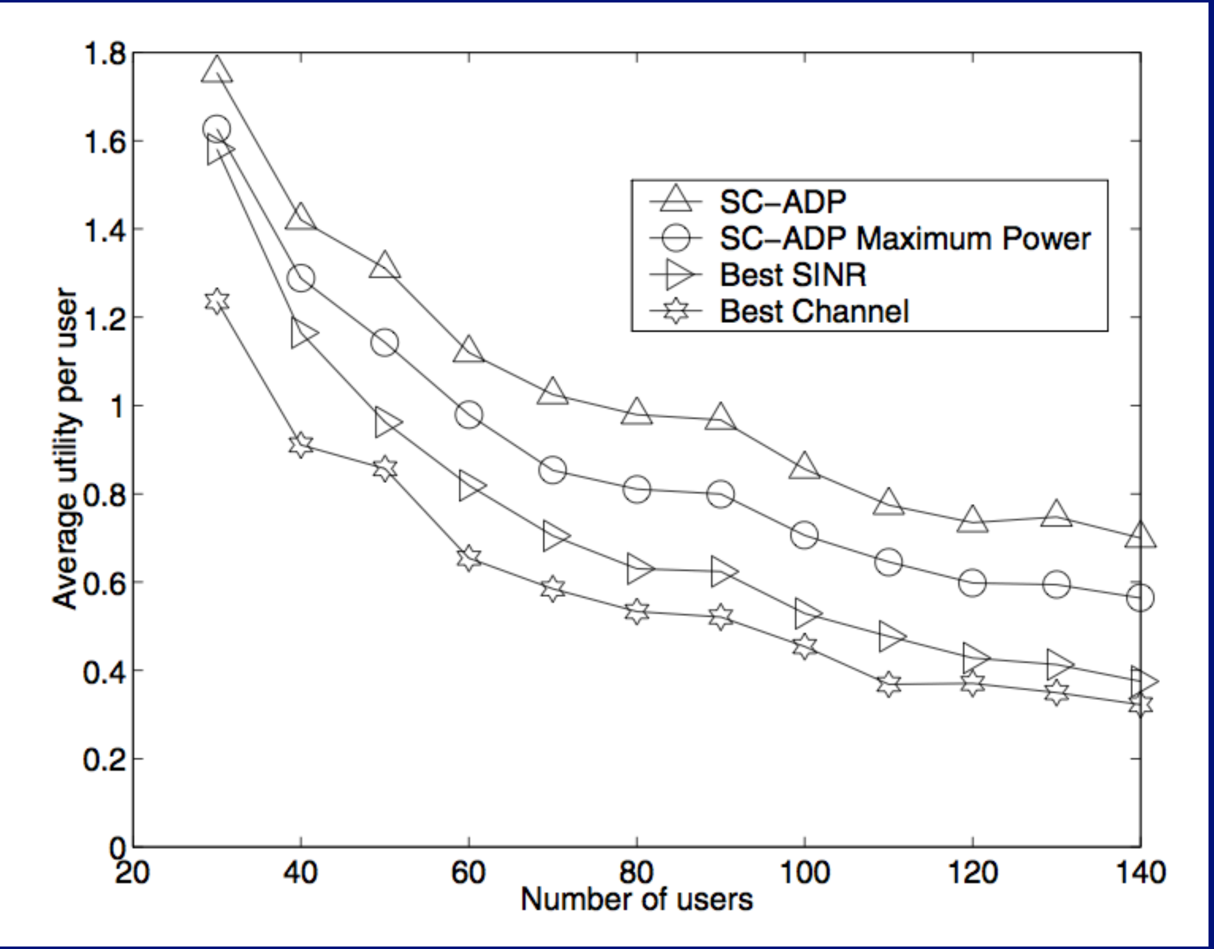
$$\phi(i) = \arg\max_{k} \frac{h_{ii}^{k}}{n_o + \sum_{j \neq i} p_j^k h_{ji}^k}$$

Performance Evaluation: SC-ADP

- **Best Channel:**
  User i transmits with maximum power in the channel with the largest channel gain

$$\phi(i) = \arg\max_k h_{ii}^k$$

Performance Evaluation: SC-ADP



- SC-ADP with continuous power control achieves significantly more utility than with only maximum power, which achieves significantly more utility than the Best SINR algorithm.
- Best Channel algorithm performs the worst since interference is not taken into account.

Multi-Channel Asynchronous Distributed Pricing (MC-ADP) Algorithm:

Maximum Power

- User i allocates max powers across K channels to maximize surplus

$$\text{Maximize:} \quad \sum_{k=1}^{K} \log(1+\gamma_i^k) - \sum_{k=1}^{K} p_i^k \sum_{j \neq i} \pi_j^k h_{ij}^k$$

Subject to:

$$\text{Total power constraint} \ \sum_{k=1}^{K} p_i^k \leq P_i^{\max}$$

Iterative Water-filling (IWF)

- **User i allocates power across K channels to maximize the rate**

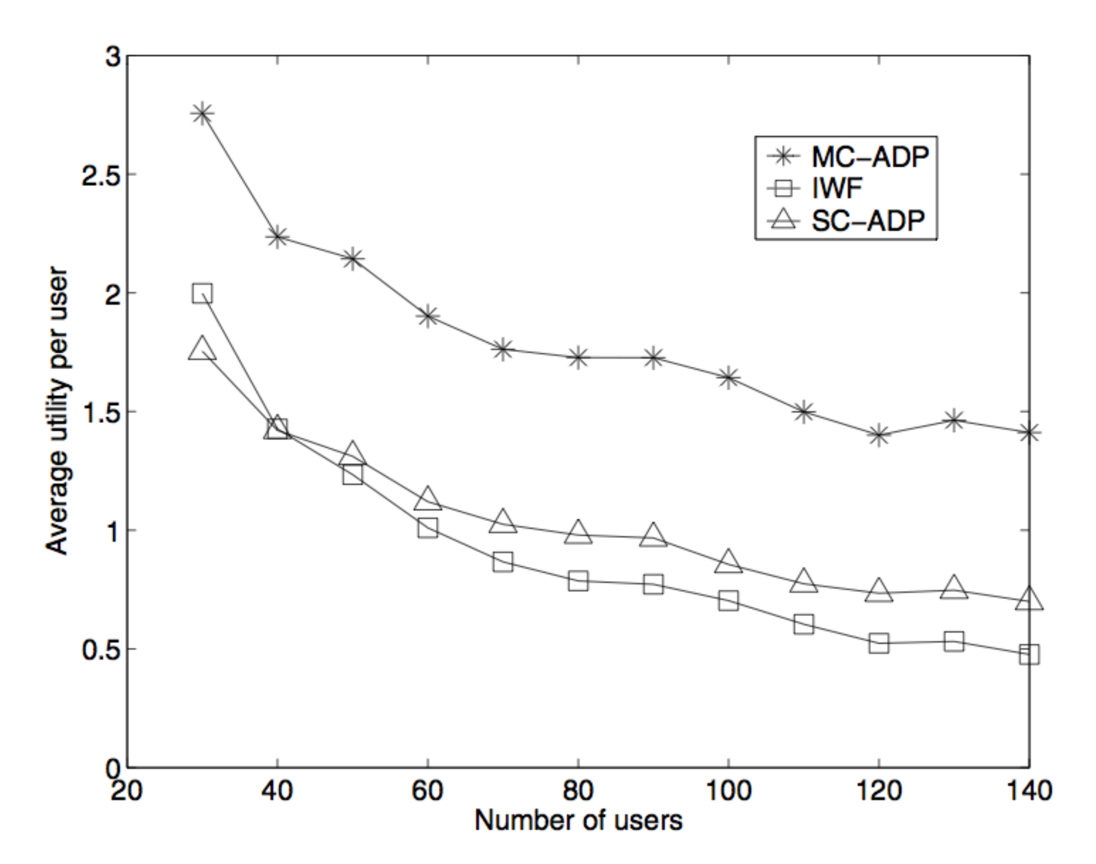
$$\text{Maximize:} \quad \sum_{k=1}^{K} \log(1+\gamma_i^k)$$

Subject to:

$$\text{Total power constraint} \quad \sum_{k=1}^{K} p_i^k \le P_i^{\max}$$

No info is exchanged among users and the power allocation across channels for each user is determined by water filling regarding the interference as noise.

Performance Evaluation: MC-ADP



- MC-ADP achieves significantly higher utility than the other algorithms, since
  - * it takes into account the interference prices, and
  - * has the flexibility of allocating power across multiple channels.

- SC-ADP algorithm outperforms IWF in a dense network (i.e., more than 40 users), where the interference prices help to mitigate the effects of interference.

SPECTRUM SHARING CLASSIFICATION

Intra-Network SS

- Centralized (Infrastruct. based)
- Distributed (Ad hoc - based)
  - Cooperative
  - Non-cooperative

Inter-Network SS

* Centralized
* Distributed

Intra-Network Spectrum Sharing - Distributed & Non-Cooperative

- Non-cooperative (or non-collaborative, selfish) solutions consider only the user itself
- Selects the channel with the objective of maximum throughput without taking other users into consideration!
- May result in reduced spectrum utilization
- Requires minimum communication among other users.

Intranetwork Spectrum Sharing: Distributed – Non-Cooperative

1. Device Centric Approach
   H. Zheng and L. Cao,
   Proc. IEEE DySPAN, Nov. 2005.

2. Belief Assisted Pricing
   J. Zhu and Ray Li,
   Proc. of IEEE SECON 2006.

DEVICE CENTRIC SPECTRUM MANAGEMENT

H. Zheng and L. Cao, "Device-centric Spectrum Management," Proc. IEEE DySPAN, pp. 56-65, Nov. 2005.

Motivations:

- Cooperation increases the number of control messages
  - Energy and BW wastage
- Users do not want to reveal spectrum usage
  - Privacy and to avoid jamming attacks
- What if CR users do not want to collaborate?

SOLUTION

- Users are allocated channels based on their observations of interference patterns and neighbors
- Compared to cooperative schemes →
  this scheme results in slightly worse performance.
- But communication overhead is reduced significantly.

Rule Based Spectrum Management

- Users sense environment conditions and neighbor activities
- Users independently adjust self-behavior following Preset Rules
- Recursive procedures
  - System reaches equilibrium within small number of steps
  - Assuming everyone is well behaved

Independent Action using Rules

- Rules tell each user which channels it should use
  - Input: Environment / Network Conditions
  - Goal: Maximize system utility
    (e.g., proportional fair utility)
  - No negotiation necessary

Independent Action using Rules

Rules based on Poverty Line Theorem

- **Poverty Line:**

  Minimum amount of spectrum that a user is entitled to, i.e., lower bound

- **Theorem:**

  In a stable and proportional fair system, the number of channels at each user n ≥ PL (n)

- PL(n) depends on the number L_n of available channels for user n and the number D_n of conflicting neighbors:

$$PL(n) = \left\lfloor \frac{L_n}{D_n + 1} \right\rfloor$$

Access Schemes

- CONFLICT FREE ACCESS SCHEME
  RULES A, B, C

- CONTENTION-BASED ACCESS SCHEME
  RULES D, E

Conflict Free Spectrum Access

- For explicit and guaranteed throughput provisioning and control over packet delay

- To prevent interference, users always select idle channels, i.e., channels unclaimed by neighbors.

- A channel is idle if the spectrum report shows no activity during the previous time period of length X, where X is a design parameter.

- To provide fairness, we limit the number of channels each user can access.

Conflict Free Spectrum Access:
Rule A: Uniform Idle Preference

- Each user adjusts its spectrum usage (# of channels) to

$$\Omega = \min_n \left\lfloor \frac{L_n}{D_n + 1} \right\rfloor \quad \text{idle channels}$$

- Rule A guarantees a conflict free spectrum allocation (See the paper!!)

Limitation

- A small number of users experiencing intensive interference from PUs (small L_n) or other SUs in a crowded area (large D_n) can limit the value of Ω

→ leading to less than ideal spectrum utilization.

Conflict Free Spectrum Access: Rule B: Poverty Exact Idle Preference

- A user n selects exactly

$$PL(n) = \left\lfloor \frac{L_n}{D_n + 1} \right\rfloor$$ channels from idle channels.

- If the number of idle channels < PL(n), it "grabs" channels from "richer" users without impacting "poor" users.

CHANNEL SELECTION PROCEDURE

- To n, a neighbor is "rich" if it uses more channels than n; otherwise it is "poor"
- Each user has the knowledge of the number of neighbors D_n, & their channel selection so that it can identify "richer" users
- To "grab" non-idle channels, a user n marks the channels occupied by "poor" neighbors as busy, and the rest as idle
- User n then selects a set of channels from the "idle" channels until its channel occupancy reaches PL(n).

Limitation

- Each user only attempts to use PL(n) channels
- Since PL(n) represents a lower bound on spectrum usage in a stable and proportional fair system, Rule B could under-utilize available spectrum.

Conflict Free Spectrum Access
Rule C: Poverty Guided Idle Preference

- Guarantees the poverty line for each user while letting some users to go beyond their poverty lines

- A user n selects channels from idle channels

- If there are not enough idle channels to reach PL(n) →
  User n "grab" channels from "richer" neighbors.

- The number of channels it can grab from any "richer" user r

$$\max\{0, \min\{C_r - PL(n), PL(n) - C_n\}\}$$

where C_n and C_r are the current number of channels of user n and r.

Rule C: Poverty Guided Idle Preference

- Rule C allows users who have attained their poverty line to grab additional idle channels

- It still allows users below their poverty line to grab channels from "richer" neighbors, but requires that each grabbing does not reduce a "richer" user's spectrum below the grabber's poverty line.

Implementation Requirements for Rules B & C

- Users (especially those below their poverty line) need to know the set of channels each neighbor currently occupies
- This is done by each node broadcasting their channel usage either embedded in beacon broadcasts or in routing hello messages

Conflict Free Spectrum Access: RECAP

Static QoS → Channel Reservation

- Rule A: Each user chooses

  $\min_{n} \left\lfloor \frac{L_n}{D_n + 1} \right\rfloor$ idle channels

- Rule B: Each user n chooses exactly

  $PL(n) = \left\lfloor \frac{L_n}{D_n + 1} \right\rfloor$ idle channels

- Rule C: Each user n chooses at least

  $PL(n) = \left\lfloor \frac{L_n}{D_n + 1} \right\rfloor$ idle channels

Theoretical Conclusion

- Using B or C, the system reaches an equilibrium after at most $O(N^2)$ local adjustments.
- At equilibrium, each user's spectrum is at least its PL (equal to PL for Rule B).

- N: # of nodes
- M: # of channels

Contention-based Spectrum Access

- Broadcasting spectrum usage to neighbors might be undesirable for a number of reasons, including privacy concerns and protection against jamming from malicious users
- Contention based spectrum access does not require knowledge of neighbors' spectrum usage

Contention-based Spectrum Access: Procedures

- On each channel, users follow a set of random access rules such as CSMA to compete fairly for channel access and avoid conflict
- Each user performs contention detection, i.e., listens to the channel before initiating any transmission
- It initiates the transmission only when the channel is idle for some given time T
- Otherwise, it backs off and delays the action for a short period

Contention-based Spectrum Access

- Penalty: Overhead of contention detection
  (even if there is only one user on the channel)

- NOTE:
  Since channels have different contention conditions, users should invoke independent contention detection and backoff process on each channel

Contention-based Spectrum Access: SHARING

- Random contention allows multiple users to share one channel but does not specify the number of channels users should use

- Users could be selfish and occupy all the channels, reducing the system to a single channel with full interference

- Therefore, we need to regulate the maximum number of channels each user can use

Contention-based Spectrum Access:

Rule D: Selfish Spectrum Contention

- Each user n can use up to the Ψ channels providing the highest throughput.
- Communication on each channel is through CSMA based time contention.

Contention-based Spectrum Access

Rule E: Poverty Guided Selfish Spectrum Contention

- The poverty line concept can provide a reference for choosing different value of Ψ for different users.

- The number of channels each user n can use is limited by

$$\Psi_n = \max(\alpha \cdot PL(n),\ 1) \qquad \alpha \geq 1.$$

Since the poverty line represents throughput attainable from conflict free spectrum usage, Ψ_n should be larger than PL(n) to account for channel contention

Contention-based Spectrum Access: Recap

Probabilistic QoS → Contention

- General rule: Listen before talk, CSMA to avoid conflict on each channel

- Rule D: Each user chooses X channels that provide the highest throughput (for this user)

- Rule E: Each user n chooses $X_n = \max(\alpha PL(n), 1)$ channels that provide the highest throughput (for this user)

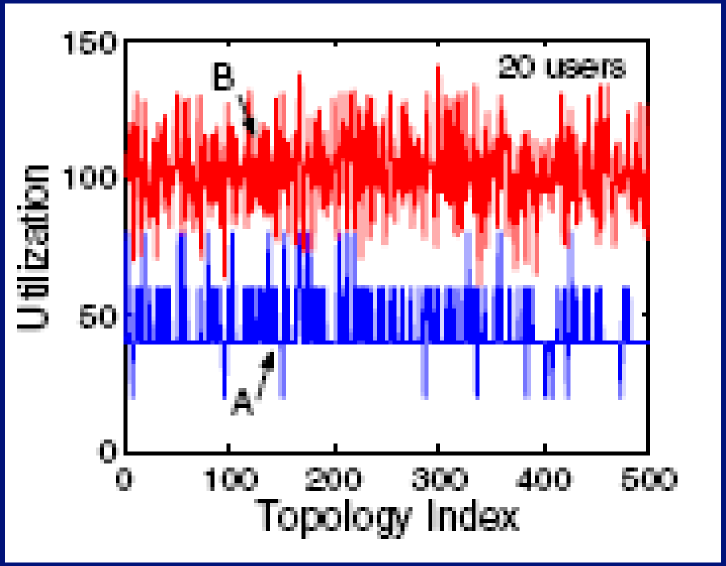
Theoretical Conclusion

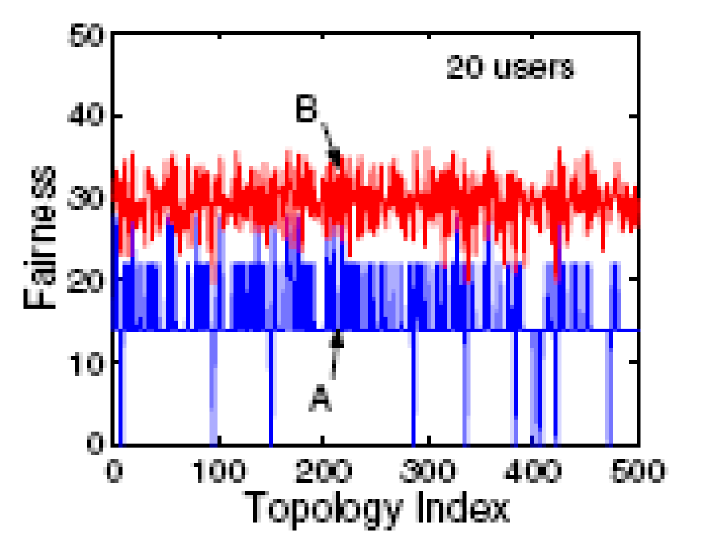
- Using D or E, the system reaches an equilibrium after at most Λ * M local adjustments, $\Lambda \le O(N^2)$

- N: # of nodes
- M: # of channels

Performance Analysis (Rule A, B)

- Rule B improves both the utilization and fairness by a factor of 2 over Rule A



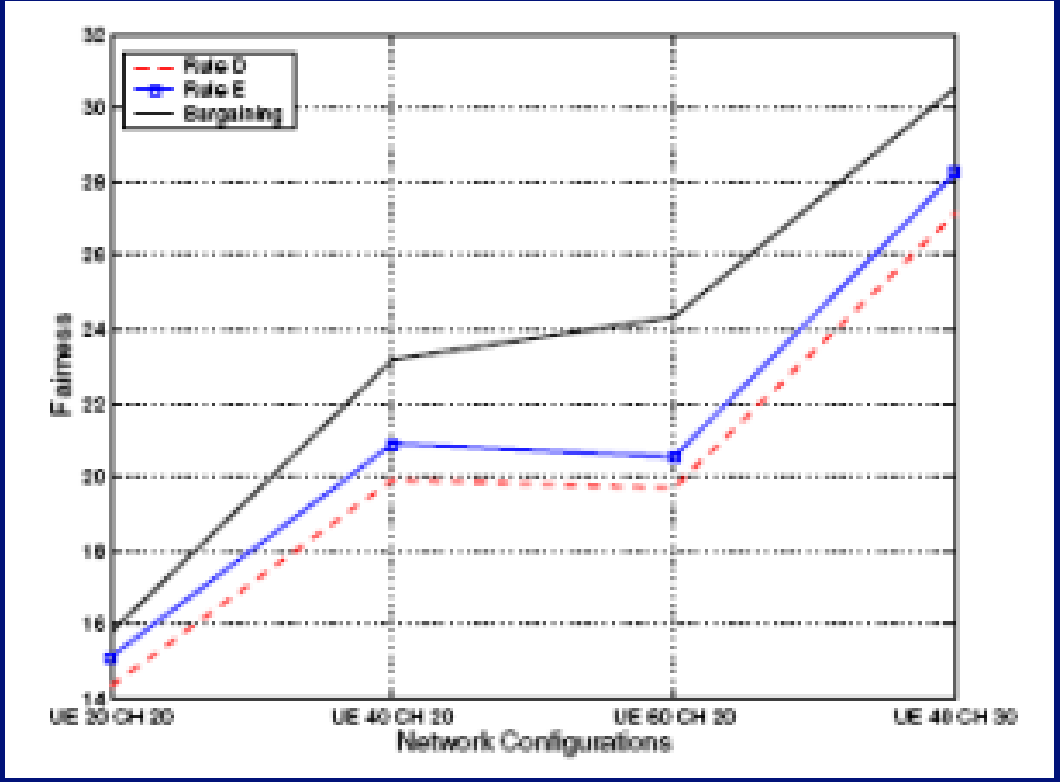


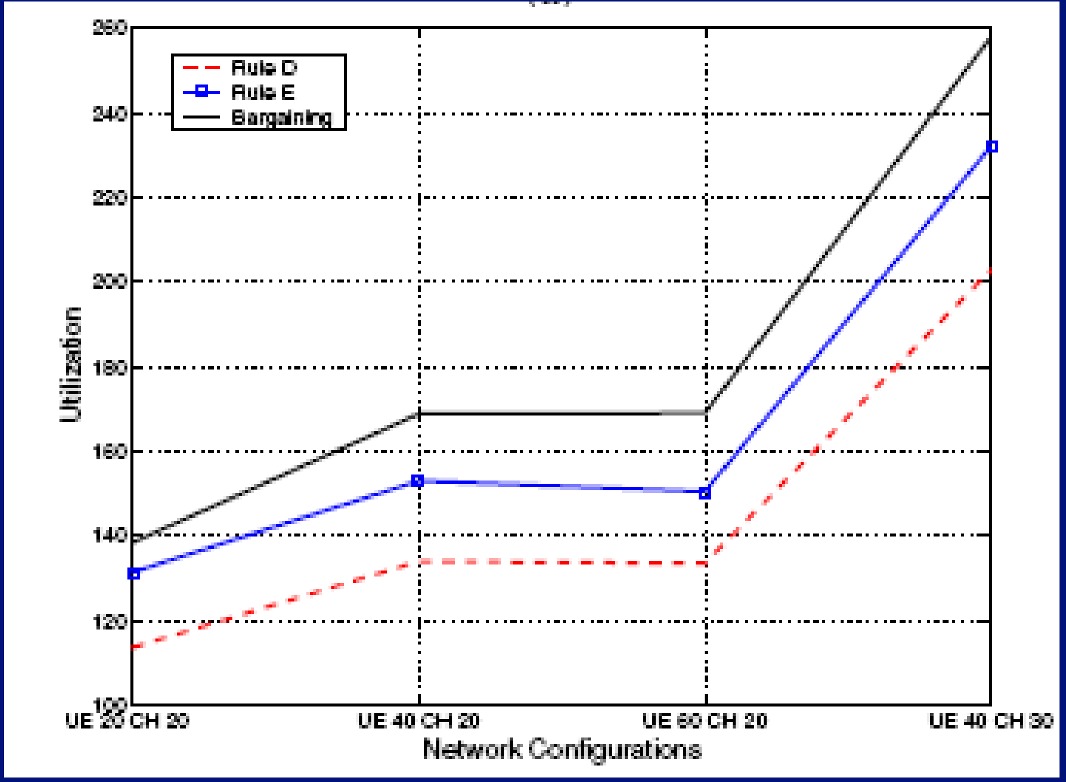
Utilization: $\sum_i \beta_i \quad (\beta_i : \#\text{ of channels at user i})$ **Fairness:** $\sum_i \log \beta_i \quad (\beta_i : \#\text{ of channels at user i})$

Performance Analysis (Rule D, E)

- Rule E outperforms Rule D

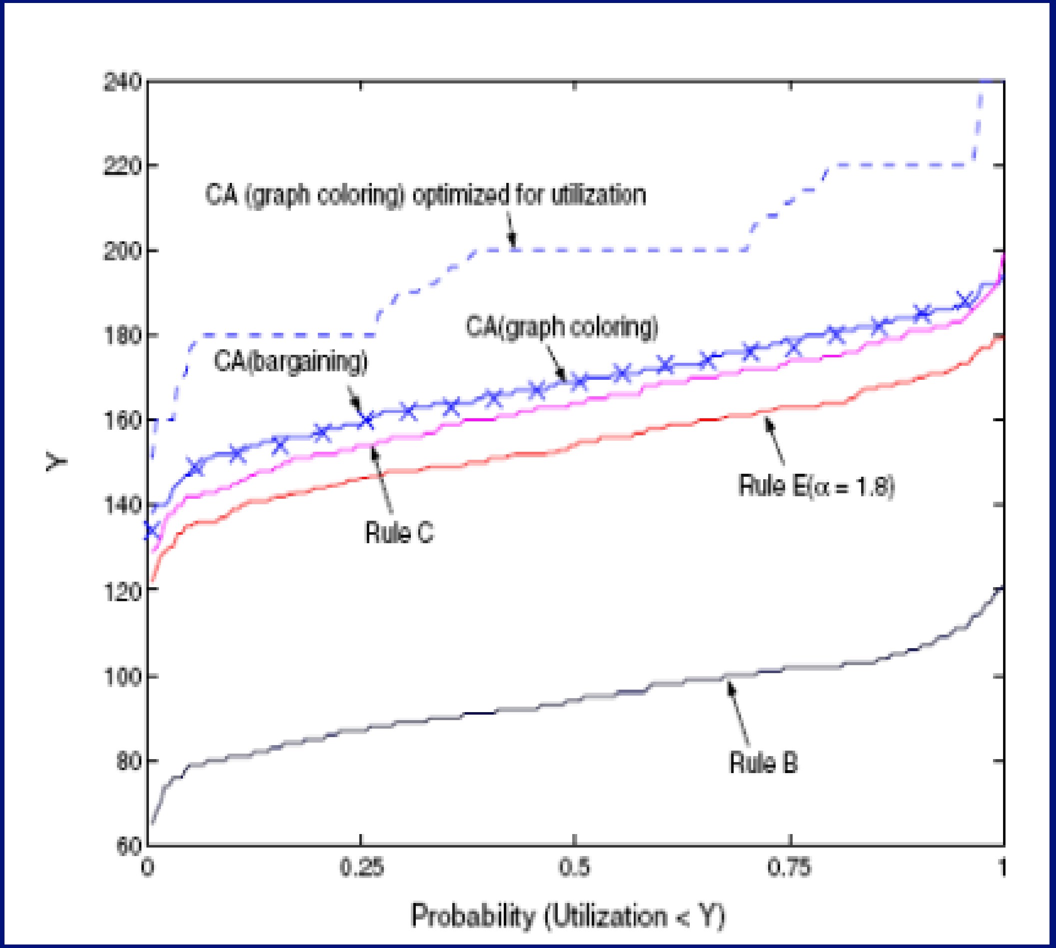
Rule E with α = 1.8 and
Rule D with optimal Ψ.





Performance Analysis (Rule B, C, E)

- Performance gap between Rule B and C shows that the poverty line is a still a loose bound on spectrum usage. (Rule B)
- By opportunistically going beyond the poverty line, users achieve better spectrum utilization (Rule C)
- Compared to the bargaining approach, Rule C leads to a graceful 8% degradation in utilization.



Performance Analysis (Rule B, C, E)

- Performance difference between Rule C and E shows that Rule E provides better fairness, as a PL(n) provides a proportional increase in spectrum usage

- Compared to the bargaining approach, Rule C leads to a 25% degradation in fairness.