CHAPTER 5.4.
MOBILITY AWARE SENSING DESIGN

A. S. Cacciapuoti, I. F. Akyildiz and L. Paura,
"Optimal Primary-User Mobility Aware Spectrum Sensing Design for Cognitive Radio Networks",
IEEE Journal on Selected Areas in Communications, Nov. 2013.

Optimal Primary-User Mobility Aware Sensing Design: Overview

– Spectrum sensing functionality by jointly

- maximizing the sensing efficiency
- satisfying the PU interference constraint

in presence of Primary-User mobility.

Conventional Sensing Scenarios

- **Large-Scale PU Networks**
  - Static PUs
  - CR users are always inside the PU range
    Their capability to sense the PU transmissions does not vary in time

Our Considered Sensing Scenarios

- Small-Scale PU Networks
  - Mobile PUs
  - CR users can be out of the PU range

    Their capability to sense the PU transmissions varies in time

Why Small-Scale PU Networks?

- Gained a lot of attention recently

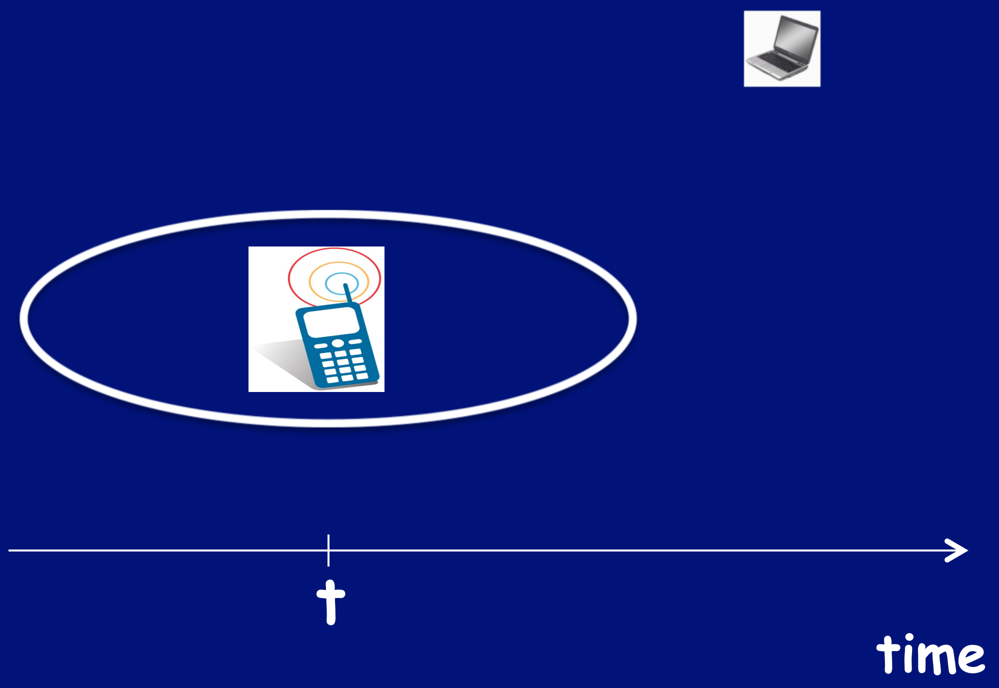
- Examples of small-scale PU networks:
  - Ad hoc networks
  - Wireless personal area networks
  - HetNets (Small Cells)
  - Wireless microphones

Why focus on Spectrum Sensing?

Mobility changes dynamically the mutual distances among PUs and CR users

- CR capability to sense the PU transmission varies in time
- An effective sensing must be aware of the mobile PU dynamics

Example:
Impact of PU Mobility on Sensing



- At time t the CR user is out of the PU range
  - it cannot sense the PU transmissions
- After the PU movement, the CR user is inside the PU range
  - it can sense the PU transmissions

Objectives

IN MOBILE PU SCENARIOS:

- Maximize the Sensing Efficiency
- Satisfy the PU Interference Avoidance

How?

Tuning the sensing time and the transmission time according to the mobile PU dynamics:

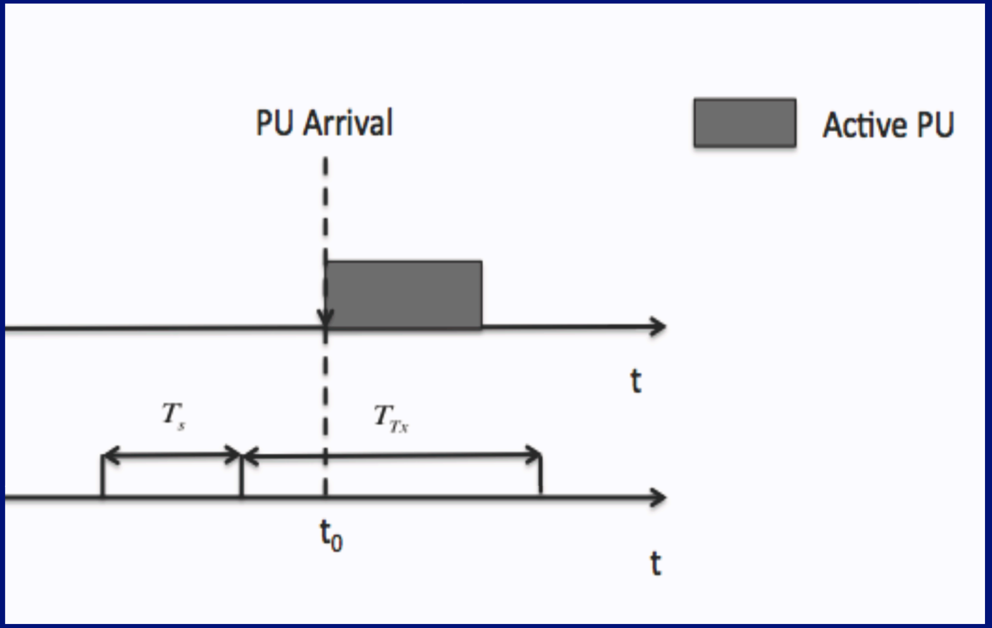
- How often must the sensing be performed in presence of PU mobility?
- How long must a spectrum band be sensed to reliably detect mobile PUs?

Why?

- Sensing time and transmission time influence both the spectrum efficiency and interference avoidance.
- Proper selection of these parameters is the most critical factor influencing the performance of CR networks.

Example: Transmission Time

- Hypothesis: Ideal Sensing
  - Static scenarios → no interference
  - Mobile Scenarios → the interference can be greater than zero
    - At the end of the sensing time T_s, CR user correctly decides to use the band
    - At t_0, an active PU arrives during the CR transmission time
    - CR user interferes with the PU, despite the perfect sensing decision



Example:
Sensing Time

- **Static Scenarios:**
  - A CR user is always inside the PU range → Sensing is mandatory
- **Mobile Scenarios:**
  - A CR user can be out of the PU range
  - If the prob of being inside the PU range is lower than the maximum interference prob tolerated by the PU

    → Sensing is useless

    - Sensing time should be set equal to zero since the CR prob to interfere the PU is lower than the PU interference constraint.

CONTRIBUTIONS

1. Optimal transmission time for a general mobility model

   Jointly maximize the sensing efficiency and satisfies the PU interference constraint.

2. Proof of a threshold behavior in the sensing accuracy as a function of the sensing time:

   in mobile scenarios the sensing accuracy decreases for [sensing times > threshold value]

SURPRISE !!!!!

In static scenarios:

longer sensing times → higher sensing accuracy → less interference

CONTRIBUTIONS

3. Closed-form expression of the optimal sensing time threshold for a general mobility model

4. Practical rules for setting the transmission time and the sensing time when the PUs move according to Random Walk Mobility Model (RWM)

5. Evaluation of the sensing efficiency → it can increase in presence of Mobile PUs

This is the first work in literature that addresses the above issues !!

Network Models

PU Mobility Model (Assumption 1):

* Memoryless mobility pattern constituted by a sequence of movement periods
* During each period, a PU does not change its direction and its velocity

Assumption 1 is assumed as general mobility model.

Network Models

PU Traffic Model:

Two state birth-death process with death rate α and birth rate β

ON (Busy) State: PU → active with probability $P_{on} = \beta/(\alpha+\beta)$

OFF (Idle) State: PU → inactive with probability $P_{off} = \alpha/(\alpha+\beta)$

* CR User Network Model

- CR users static, uniformly distributed in the network region A, assumed either as a line or as a square.

Mobility-Aware Sensing Design: Overview

- First Part of the paper:
  - Characterization of the PU inter-arrival time for a general mobility model
- Second Part of the paper:
  - Derivation of the optimal sensing time parameters (sensing time and transmission time), by exploiting the results of the first part
- Third Part of the paper:
  - Specialization of the previously derived results for RWM.

Mobility-Aware Sensing Design: Some Definitions

- **PU Protection Range R**
  - To avoid any interference on PUs, CR users detect active PUs within a range R, called PU Protection Range (determined by PU transmission range and by CR interference range)

- **Maximum Interference Prob P_{int}**
  - P_{int} → max. value of the interference prob. that a PU can tolerate

- **Event I:** CR user is inside the PU protection range

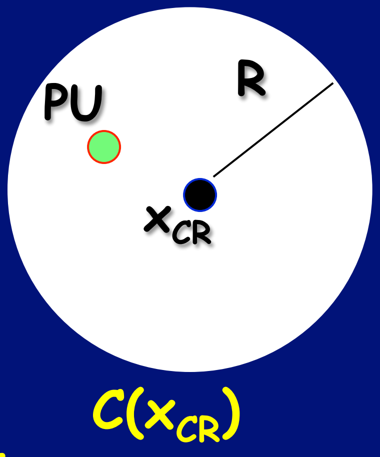
- **Event 0:** CR user is out of the PU protection range

Mobility-Aware Sensing Design: Some Definitions

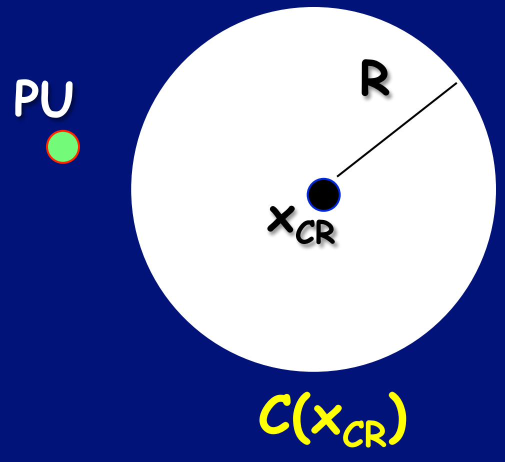
CR Interference Region:

- $C(x_{CR})$ is a disk of radius R (PU protection range) around the CR user location x_{CR}.
- Any CR user is inside the PU protection range R (Event I occurs), if the PU is placed within the CR interference region $C(x_{CR})$, i.e., if the Euclidean distance between the CR user and the PU is not greater than R.

Event I occurs:

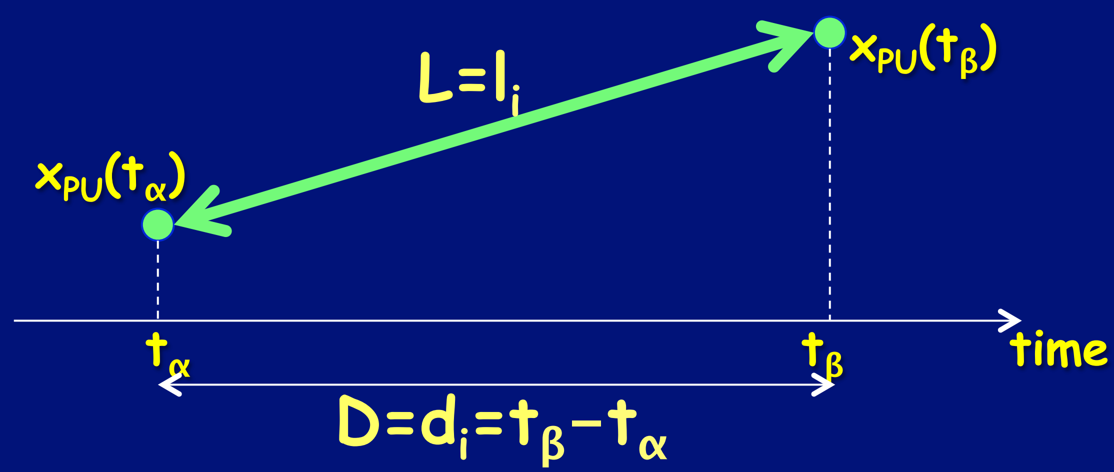


Event O occurs:



Mobility-Aware Sensing Design: Some Definitions

- **MOVEMENT LENGTH L:**
  RV L → Euclidean distance covered by a PU during a movement period
- **MOVEMENT DURATION D:**
  RV D → time spent by a PU to complete a movement period



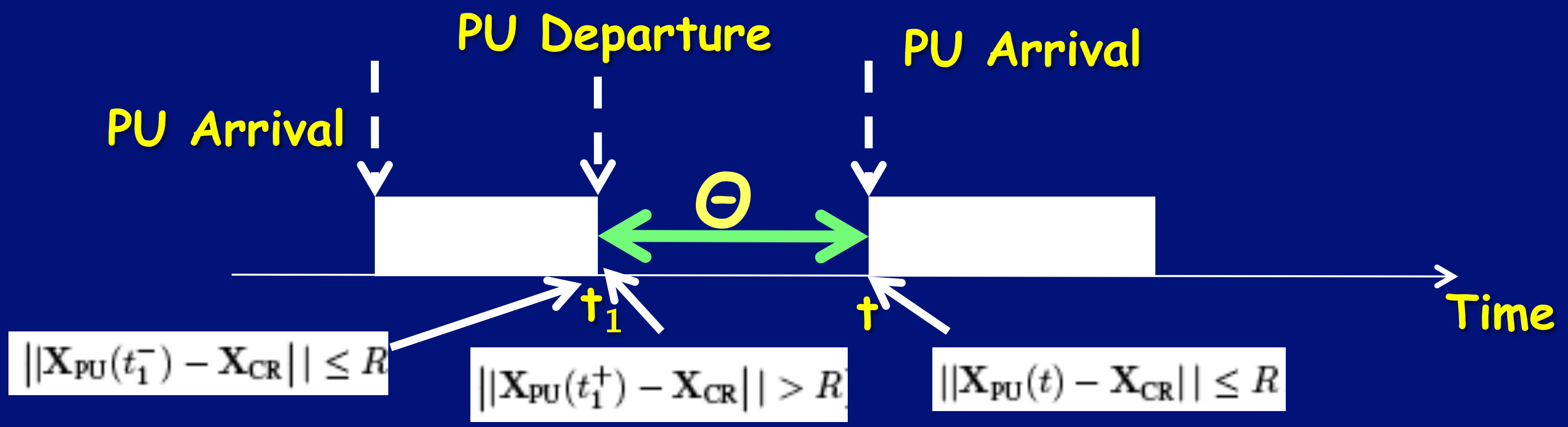
Mobility-Aware Sensing Design: Some Definitions

- **Out Time Θ:**

  Time interval a PU (starting from its steady-state spatial distribution) spends out of the interference region of an arbitrary CR user:

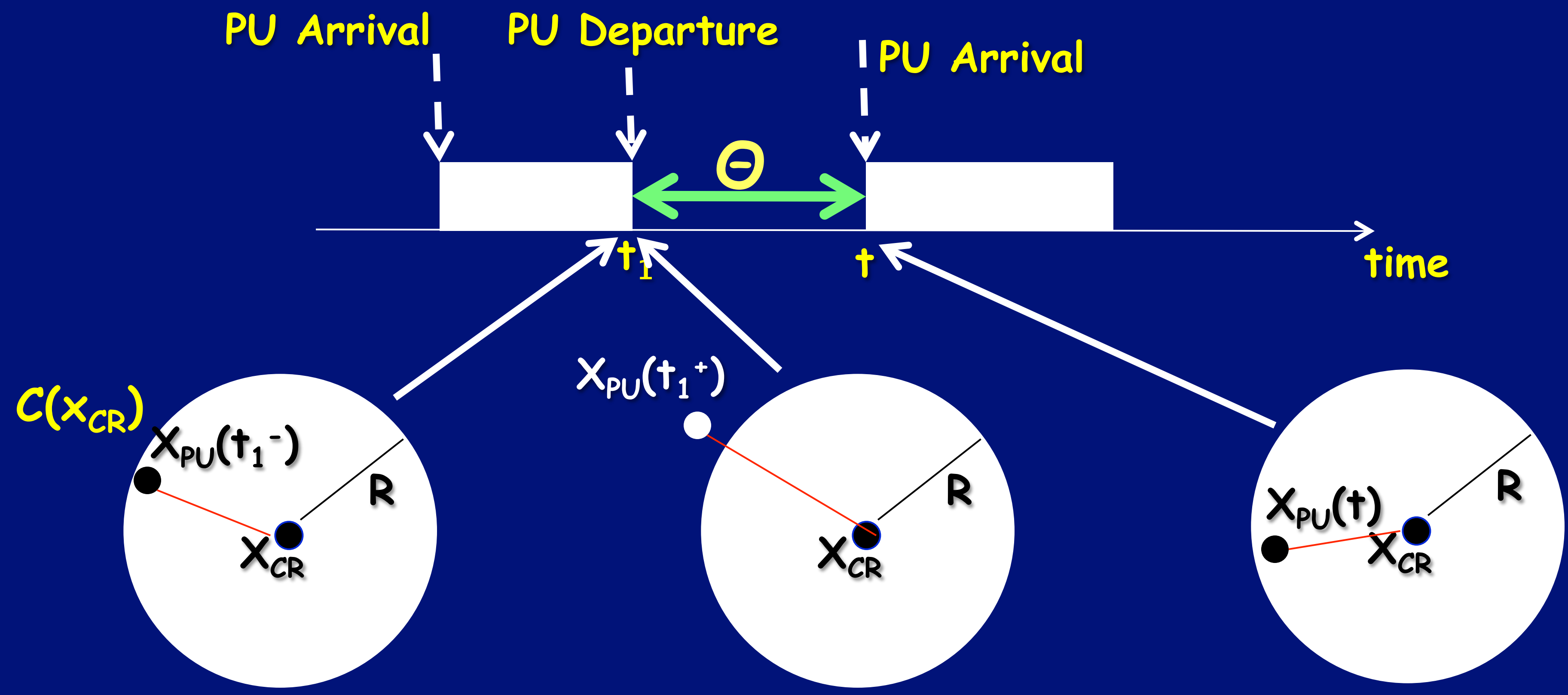
$$\Theta \triangleq \inf_{t>t_1}\{t - t_1 : ||\mathbf{X}_{\mathrm{PU}}(t) - \mathbf{X}_{\mathrm{CR}}|| \le R \wedge ||\mathbf{X}_{\mathrm{PU}}(t_1^-) - \mathbf{X}_{\mathrm{CR}}|| \le R \wedge ||\mathbf{X}_{\mathrm{PU}}(t_1^+) - \mathbf{X}_{\mathrm{CR}}|| > R\}$$

||•|| denotes the Euclidean distance
$\wedge$ denotes the logical operator "*and*".



Mobility-Aware Sensing Design: Some Definitions

Out Time Θ: Time interval a PU spends out of the CR interference region



$||\mathbf{X}_{PU}(t_1^-) - \mathbf{X}_{CR}|| \leq R$

$||\mathbf{X}_{PU}(t_1^+) - \mathbf{X}_{CR}|| > R$

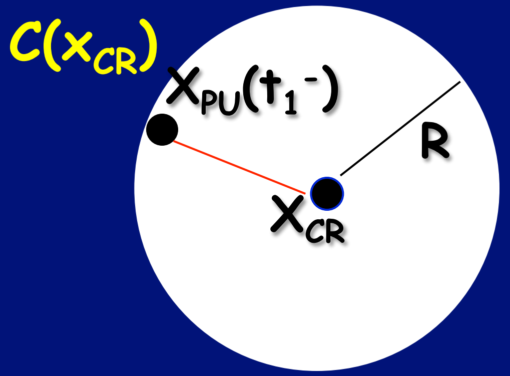
$||\mathbf{X}_{PU}(t) - \mathbf{X}_{CR}|| \leq R$

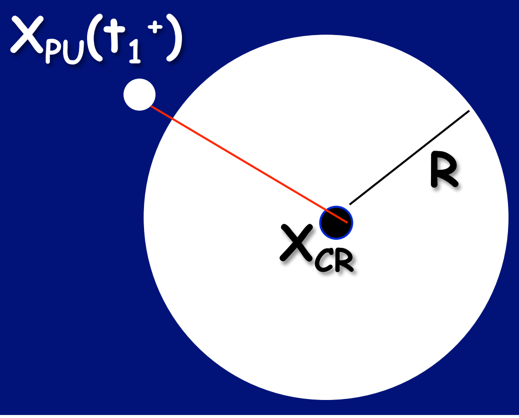
Out Time: Numerical Example

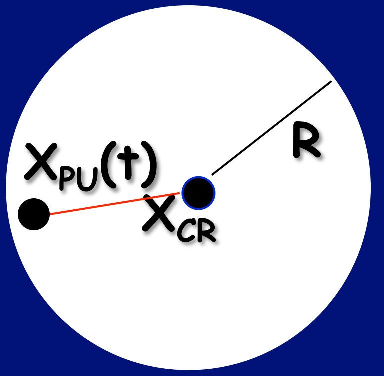
Out Time Θ:

$$\Theta \triangleq \inf_{t>t_1} \{t - t_1 : ||\mathbf{X}_{\mathrm{PU}}(t) - \mathbf{X}_{\mathrm{CR}}|| \leq R \wedge ||\mathbf{X}_{\mathrm{PU}}(t_1^-) - \mathbf{X}_{\mathrm{CR}}|| \leq R \wedge ||\mathbf{X}_{\mathrm{PU}}(t_1^+) - \mathbf{X}_{\mathrm{CR}}|| > R\}$$

- Assume:
  R=30m, t_1= 3 sec., t= 7 sec. ⟶ Θ = (7-3) sec= 4 sec.







$$3^- = t_1^- \stackrel{\text{def}}{=} \lim_{\varepsilon \to 0}(t_1 - \varepsilon)$$

$$||X_{PU}(3^-) - X_{CR}|| \leq 30m$$

$$3^+ = t_1^+ \stackrel{\text{def}}{=} \lim_{\varepsilon \to 0}(t_1 + \varepsilon)$$

$$||X_{PU}(3^+) - X_{CR}|| > 30m$$

$$7 = t$$

$$||X_{PU}(7) - X_{CR}|| \leq 30m$$

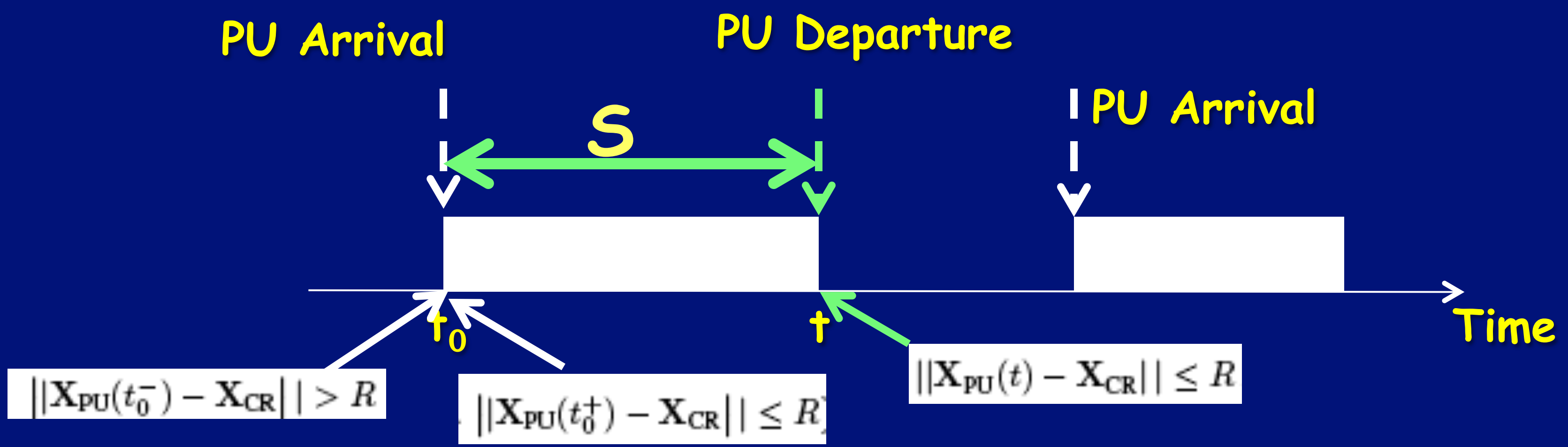
Mobility-Aware Sensing Design: Some Definitions

Sojourn Time S:

Time interval a PU (starting from its steady-state spatial distribution) spends inside a CR interference region:

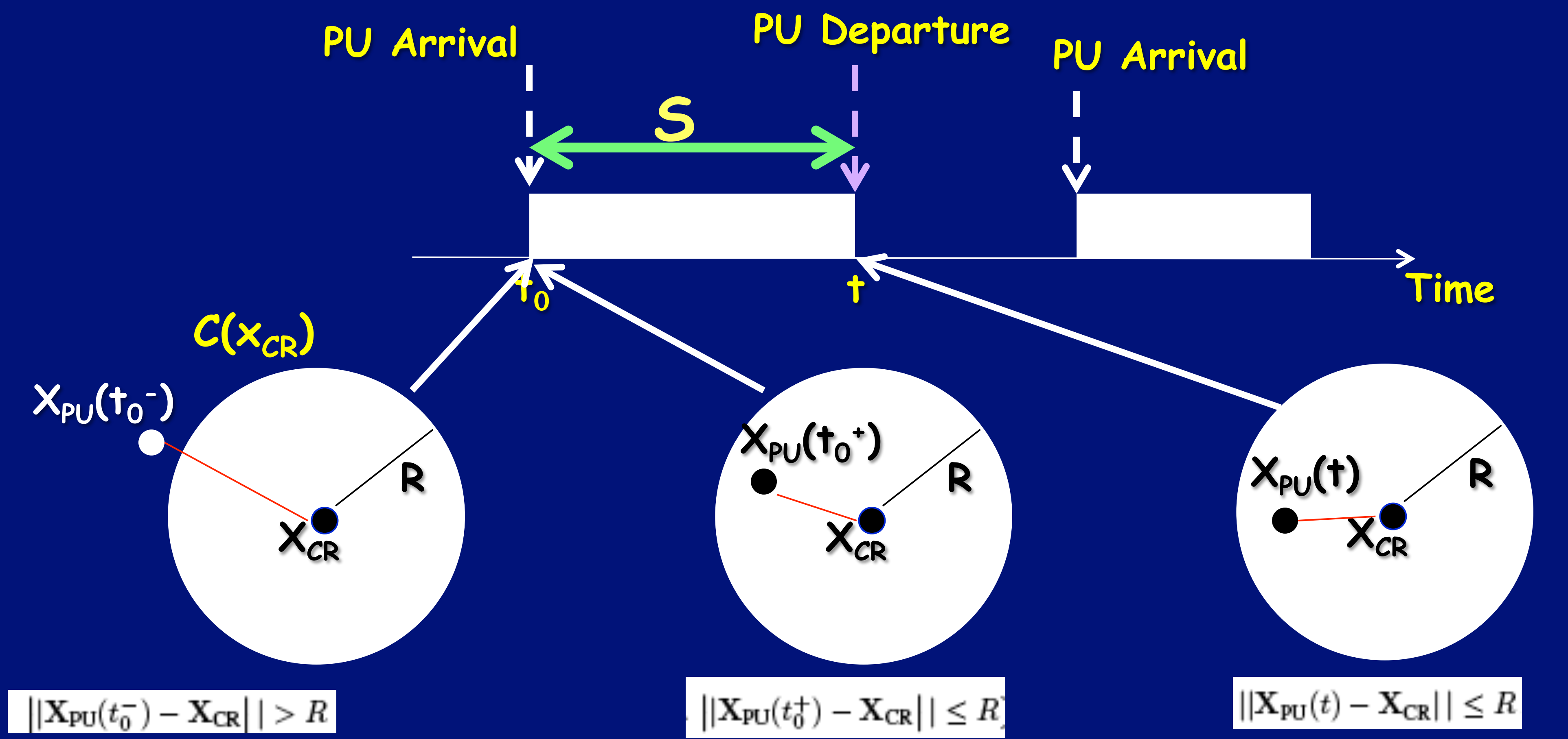
$$S \triangleq \sup_{t>t_0}\{t - t_0 : ||\mathbf{X}_{\mathrm{PU}}(t) - \mathbf{X}_{\mathrm{CR}}|| \leq R\} \wedge$$
$$||\mathbf{X}_{\mathrm{PU}}(t_0^-) - \mathbf{X}_{\mathrm{CR}}|| > R \wedge ||\mathbf{X}_{\mathrm{PU}}(t_0^+) - \mathbf{X}_{\mathrm{CR}}|| \leq R\}$$

$||\bullet||$ denotes the Euclidean distance
$\wedge$ denotes the logical operator "*and*".



Mobility-Aware Sensing Design: Some Definitions

Sojourn Time S: Time interval a PU spends inside a CR interference region



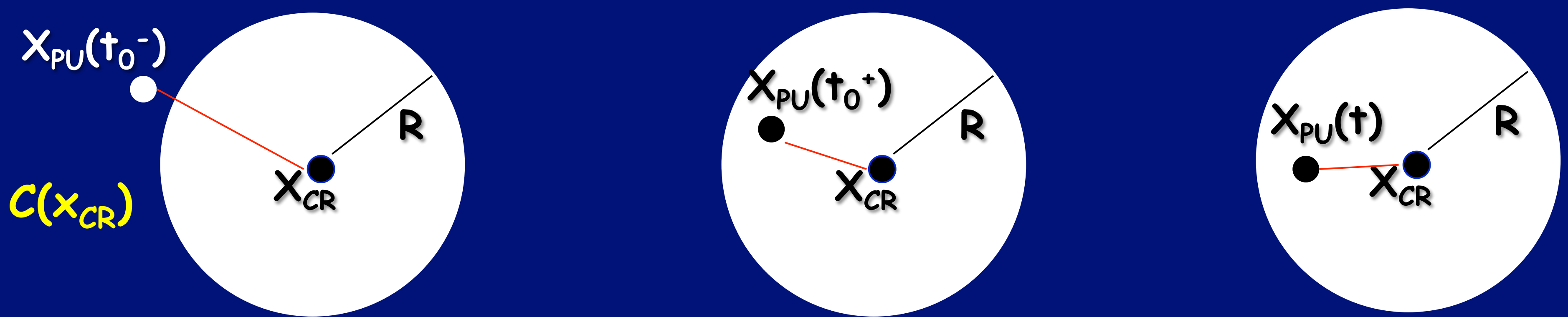
Sojourn Time: Numerical Example

- **Sojourn Time S:**

$$S \triangleq \sup_{t>t_0}\{t - t_0 : ||\mathbf{X}_{\mathrm{PU}}(t) - \mathbf{X}_{\mathrm{CR}}|| \leq R\} \wedge ||\mathbf{X}_{\mathrm{PU}}(t_0^-) - \mathbf{X}_{\mathrm{CR}}|| > R \wedge ||\mathbf{X}_{\mathrm{PU}}(t_0^+) - \mathbf{X}_{\mathrm{CR}}|| \leq R\}$$

  - Assume:

    R=30m, t_0= 10sec, t= 11sec → S = (11-10) sec = 1sec.



$$10^- = t_0^- \stackrel{\text{def}}{=} \lim_{\varepsilon\to 0}(t_0 - \varepsilon)$$

$$\|X_{PU}(10^-) - X_{CR}\| > 30m$$

$$10^+ = t_0^+ \stackrel{\text{def}}{=} \lim_{\varepsilon\to 0}(t_0 + \varepsilon)$$

$$\|X_{PU}(10^+) - X_{CR}\| \leq 30m$$

$$11 = t$$

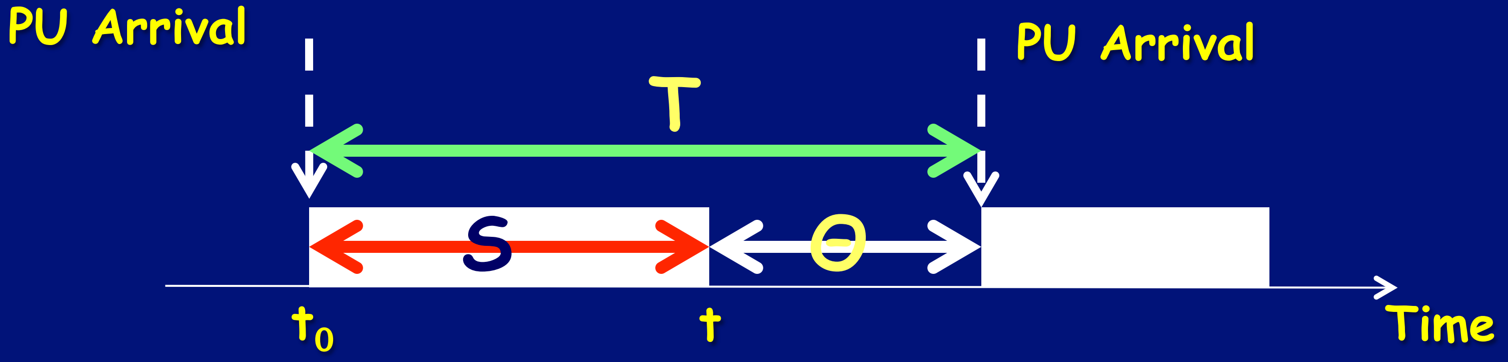
$$\|X_{PU}(11) - X_{CR}\| \leq 30m$$

Mobility-Aware Sensing Design: Some Definitions

- **Inter-Arrival Time $\mathcal{T}$:**
  Time interval between two consecutive arrivals of the PU in the interference region of a CR user.

  Inter-arrival time $\mathcal{T}$ is equal to the sum of the sojourn time and of the out time:

$$\mathcal{T} = \mathcal{S} + \Theta$$



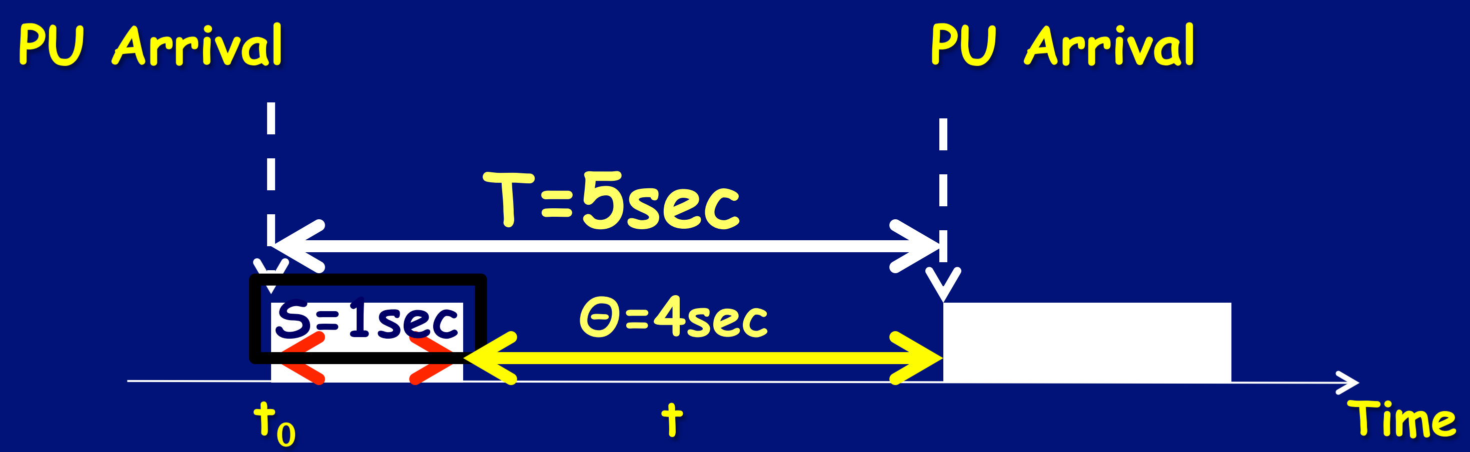
Inter-Arrival Time: Numerical Example

Inter-Arrival Time $\mathcal{T}$:

$$\mathcal{T} = \mathcal{S} + \Theta$$

From examples on Slides 23 and 26:

$S = 1\text{sec}$ and $\Theta = 4\text{sec}$ $\Rightarrow$ $T = (1+4)\text{sec} = 5\text{sec}$



Mobility-Aware Sensing Design: Overview

- First Part of the paper:
  - Characterization of the PU inter-arrival time for a general mobility model
- Second Part of the paper:
  - Derivation of the optimal sensing time parameters (sensing time and transmission time), by exploiting the results of the first part
- Third Part of the paper:
  - Specialization of the previously derived results for the RWM

Primary-User Inter-Arrival Process: Average Value of the Inter-Arrival Time

Theorem 1: Average inter-arrival time $\overline{\mathcal{T}} \triangleq E[\mathcal{T}]$ of a PU roaming within a network region A according to a general mobility model is:

$$\overline{\mathcal{T}} = \frac{\overline{\Theta}}{1 - P(\mathcal{I})} = \frac{\overline{D} \int_{\mathrm{A}} \frac{f_{\mathbf{X}_{CR}}(\mathbf{x}_{CR}) d\mathbf{x}_{CR}}{\int_{\mathcal{L}} \int_{\mathrm{C}(\mathbf{x}_{CR}, l)} f_{\mathbf{X}_{PU}}(\mathbf{x}_{PU}) d\mathbf{x}_{PU} \, f_L(l) dl}}{1 - P(\mathcal{I})}$$

with:

* P(I) is the prob of event I.; i.e., % of time that a CR user is located within PU protection range R
* the average out time is equal to $\overline{\Theta} = \overline{D} \int_{\mathrm{A}} \frac{f_{\mathbf{X}_{CR}}(\mathbf{x}_{CR}) d\mathbf{x}_{CR}}{\int_{\mathcal{L}} \int_{\mathrm{C}(\mathbf{x}_{CR}, l)} f_{\mathbf{X}_{PU}}(\mathbf{x}_{PU}) d\mathbf{x}_{PU} \, f_L(l) dl}$
* $f_{\mathbf{X}_{CR}}(\mathbf{x}_{CR})$ pdf of the CR user spatial distribution
* $f_{\mathbf{X}_{PU}}(\mathbf{x}_{PU})$ pdf of the PU steady-state spatial distribution, i.e., the PU spatial distribution after the transition
* $f_L(l)$ pdf of the RV L (Defined in slide 20)
* $\overline{D}$ average value of the RV D (Defined in slide 20)

Primary-User Inter-Arrival Process

- **Average value of the Inter-Arrival Time**
  - holds for every mobility model satisfying Assumption 1 (Slide 15).
  - Depends on three factors:
    - PU mobility model, through the PU steady-state spatial distribution $f_{X_{PU}}(x_{PU})$, $f_L(l)$, and $\overline{D}$
    - CR spatial distribution $f_{X_{CR}}(x_{CR})$
    - Prob P(I) of an arbitrary CR user being inside the PU protection range.

Average Value of the Inter-Arrival Time: Numerical Example

- **Theorem 1:**

$$\overline{\mathcal{T}} = \frac{\overline{\Theta}}{1 - P(\mathcal{I})} = \frac{\overline{D} \int_{\mathrm{A}} \frac{f_{\mathrm{X}_{CR}}(\mathrm{x}_{CR})d\mathrm{x}_{CR}}{\int_{\mathcal{L}} \int_{\mathrm{C}(\mathrm{x}_{CR},l)} f_{\mathrm{X}_{PU}}(\mathrm{x}_{PU})d\mathrm{x}_{PU}\, f_L(l)dl}}{1 - P(\mathcal{I})}$$

Assume: P(I) = 0.03 and $\overline{\Theta} = 10\ sec$

$$\overline{\mathcal{T}} = \frac{\overline{\Theta}}{1 - P(I)} = \frac{10}{1 - 0.03} = 10.31\ sec$$

With this setting, the average time between two consecutive arrivals of a PU inside the CR interference region is roughly 10.3 sec. This means a CR user waits in average 10.3 sec before to meet again the PU.

Primary-User Inter-Arrival Process:
Cumulative Distribution Function (CDF) of the Inter-Arrival Time

- **Theorem 2:** CDF of the inter-arrival time of a PU roaming within a network region A according to a general mobility model is bounded by:

$$\mathcal{F}_{\mathcal{T}}(t) \triangleq P(\mathcal{T} \leq t) \leq 1 - \int_{\mathbf{A}} e^{-\frac{t}{\bar{D}} P_g(\mathbf{x}_{CR})} f_{\mathbf{X}_{CR}}(\mathbf{x}_{CR}) d\mathbf{x}_{CR}$$

where $\bar{D}$ is the average value of the RV D and Pg(xCR) is the prob that a mobile PU meets the CR user (with location x_{CR}) during a PU movement and given by:

$$P_g(\mathbf{x}_{CR}) = \int_{\mathcal{L}} \int_{\mathbf{C}(\mathbf{x}_{CR}, l)} f_{\mathbf{X}_{PU}}(\mathbf{x}_{PU}) d\mathbf{x}_{PU} \, f_L(l) dl$$

CDF of the Inter-Arrival Time: Numerical Example

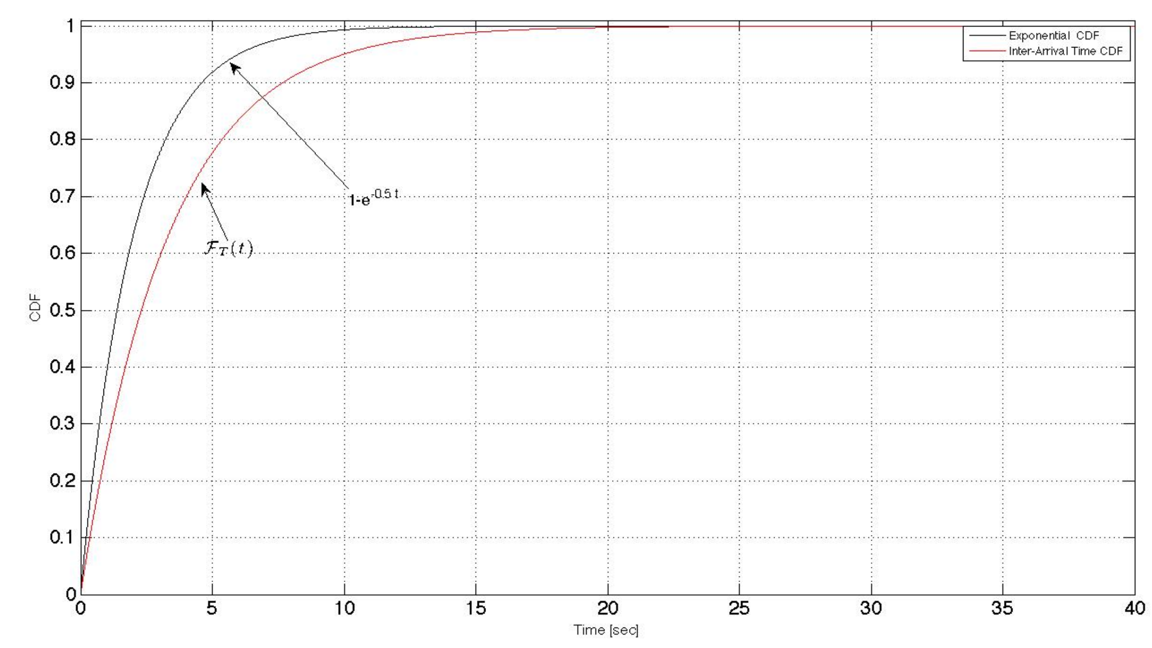
– Theorem 2: $$\mathcal{F}_{\mathcal{T}}(t) \triangleq P(\mathcal{T} \leq t) \leq 1 - \int_{\mathbf{A}} e^{-\frac{t}{\bar{D}} P_g(\mathbf{x}_{CR})} f_{\mathbf{X}_{CR}}(\mathbf{x}_{CR}) d\mathbf{x}_{CR}$$

Consider a squared network region A=[0,a]x[0,a] and assume $P_g(x_{CR})$ independent from the CR user location with

$$\frac{P_g(\mathbf{x}_{CR})}{\bar{D}} = 0.5$$

since $f_{\mathbf{X}_{CR}}(\mathbf{x}_{CR}) = \frac{1}{a^2}$, for all $\mathbf{x}_{CR} \in \mathbf{A} = [0,a] \; x \; [0,a]$, it results:

$$\mathcal{F}_{\mathcal{T}}(t) \triangleq P(\mathcal{T} \leq t) \leq 1 - e^{-0.5\,t}$$



Mobility-Aware Sensing Design: Overview

- First Part of the paper:
  - Characterization of the PU inter-arrival time for a general mobility model
- Second Part of the paper:
  - Derivation of the optimal sensing time parameters (sensing time and transmission time), by exploiting the results of the first part
- Third Part of the paper:
  - Specialization of the previously derived results for the RWM

Optimal Mobility Aware Sensing Parameters

- **Find Optimal Mobility Aware Sensing Parameters**
  (Sensing Time T_s and Transmission Time T_{TX})

STEPS:

- Optimal Mobility-Aware Transmission Time
- Optimal Mobility-Aware Sensing Time Threshold
- Mobility-Aware Sensing Time

Optimal Mobility-Aware Transmission Time

- **Definition:** $T_{\mathrm{Tx}}^{\mathrm{opt}}$ is the transmission time that:

  * allows a CR user to respect the PU interference constraint

  * maximizes the sensing efficiency for a given value of the sensing time.

Optimal Mobility-Aware Transmission Time

Theorem 3: Consider a PU roaming in a network region A according to a general mobility model, then

$$T_{Tx}^{opt} = \mathcal{F}_{\mathcal{T}}^{-1}\left(\frac{P_{int}}{P_{on}}\right)$$

where:

- CDF of the PU inter-arrival time (Theorem 2)
- $\mathcal{F}_{\mathcal{T}}(t)$ P_{int} maximum interference probability
- P_{on} PU on-state probability

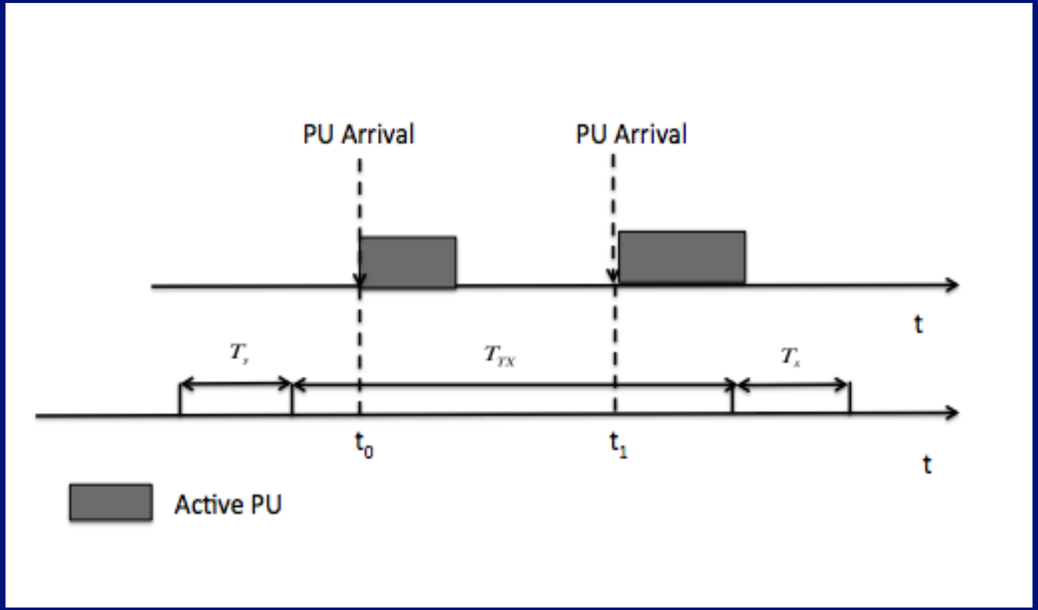
Explanation

- During two PU arrival events, a CR can interfere an active PU during T_{Tx}

  - ✓ T_{Tx} cannot exceed the maximum interference time an active PU can tolerate between two arrival events, i.e.:

$$\mathcal{F}_{\mathcal{T}}(T_{\mathrm{Tx}})P_{\mathrm{on}} = P\left(\mathcal{T} \leq T_{\mathrm{Tx}}\right)P_{\mathrm{on}} \leq P_{\mathrm{int}} \Leftrightarrow T_{\mathrm{Tx}} \leq \mathcal{F}_{\mathcal{T}}^{-1}\left(\frac{P_{\mathrm{int}}}{P_{\mathrm{on}}}\right)$$

- Optimal transmission time is the maximum T_{Tx} satisfying this equation



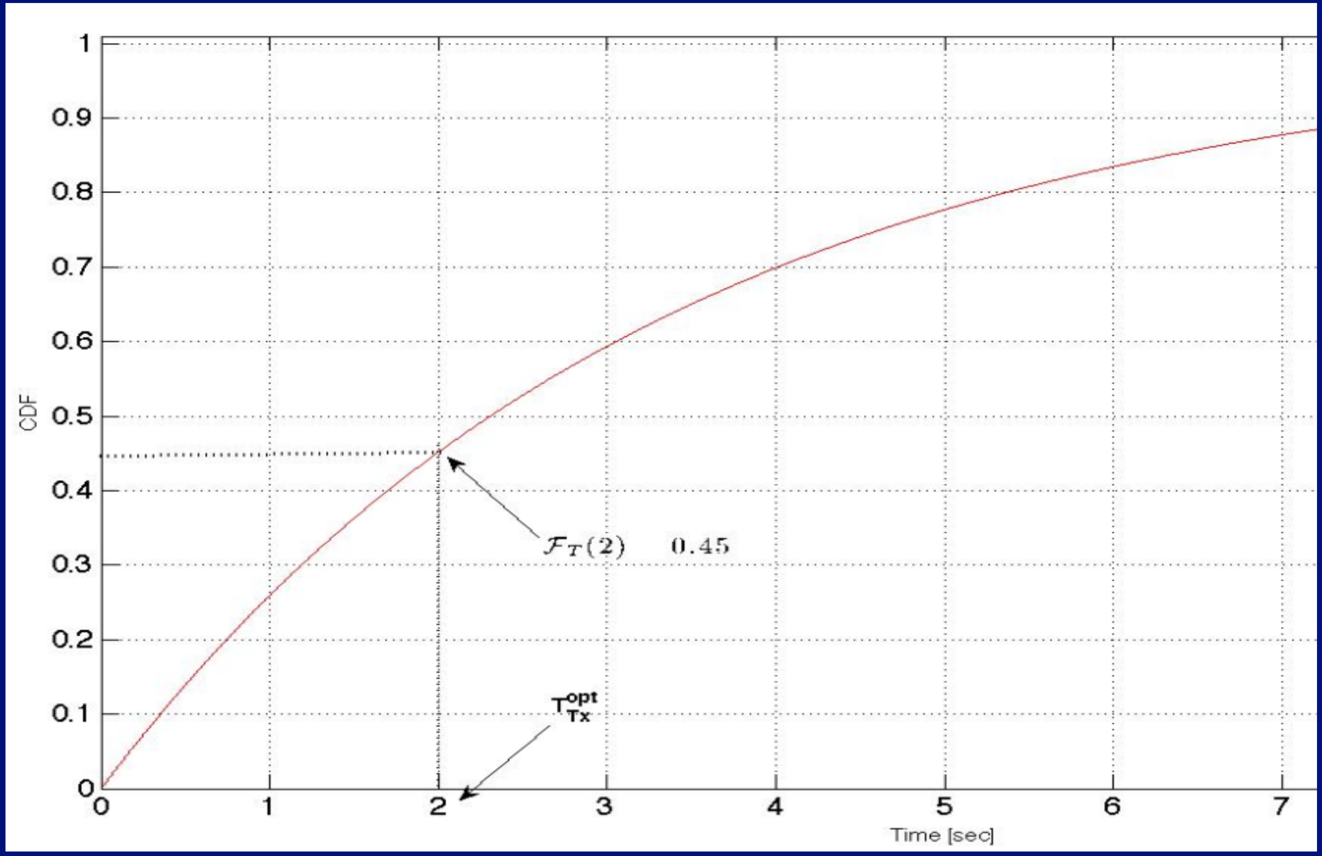
Optimal Mobility-Aware Transmission Time: Numerical Example

- **Theorem 3:** $T_{Tx}^{opt} = \mathcal{F}_{\mathcal{T}}^{-1}\left(\frac{P_{int}}{P_{on}}\right)$

Given: $P_{int}/P_{on} = 0.45$ and $\mathcal{F}_{\mathcal{T}}(t)$ reported in Figure

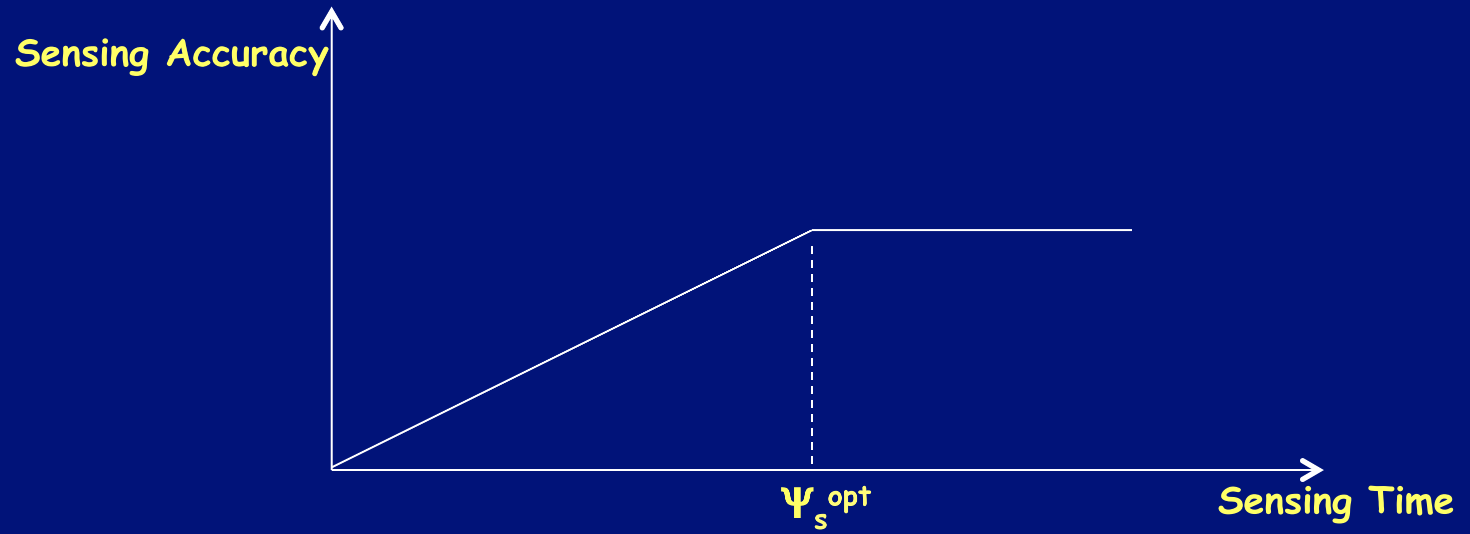
$$T_{Tx}^{opt} = \mathcal{F}_{\mathcal{T}}^{-1}(0.45) = 2\ sec$$

A CR User can transmits for 2 sec, by satisfying the PU interference constraint



Optimal Mobility-Aware Sensing Time Threshold

- **Definition:** Optimal sensing time threshold ψ_s^{opt} is the maximum value of the sensing time assuring that:
  - ✓ the sensing accuracy does not increase by observing the band for times longer than ψ_s^{opt}, regardless of the adopted sensing technique.



Optimal Mobility-Aware Sensing Time Threshold

Theorem 4: The optimal sensing time threshold ψ_s^{opt} is equal to the average sojourn time $\overline{S}$ of a mobile PU inside the CR interference region:

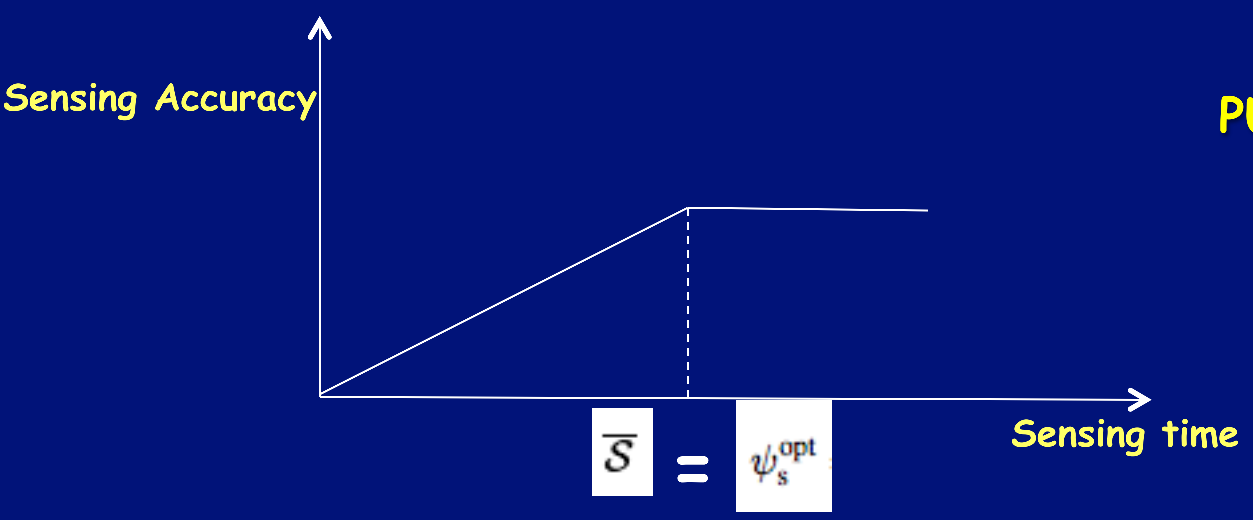
$$\psi_s^{\text{opt}} = \overline{S} = P(\mathcal{I})\,\overline{\mathcal{T}} = \frac{P(\mathcal{I})}{1 - P(\mathcal{I})}\,\overline{\Theta}$$

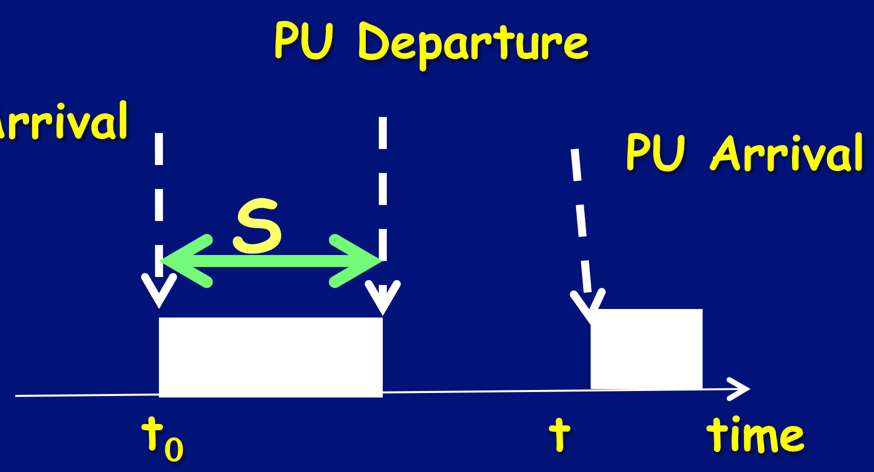
where:

- ✓ $\overline{\mathcal{T}}$ and $\overline{\Theta}$ given in Theorem 1
- ✓ P(I): prob. of a CR user being inside the PU protection range, i.e., % of time that a CR user is located within R.

Optimal Mobility-Aware Sensing Time Threshold: Graphical Representation

Theorem 4: ψ_s^{opt} is equal to the average sojourn time $\overline{S}$





Optimal Mobility-Aware Sensing Time Threshold: Numerical Example

Theorem 4:

$$\psi_s^{\text{opt}} = \overline{S} = P(\mathcal{I})\,\overline{\mathcal{T}} = \frac{P(\mathcal{I})}{1 - P(\mathcal{I})}\,\overline{\Theta}$$

✓ Utilize the same values of the prev. example:
P(I)=0.03, $\bar{\mathcal{T}} = 10.31\ sec$

$$\psi_s^{opt} = \bar{S} = P(I)\bar{\mathcal{T}} = 0.03 \cdot 10.31 = 0.31\ sec$$

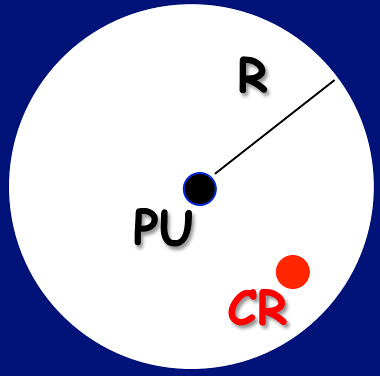
The maximum value of the sensing time, i.e. the sensing time threshold, ψ_s^{opt}, is equal to 0.31 sec.

Explanation

H_0: no PU signal, H_1: PU signal

Hypothesis Test for CR users under Event I:

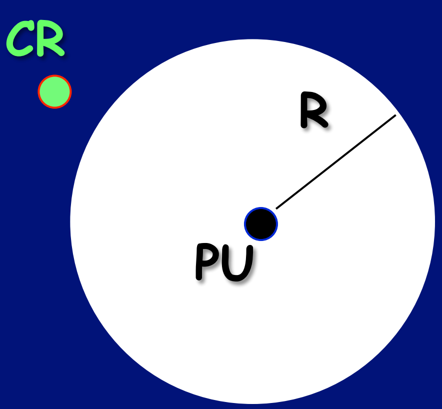
$$x(t) = \begin{cases} v(t) & \mathcal{H}_0 \\ g(t)\, s(t) + v(t) & \mathcal{H}_1 \end{cases}$$



CR user can sense the PU transmissions

Hypothesis Test for CR users under Event 0:

$$x(t) = \begin{cases} v(t) & \mathcal{H}_0 \end{cases}$$



CR user cannot sense the PU transmissions

Explanation

- Hence the Detection and the False Alarm Probabilities:

$$P_d = P(Y > \gamma|\mathcal{H}_1, \mathcal{I})P(\mathcal{I}) + \underbrace{P(Y > \gamma|\mathcal{H}_1, \mathcal{O})}_{=0}P(\mathcal{O}) = P(Y > \gamma|\mathcal{H}_1, \mathcal{I})P(\mathcal{I})$$

If the event O occurs, the CR user cannot sense the PU

$$P_f = P(Y > \gamma|\mathcal{H}_0, \mathcal{I})P(\mathcal{I}) + P(Y > \gamma|\mathcal{H}_0, \mathcal{O})P(\mathcal{O}) \geq P(Y > \gamma|\mathcal{H}_0, \mathcal{I})P(\mathcal{I})$$

Y and γ denote the decision variables and the threshold of the generic adopted sensing technique, respectively.

Explanation

- Consequently
  - P_d affected only by the event $\mathcal{I}$
  - P_f affected by both the events $\mathcal{I}$ and $\mathcal{O}$ with $P(\mathcal{I}) = \overline{S}/\overline{T}$
- From P_d and P_f it follows that observing the spectrum for a time greater than the average sojourn time S has two effects:
  - P_d does not improve
  - P_f can increase

According to the definition of ψ_s^{opt}, $\psi_s^{opt} = \overline{S}$

- $\psi_s^{opt} = \overline{S}$ agrees with the intuition: if the event O occurs, the CR user can use the spectrum without interfering the PU
  - it is useless to waste time by sensing a free spectrum.

Mobility-Aware Sensing Time

- **Corollary 1:** Consider a PU roaming within a network region A according to a general mobility model.

  The mobility-aware sensing time T_s must be set:

$$T_s \leq \overline{S}$$

  - ✓ Direct consequence of Theorem 4

Mobility-Aware Sensing Time: Numerical Example

- **Corollary 1:** $T_s \leq \overline{S}$

From the example:

$$\psi_s^{opt} = \bar{S} = P(I)\bar{T} = 0.03 \cdot 10.31 = 0.31\ sec$$

$$T_s \leq \bar{S} = 0.31\ sec$$

A CR user senses the spectrum for a time not longer than 0.31 sec.

Mobility-Aware Sensing Time

- **Insight:** The amount $v(P_d;P_f)$ of the average sojourn time used for sensing depends on:
  - required detection accuracy (P_d; P_f) &
  - adopted sensing technique.

Hence

$$T_s = \nu_{(P_d,P_f)}\,\overline{S} = \nu_{(P_d,P_f)}\,P(\mathcal{I})\,\overline{\mathcal{T}} = \nu_{(P_d,P_f)}\,\frac{P(\mathcal{I})}{1-P(\mathcal{I})}\,\overline{\Theta}, \quad \nu_{(P_d,P_f)} \in (0,1]$$

$v(P_d;P_f)$ accounts for the targeted detection accuracy and the adopted sensing technique characteristics.

Mobility-Aware Sensing Time

Insight:

- If $\overline{S} \to 0$ then $T_s \to 0$

  if the CR is never in the PU protection range, it is useless to sense the spectrum

- If $\overline{\Theta} \to 0$ i.e., if CR is always in the PU protection range, T_s must be set according to the static scenario rules or duality if the PU is always in the CR interference region

Mobility-Aware Sensing Design: Overview

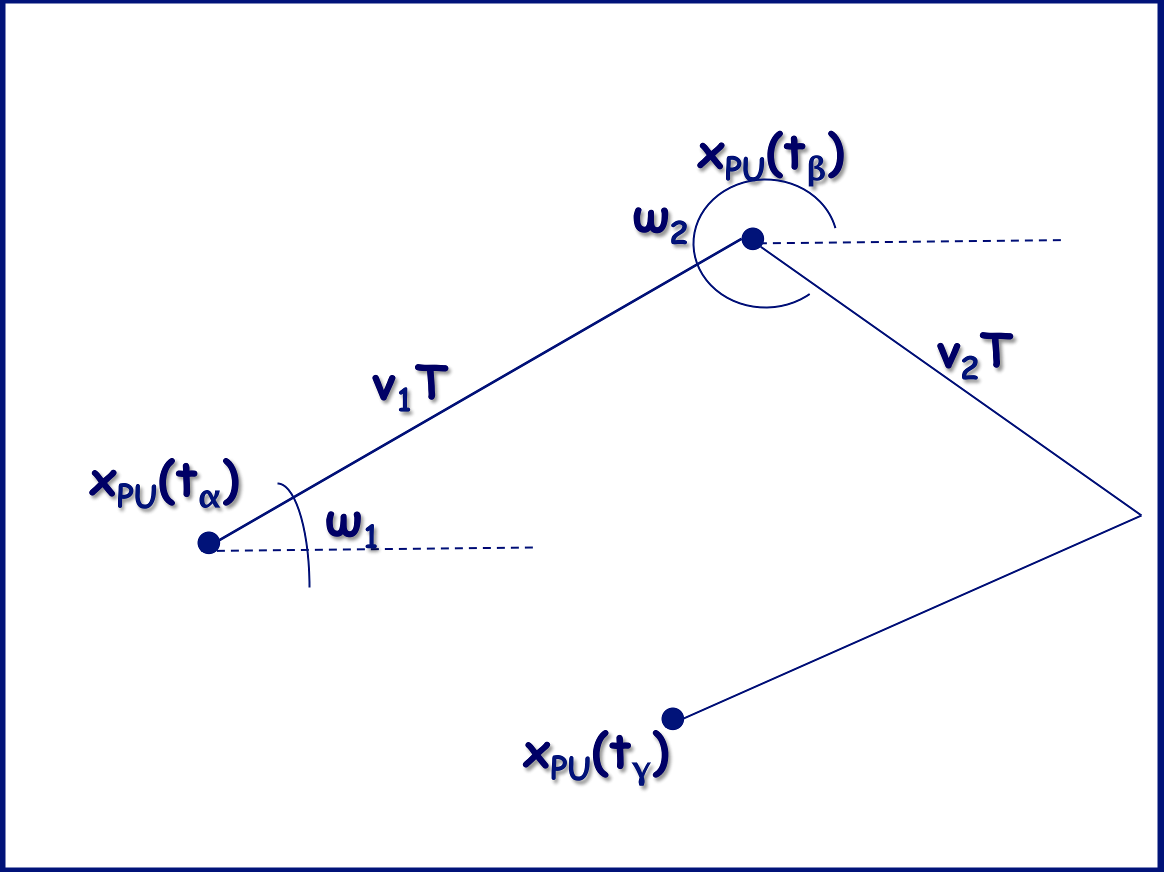
- First Part of the paper:
  - Characterization of the PU inter-arrival time for a general mobility model
- Second Part of the paper:
  - Derivation of the optimal sensing time parameters (sensing time and transmission time), by exploiting the results of the first part
- Third Part of the paper:
  - Specialization of the previously derived results for RWM

Random Walk Mobility Model (RWM)

- A PU chooses uniformly at random both a direction and a velocity in the intervals $[0; 2\pi]$ and $[v_{min}; v_{max}]$ m/s
- Each movement occurs in a constant time
- When the edge of the network region A is reached, the PU is bounced back to the region A.
- Uniform steady-state spatial distribution regardless of the average PU speed

Random Walk Mobility Model (RWM): Graphical Representation

Network Region A



- At time t_α , the PU in position $x_{PU}(t_\alpha)$ randomly selects a direction ω_1 and a velocity v_1
- Then PU moves for a constant time T, reaching the position $x_{PU}(t_\beta)$ at time t_β
- Here, PU selects a new direction ω_2 and speed v_2 according to the same rule
- Since during this movement, PU reaches the edge of A & is bounced back to the region until reaching the position $x_{PU}(t_\gamma)$ at time t_γ

Primary-User Inter-Arrival Process for RWM

- **Theorem 1 for RWM:** The average inter-arrival time of a PU roaming within a network region A according to the RWM is:

$$\overline{T}_{\mathrm{RWM}} = \frac{\overline{\Theta}_{\mathrm{RWM}}}{1 - P_{\mathrm{RWM}}(\mathcal{I})} = \begin{cases} \frac{a}{\overline{v}_{\mathrm{RWM}}(1-P_{\mathrm{1D\text{-}RWM}}(\mathcal{I}))}, & A = [0,a] \\ \frac{a^2}{2\,R\,\overline{v}_{\mathrm{RWM}}(1-P_{\mathrm{2D\text{-}RWM}}(\mathcal{I}))}, & \mathbf{A} = [0,a] \times [0,a] \end{cases}$$

One-dimensional Network region

Bi-dimensional Network region

- Θ is the average out time (average time interval a PU spends out of interference region of a CR user)
- v_{RWM} is the average PU velocity
- $P_{RWM}(I)$ is the prob of event I (% of time a CR user is located within R) for one-dimensional and bi-dimensional network regions, respectively

Primary-User Inter-Arrival Process for RWM

$\overline{T}_{\mathrm{RWM}}$ depends on three factors:

- Average PU velocity $\overline{v}_{\mathrm{RWM}} = \frac{v_{\min} + v_{\max}}{2}$
- Normalized protection radius R/a
- Size of the network region A

Average Value of the RWM Inter-Arrival Time: Numerical Example

Theorem 1 for RWM:

$$\overline{\mathcal{T}}_{\mathrm{RWM}} = \frac{\overline{\Theta}_{\mathrm{RWM}}}{1 - P_{\mathrm{RWM}}(\mathcal{I})} = \begin{cases} \frac{a}{\overline{v}_{\mathrm{RWM}}(1 - P_{\mathrm{1D\text{-}RWM}}(\mathcal{I}))}, & A = [0, a] \\ \frac{a^2}{2\,R\,\overline{v}_{\mathrm{RWM}}(1 - P_{\mathrm{2D\text{-}RWM}}(\mathcal{I}))}, & \mathbf{A} = [0, a] \times [0, a] \end{cases}$$

Assume:

A squared network region A=[0,a]x[0,a]

R/a=0.01

- $\frac{\bar{v}_{\mathrm{RWM}}}{a} = 0.2$
- $P_{2D-RWM}(I) = \pi \cdot 10^{-4}$

$$\bar{\mathcal{T}}_{\mathrm{RWM}} = \frac{1}{(2 \cdot 0.01 \cdot 0.2 \cdot (1 - \pi \cdot 10^{-4}))} = 250.08\ sec$$

i.e, CR user waits on the average roughly 250 sec before to meet again a PU that is moving according to the RWM.

Primary-User Inter-Arrival Process for RWM

Theorem 2 for RWM:

The CDF of the inter-arrival time of a PU roaming within a network region A according to the RWM is bounded by an exponential distribution:

$$\mathcal{F}_{\mathcal{T}_{\mathrm{RWM}}}(t) \stackrel{\triangle}{=} P(\mathcal{T}_{\mathrm{RWM}} \leq t) \leq 1 - e^{-\Omega_{\mathrm{RWM}}\, t} \stackrel{\triangle}{=} \mathcal{E}_{\Omega_{\mathrm{RWM}}}(t)$$

$$\Omega_{\mathrm{RWM}} \stackrel{\triangle}{=} \frac{1}{\overline{\Theta}_{\mathrm{RWM}}} = \begin{cases} \frac{\overline{v}_{\mathrm{RWM}}}{a}, & A = [0, a] \\ \frac{2\,R\,\overline{v}_{\mathrm{RWM}}}{a^2}, & \mathbf{A} = [0, a] \times [0, a] \end{cases}$$

One-dimensional Network region

Bi-dimensional Network region

CDF of the Inter-Arrival Time for the RWM: Numerical Example

- **Theorem 2 for RWM:** $\mathcal{F}_{\mathcal{T}_{\mathrm{RWM}}}(t) \triangleq P(\mathcal{T}_{\mathrm{RWM}} \leq t) \leq 1 - e^{-\Omega_{\mathrm{RWM}}\, t} \triangleq \mathcal{E}_{\Omega_{\mathrm{RWM}}}(t)$

Assume:

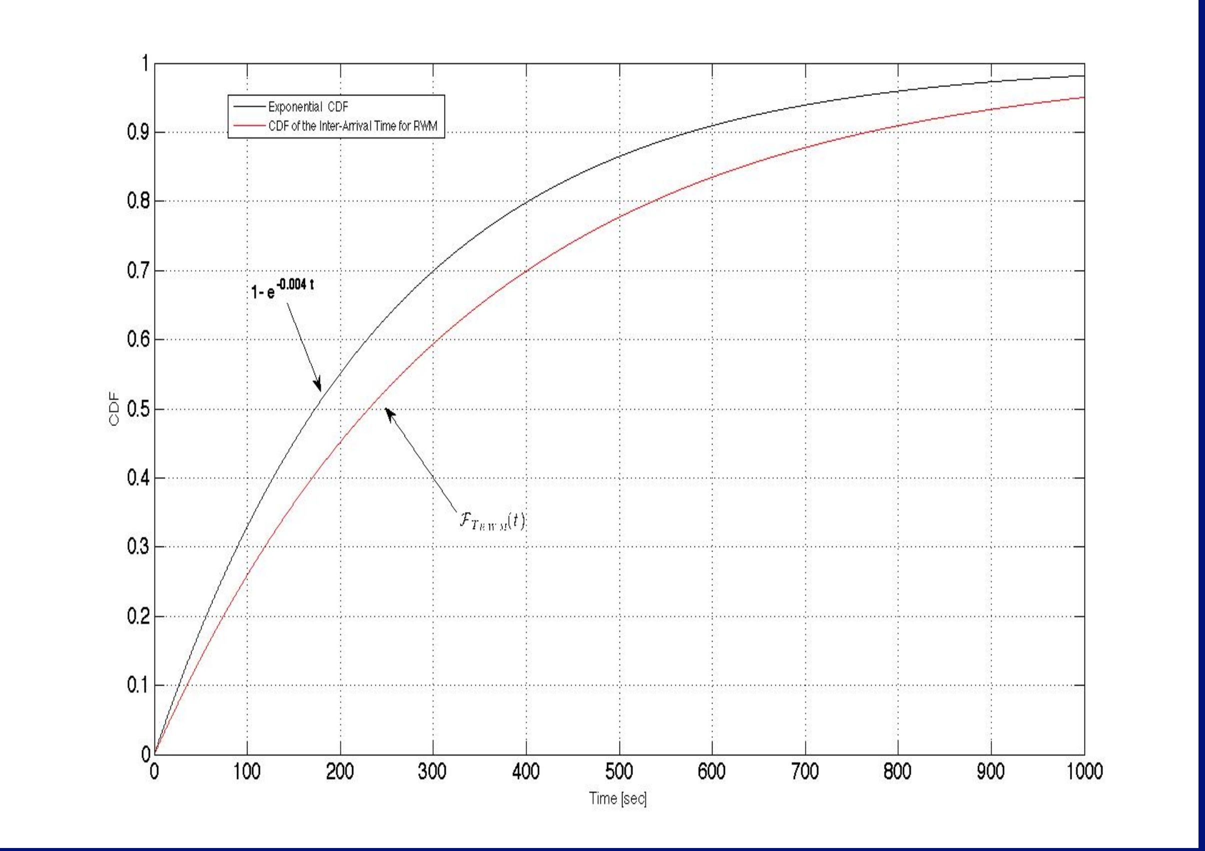
A squared network region A=[0,a]x[0,a]

R/a=0.01

$$\frac{v_{\mathrm{RWM}}}{a} = 0.2$$

$$\Omega_{\mathrm{RWM}} = 0.004$$

$$\mathcal{F}_{\mathcal{T}_{\mathrm{RWM}}}(t) \leq 1 - e^{-0.004\, t}$$



Optimal Mobility-Aware Transmission Time for RWM

- According to Theorem 3, the optimal mobility-aware transmission time for RWM is:

$$T^{opt}_{\mathrm{Tx-RWM}} = \mathcal{F}^{-1}_{\mathcal{T}_{\mathrm{RWM}}}\left(\frac{P_{\mathrm{int}}}{P_{\mathrm{on}}}\right)$$

where $\mathcal{F}_{\mathcal{T}_{\mathrm{RWM}}}(t)$ is given by Theorem 2 for RWM: $\mathcal{F}_{\mathcal{T}_{\mathrm{RWM}}}(t) \triangleq P(\mathcal{T}_{\mathrm{RWM}} \leq t) \leq 1 - e^{-\Omega_{\mathrm{RWM}} t} \triangleq \mathcal{E}_{\Omega_{\mathrm{RWM}}}(t)$

- By combining these two results, the following bound is derived:

$$T^{opt}_{Tx\text{-}RWM} \geq \overline{\Theta}_{RWM} \log\left(\frac{P_{on}}{P_{on} - P_{int}}\right), \quad P_{on} \geq P_{int}$$

where θ is the average out time for the RWM.

According to this result, a CR user can transmit for a longer time than the product between the average out-time and $\log\left(\frac{P_{on}}{P_{on} - P_{int}}\right)$

Optimal Mobility-Aware Transmission Time for RWM

The lower bound provides a practical rule for setting T_{TX} in mobile scenarios, in fact

- T_{TX} shorter than the derived bound causes sensing inefficiency
- T_{TX} longer than the derived bound can violate the PU interference constraint.

$$T_{\text{Tx-RWM}} = \overline{\Theta}_{\text{RWM}} \log\left(\frac{P_{\text{on}}}{P_{\text{on}} - P_{\text{int}}}\right)$$

Optimal Mobility-Aware Transmission Time for RWM:
Numerical Example

Assume:

- $A=[0,a]\times[0,a]$
- $R/a=0.005$
- $P_{int}=10^{-2}$
- $P_{on} = 0.2$
- $\dfrac{\bar{v}_{\mathrm{RWM}}}{a} = 0.5$

$$\bar{\Theta}_{\mathrm{RWM}} = \frac{a^2}{2\,R\,\bar{v}_{\mathrm{RWM}}} = 200\ sec$$

$$T^{opt}_{\mathrm{Tx-RWM}} = \bar{\Theta}_{\mathrm{RWM}} \log\left(\frac{P_{on}}{P_{on} - P_{int}}\right) = (200 \cdot 0.0513)sec = 10.26\ sec$$

A CR user can transmit for 10.26 sec

Mobility-Aware Sensing Time for RWM

According to Corollary 1 & the related insight, the mobility-aware sensing time for the RWM is

$$T_{s-\mathrm{RWM}} = \nu(P_d, P_f)\bar{S}_{\mathrm{RWM}} = \nu(P_d, P_f)P_{\mathrm{RWM}}(I)\bar{T}_{\mathrm{RWM}}$$

where $\bar{T}_{\mathrm{RWM}}$ is given by Theorem 1 for RWM:

$$\overline{T}_{\mathrm{RWM}} = \frac{\overline{\Theta}_{\mathrm{RWM}}}{1 - P_{\mathrm{RWM}}(\mathcal{I})} = \begin{cases} \frac{a}{\overline{v}_{\mathrm{RWM}}(1 - P_{\mathrm{1D\text{-}RWM}}(\mathcal{I}))}, & A = [0, a] \\ \frac{a^2}{2\,R\,\overline{v}_{\mathrm{RWM}}(1 - P_{\mathrm{2D\text{-}RWM}}(\mathcal{I}))}, & \mathbf{A} = [0, a] \times [0, a] \end{cases}$$

By combining these two results, the mobility-aware sensing time for the RWM is given by:

$$T_{s-\mathrm{RWM}} = \begin{cases} \frac{\nu(P_d, P_f)P_{1\mathrm{D-RWM}}(I)}{1 - P_{1\mathrm{D-RWM}}(I)}\frac{a}{\bar{v}_{\mathrm{RWM}}}, & A = [0, a] \\ \frac{\nu(P_d, P_f)P_{2\mathrm{D-RWM}}(I)}{1 - P_{2\mathrm{D-RWM}}(I)}\frac{a^2}{2\,R\,\bar{v}_{\mathrm{RWM}}}, & A = [0, a]x[o, a] \end{cases}$$

$A = [0, a]$ → One-dimensional Network region

$A = [0, a]x[o, a]$ → Bi-dimensional Network region

Mobility-Aware Sensing Time for RWM: Numerical Example

$$T_{s-\mathrm{RWM}} = \begin{cases} \dfrac{\nu(P_d, P_f) P_{1D-\mathrm{RWM}}(I)}{1 - P_{1D-\mathrm{RWM}}(I)} \dfrac{a}{\bar{v}_{\mathrm{RWM}}}, & A = [0, a] \\ \dfrac{\nu(P_d, P_f) P_{2D-\mathrm{RWM}}(I)}{1 - P_{2D-\mathrm{RWM}}(I)} \dfrac{a^2}{2\, R\, \bar{v}_{\mathrm{RWM}}}, & A = [0, a]x[o, a] \end{cases}$$

Continue the Example:

- A=[0,a]x[0,a]
- R/a=0.005
- $v(P_d, P_f) = 1$
- $\dfrac{\bar{v}_{\mathrm{RWM}}}{a} = 0.5$
- $P_{2D-RWM}(I) = 7.85 \cdot 10^{-5}$

$$T_{s-\mathrm{RWM}} = \frac{1 \cdot 7.85 \cdot 10^{-5}}{1 - 7.85 \cdot 10^{-5}} \frac{1}{2 \cdot 0.005 \cdot 0.5} = 0.016\ sec$$

Since PU is very fast and at the same time has a small protection range compared to the network region A, a CR user must sense the channel for just 0.016 sec.
It can transmit for 10.26 sec.

High sensing efficiency by respectingat the same time the PU interference constraint

Mobility-Aware Sensing Time for RWM: Some Considerations

T_s depends on:

- normalized PU protection range R/a
- extension of the network area A
- average PU velocity
- sensing accuracy $v(P_d, P_f)$

Mobility-Aware Sensing Efficiency

- **Definition:** The mobility-aware sensing efficiency is the ratio of the optimal transmission time over the entire sensing period

$$\eta_{\mathrm{mob}} \triangleq \frac{T_{\mathrm{Tx}}^{\mathrm{opt}}}{T_{\mathrm{sp}}} = \frac{T_{\mathrm{Tx}}^{\mathrm{opt}}}{T_{\mathrm{s}} + T_{\mathrm{Tx}}^{\mathrm{opt}}}$$

By using the derived results, a lower bound of the sensing efficiency is evaluated.

Mobility-Aware Sensing Efficiency

$$\eta_{mob} = 1 - \frac{1}{1 + \frac{T_{Tx}^{opt}}{\nu_{(P_d,P_f)} \frac{P(\mathcal{I})}{1-P(\mathcal{I})}\Theta}} \geq 1 - \frac{1}{1+\frac{\log\left(\frac{P_{on}}{P_{on}-P_{int}}\right)}{\nu_{(P_d,P_f)}}\frac{1-P_{RWM}(\mathcal{I})}{P_{RWM}(\mathcal{I})}},$$

- η_{mob} depends on 3 factors:
  - PU Interference Constraint P_{int}
  - PU Mobility Model
  - PU Traffic
- η_{mob} reflects the dynamic nature of
  - PU topology through P(I)
  - PU traffic through P_{on}.

- Given T_s, P_{int} and P_{on}, η_{mob} increases when $\overline{\Theta}$ increases

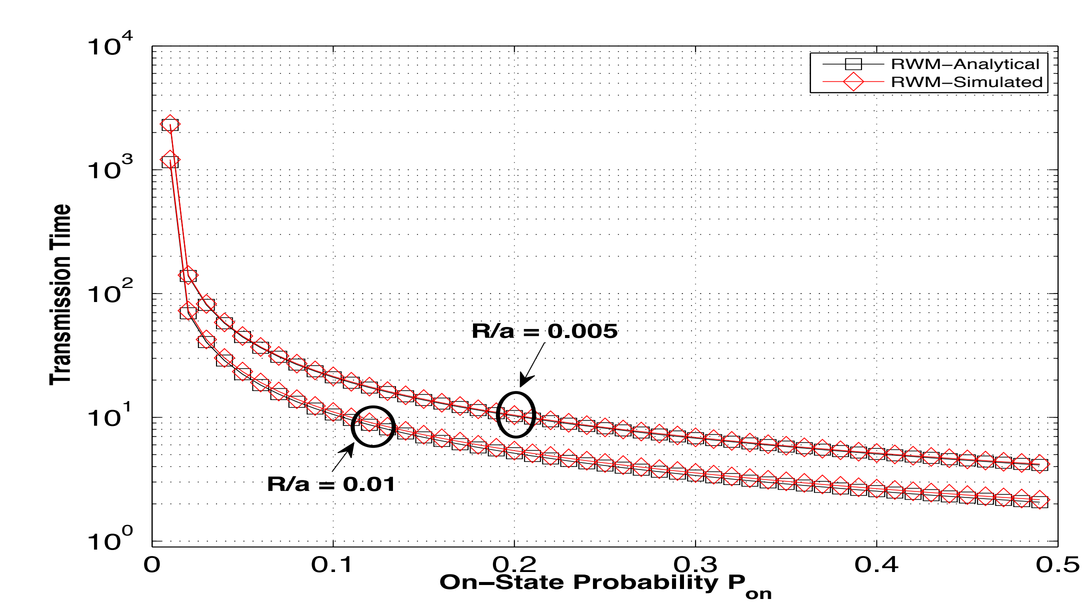
  CR user spends more time out of the PU range and thus it can use the spectrum for a longer time with the same P_{int}

- Given T_s, P_{int} and $\overline{\Theta}$, if $P_{on} \rightarrow P_{int}$, $\eta_{mob} \rightarrow 1$

  CR user can transmit in an arbitrarily long time interval

Results: Optimal Mobility-Aware Transmission Time

- T_{Tx} is set according to the lower bounds, P_{int}= 10^{-2}, v_{min}/a=0.1, v_{max}/a= 0.9



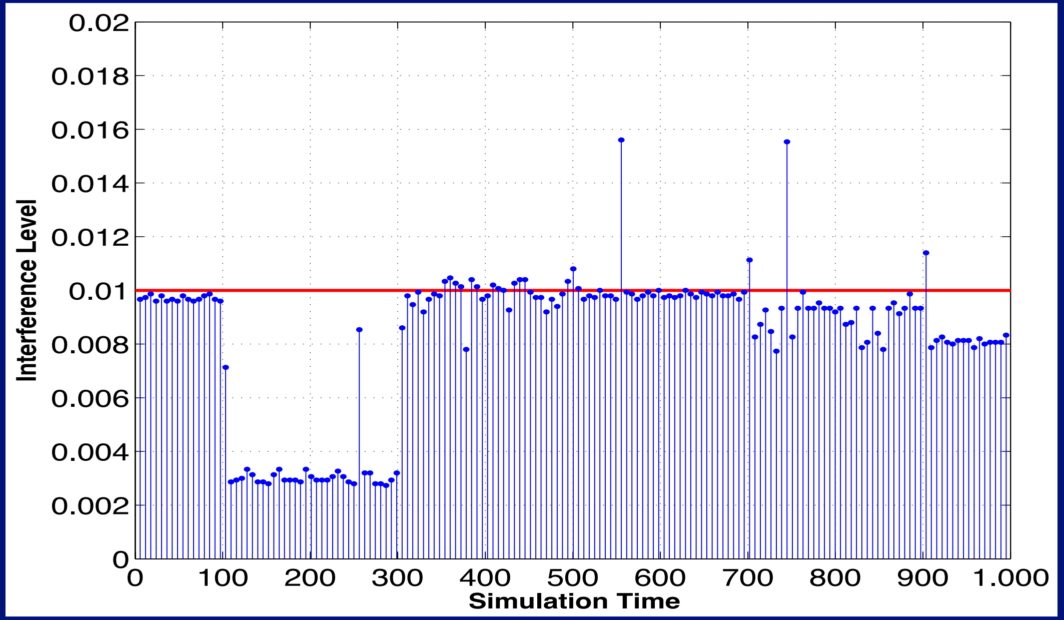
Theoretical results match well the simulation results

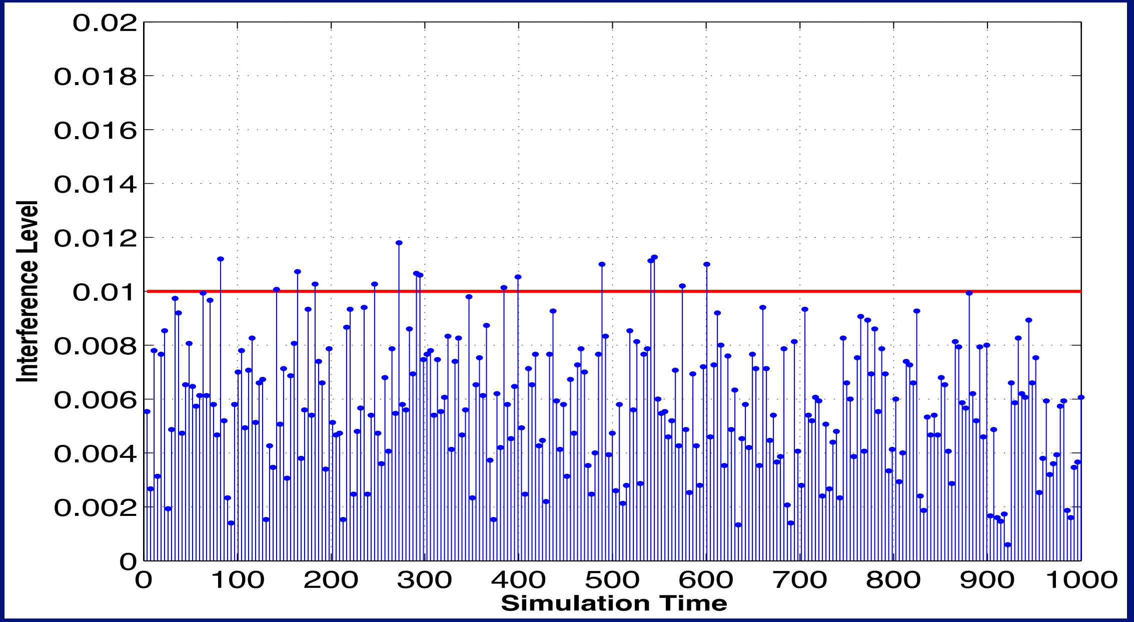
When R/a increases, T_{Tx} decreases → CR spends more time in the PU range

When P_{on} increases, T_{Tx} decreases → PU traffic dynamics increase

Results: Optimal Mobility-Aware Transmission Time

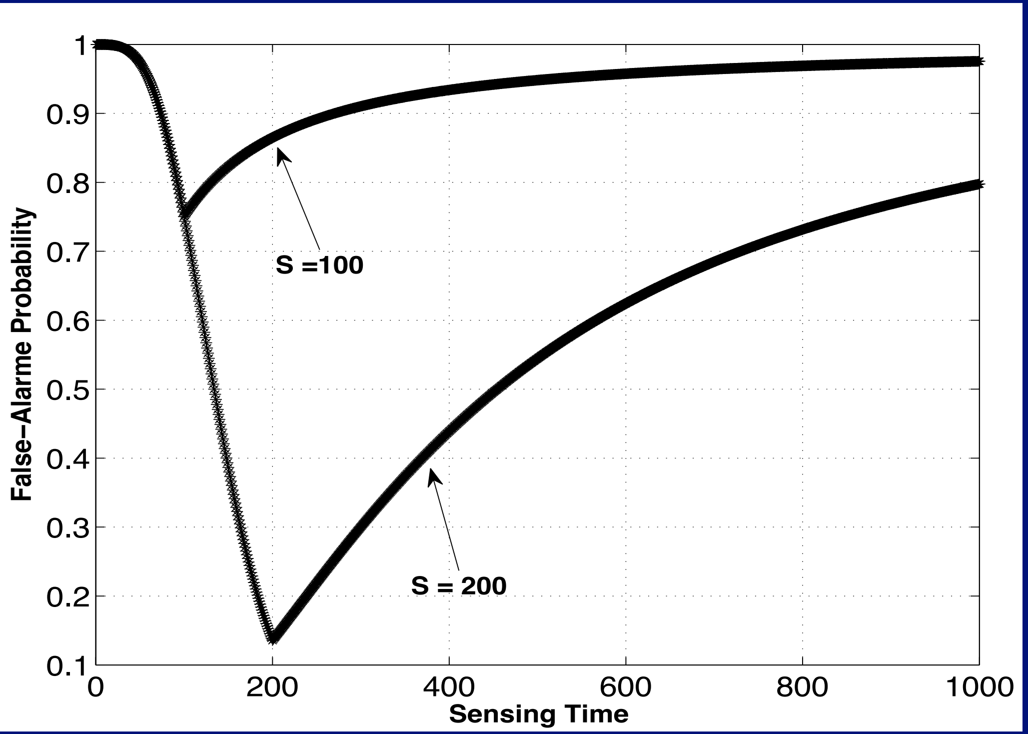
- Instantaneous Interference Level on the PU transmissions, $P_{int} = 10^{-2}$ (represented in the figures with the red lines), $P_{on} = 1/3$





✓ The results confirm the benefits of setting T_{Tx} according to our results: → the average interference levels on the PU for RWM are $9 \cdot 10^{-3} < P_{int}$ and $6 \cdot 10^{-3} < P_{int}$

Results: Optimal Mobility-aware Sensing Time Threshold



False-alarm probabilities versus T_s,
SNR= -5 dB, Energy Detector, P_d=0.999

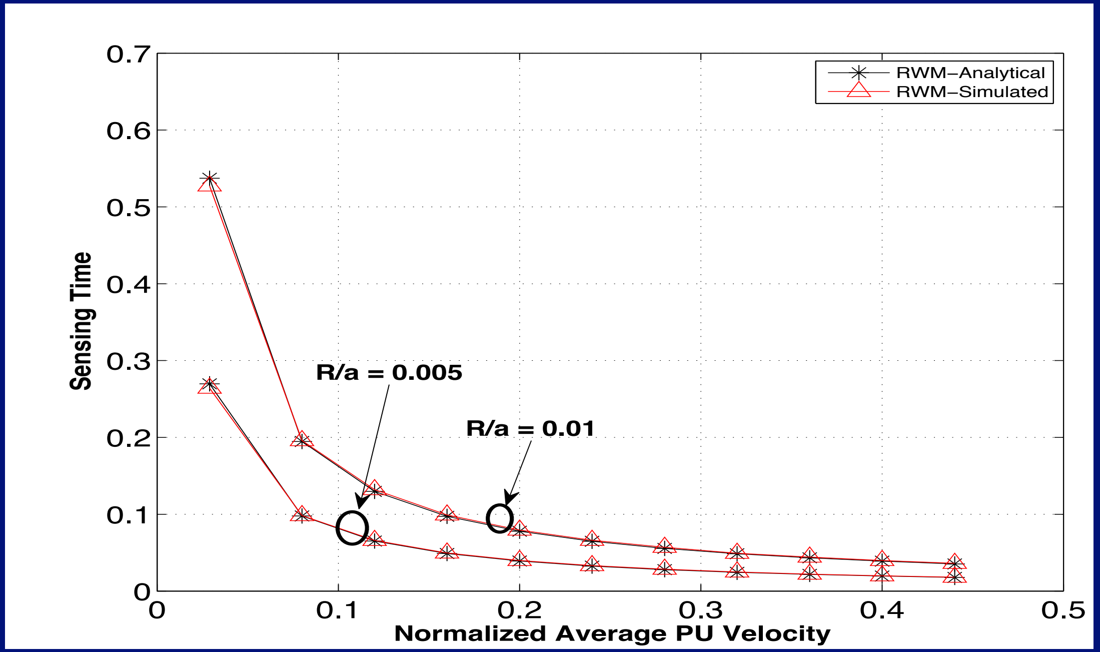
- The results validate the analysis:
  - ✓ for T_s longer than the average sojourn time S, P_f increases

↓

Threshold behavior in the sensing accuracy

Results: Mobility-Aware Sensing Time

- T_s is set according to the analytical results for $v(P_d,P_f)=1$



- ✓ theoretical results match well the simulation results
- ✓ when R/a increases, T_s increases → CR probability of being inside the PU range increases
- ✓ when v/a increases, T_s decreases → CR spends more time out of the PU range

Conclusions

- Mobile PU dynamics force researchers to revise the current design of the sensing functionality for jointly
  - maximizing the sensing efficiency
  - satisfying the PU interference constraint

- Sensing Time and Transmission Time Optimization
  - Two fundamental questions are answered:
    - How often must the sensing be performed in presence of PU mobility?
    - How long must a spectrum band be sensed to reliably detect mobile PUs?
  - A threshold behavior in the sensing accuracy as a function of the sensing time

Conclusions

- The developed optimal mobility-aware sensing design exhibits a very attractive feature:
  - It does not depend on the instantaneous values of the PU mobility pattern but only on the average statistics such as average PU sojourn time and the average PU out time