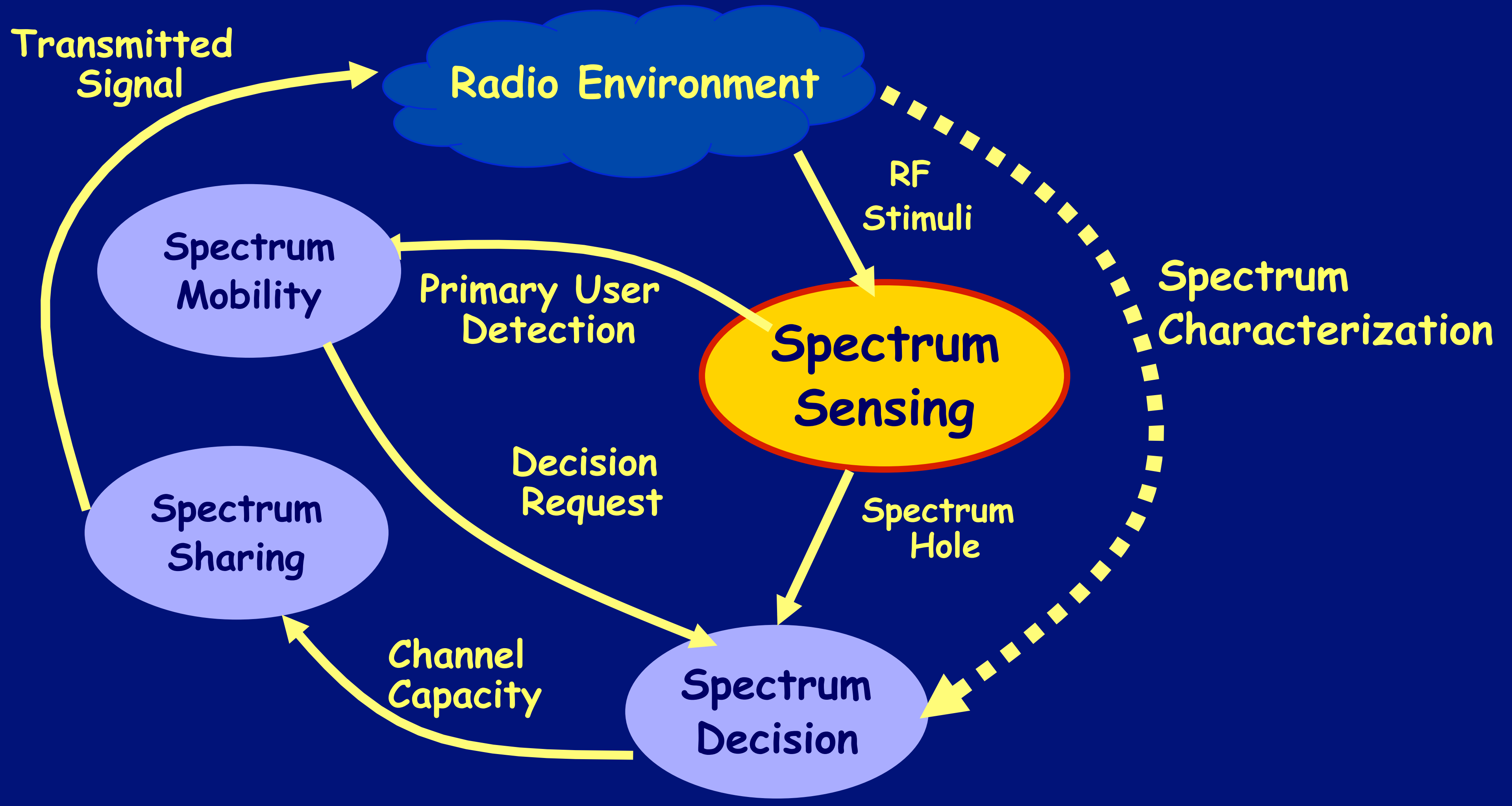
CHAPTER 5.1.
SPECTRUM SENSING

Spectrum Sensing



Classification of Spectrum Sensing Techniques

Spectrum Sensing

- Transmitter Detection
  - Matched Filter Detection
  - Energy Detection
  - Cyclostationary Feature Detection
  - Covariance Matrix-based Detection
  - Wavelet Detection
  - Compressed Sensing
- Receiver Detection
- Interference Temperature Management

Binary Hypothesis Testing for Transmitter Detection

The received signal $x(t)$ to be detected by a CR user is

$$x(t) = \begin{cases} n(t) & H_0 \\ h(t)s(t) + n(t) & H_1 \end{cases}$$

where $n(t)$ → Additive white Gaussian noise (AWGN)
$s(t)$ → Transmitted signal of a PU
$h(t)$ → Amplitude gain of the channel

H_0: Null hypothesis → No licensed user signal in a certain spectrum band
H_1: Alternative hypothesis → There exists some licensed user signal

Spectrum Sensing Performance

Based on the hypothesis testing, spectrum sensing performance is evaluated by Prob. of Detection P_d and Prob. of False Alarm P_f:

$$P_d = P\{decision = H_1 \mid H_1\}$$

$$P_f = P\{decision = H_1 \mid H_0\}$$

P_d: Probability of CR user claiming H_1 when the PU is present (H_1)
P_f: Probability of CR user claiming H_1 when the PU is absent (H_0)

What do P_d and P_f mean?

- A low P_d → Missing the correct detection of the presence of the PU with high probability → increases the interference to the PU
- A high P_f → low spectrum utilization (since false alarms increase the number of missed opportunities (white spaces)).

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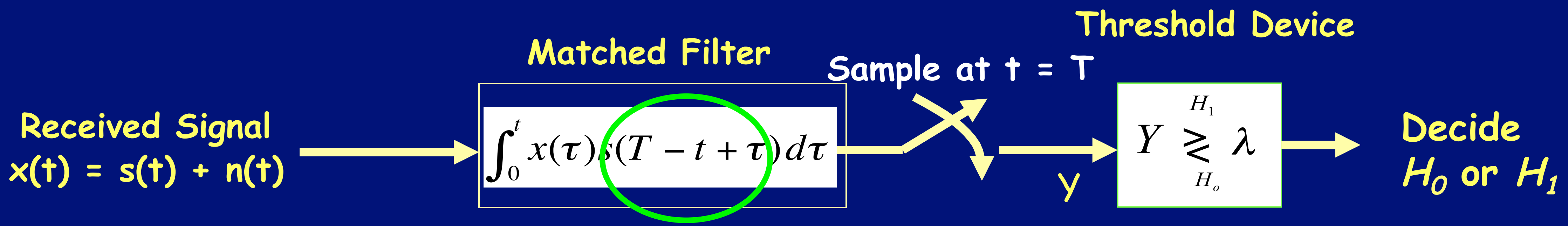
What is a Matched Filter?

DEFINITION:

A matched filter is a linear filter whose impulse response *h(t)* is a conjugated, time-reversed, delayed version of the known signal *s(t)*

- **Detection of *s(t)* by a "matched filter" receiver**
  - Received signal *x(t)* is first convolved with *h(t)*
  - Output of the filter is sampled at *t=T*
  - The result is then sent to a threshold device for detection

Matched Filter for CR



s(t): the transmitted signal of the PU

n(t): AWGN

T: Symbol interval

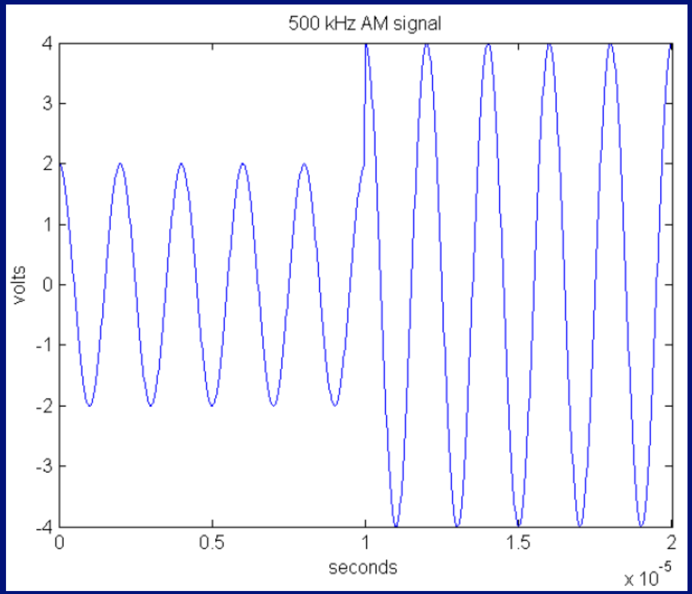
λ : Threshold

Need

- Transmitted signal information s(t)
- Synchronization for sampling timing (t=T)

Example: Matched Filter Detection

Primary AM Signal (no noise)
$x(t) = A \cos(2 \pi f_c t)$

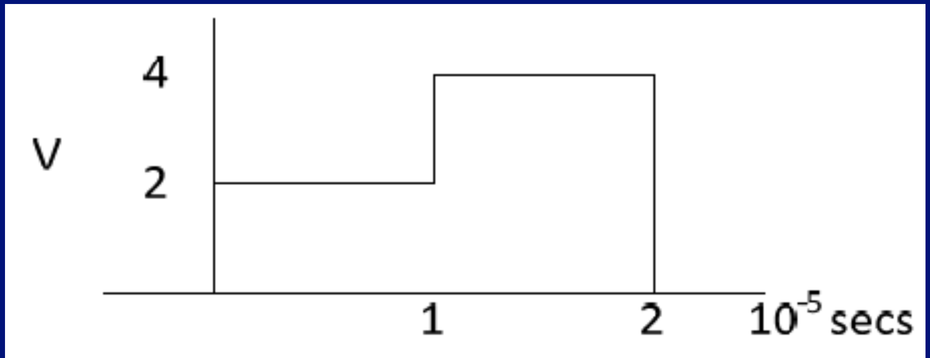


Baseband waveform

$x(t) \cos(2 \pi f_c t) = A/2 + A/2 \cos(4 \pi f_c t)$

→ Low-pass filter: A

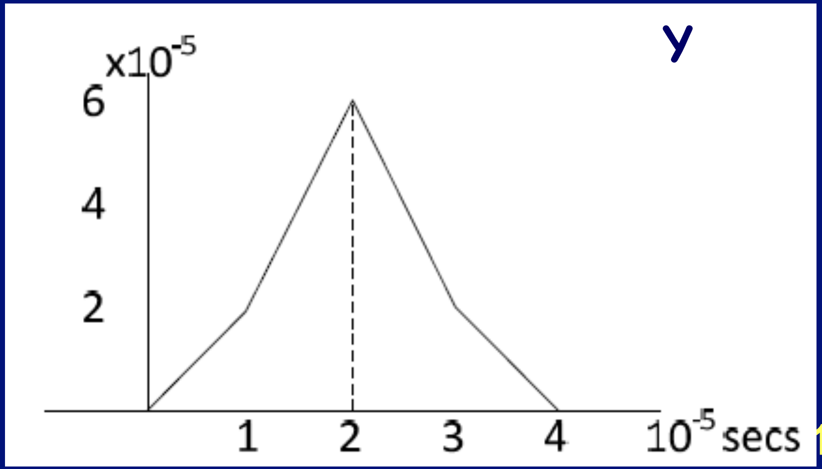
Total energy: $E = P*t = V^2 *t = 200\ \mu J$



Sample at t = T → Output of the matched filter:

$T = 2*10^{-5}$ sec

Y_{max}=Peak value = total area = $6*10^{-5}$



Matched Filter Detection for CR

A. Sahai, N. Hoven and R. Tandra,
"Some Fundamental Limits in Cognitive Radio",
Proc. Allerton Conf. on Comm., Control and Computing, 2004.

When the characteristics of the PU signal is known to the CR user →

Optimal for maximizing SNR in the presence of AWGN

Disadvantages of Matched Filter Detection

- It requires a priori knowledge of the PU signal. e.g.,
  - * modulation type and order
  - * pulse shape/signal waveform
  - * packet format

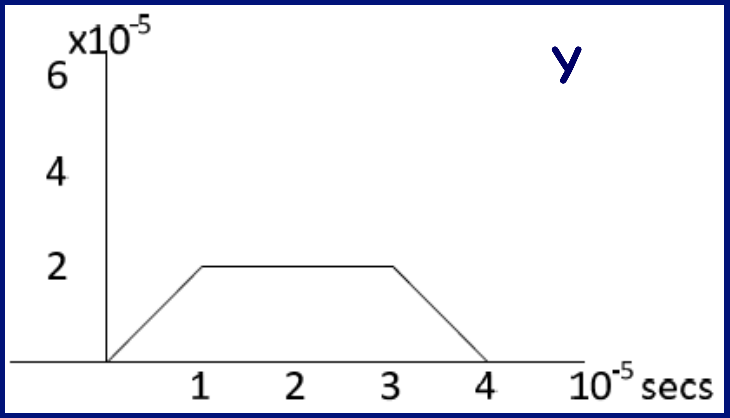
* Performs poorly if a priori knowledge is not accurate

- One matched filter is required for detecting each type of PUs → high implementation cost in heterogeneous PUs

Example: Matched Filter Detection

If the PU's exact waveform is unknown → matched filter does not work

Filter Output



- No peak value at the sampling period
  → primary signal is not detected

Advantages of Matched Filter Detection

Most wireless network systems have

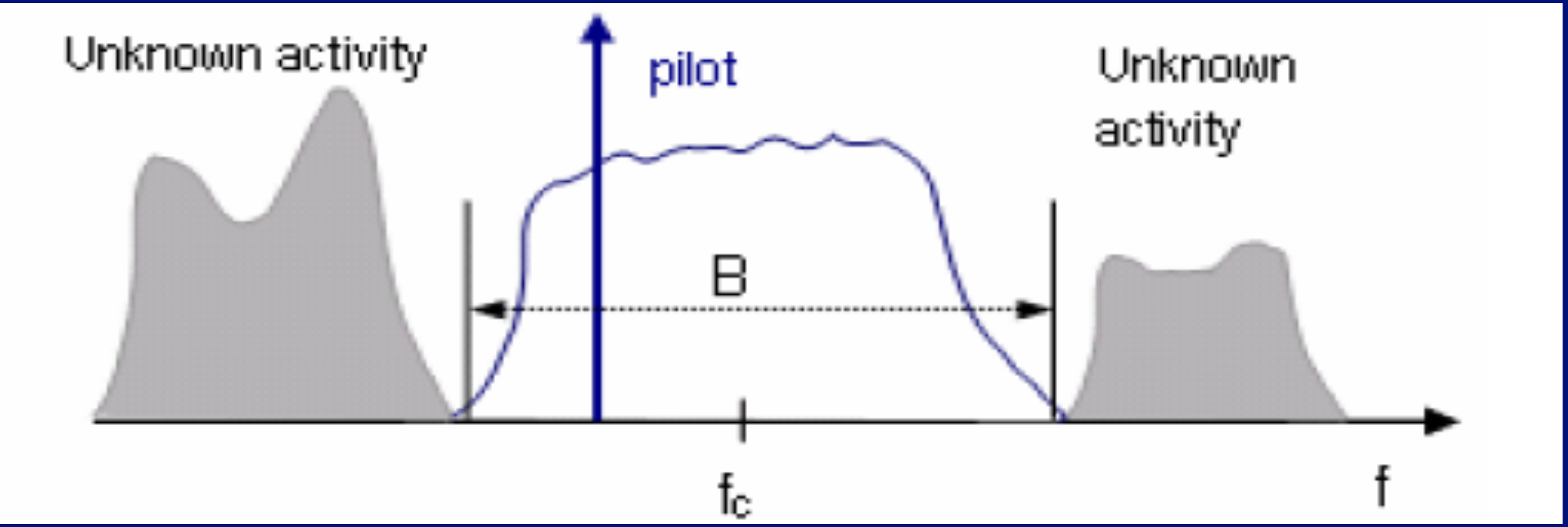
* pilot
* preambles
* synchronization word
* spreading codes

These can be used for coherent detection.

Due to coherency, it requires less time to achieve high processing gain, i.e., detection can be very fast.

Matched Filter Detection When Pilot Signal is Transmitted

D. Cabric, A. Tkachenko and R. W. Brodersen,
"Spectrum Sensing Measurements of Pilot, Energy, and Collaborative Detection",
Berkeley Wireless Research Center, UC Berkeley, 2006



- Assume PU transmitter sends a pilot signal
- Signal power is confined inside a priori known BW B around f_c
- Also, sensing receiver has a perfect knowledge of the pilot signal

Matched Filter Detection When Pilot Signal is Transmitted

Detection is the test of the following two hypotheses:

$$x(t) = \begin{cases} n(t) & H_0 \\ X_p(t) + n(t) & H_1 \end{cases}$$

Signal Absent

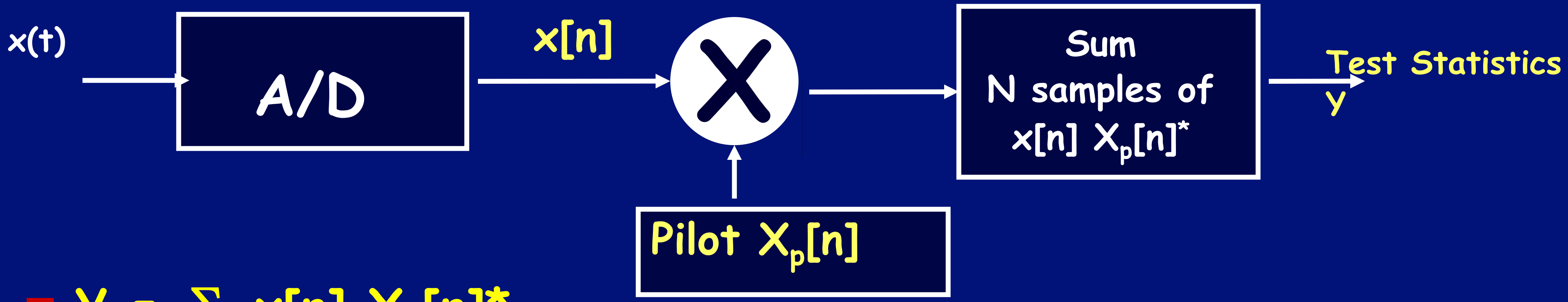
Signal Present

where T is the observation interval and t = 1,…,T

$X_p[t]$: Pilot Data

$n[t]$: AWGN

Matched Filter Using Pilot Signal



- $Y = \Sigma \, x[n] \, X_p[n]^*$

- Compare with threshold
  - $Y > \lambda$ Decide Signal Present
  - $Y < \lambda$ Decide Signal Absent

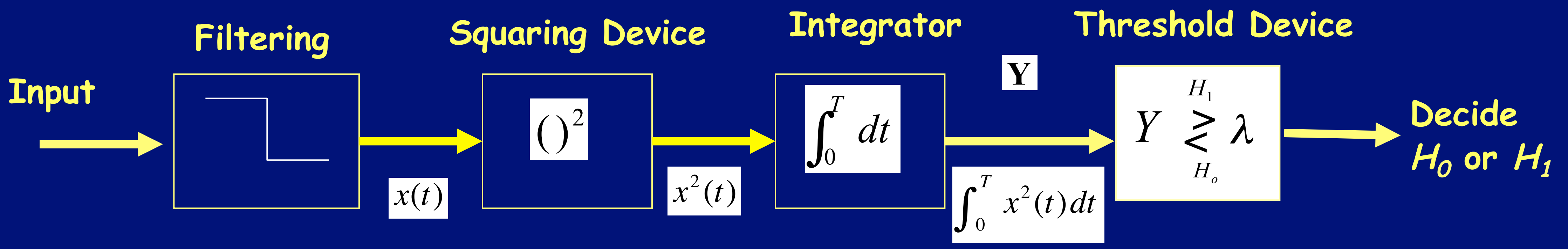
Energy Detection

D. Cabric, S. M. Mishra, and R. W. Brodersen,
"Implementation Issues in Spectrum Sensing for Cognitive Radios,"
Proc. 38th Asilomar Conference on Signals, Systems and Computers, Nov. 2004.
H. Tang,
"Some Physical Layer Issues of Wideband Cognitive Radio System,"
Proc. IEEE DySPAN, Nov. 2005.

- If the CR user cannot obtain information about the PU signal s(t), the matched filter is not suitable
- Energy detector is a non-coherent detector that does not require synchronization at the receivers like matched filter

Energy Detection

A. Ghasemi and E. S. Sousa, "Collaborative Spectrum Sensing for Opportunistic Access in Fading Environment," in Proc. IEEE DySPAN, Nov. 2005



Output of bandpass filter with BW *W* is squared and integrated over the observation (sensing) time interval *T*
→ to measure the energy of the received signal

Output of the integrator, *Y*, is compared with a threshold, *λ*, to decide whether a PU is present or not

Example
Energy Detection

Assume the filtered input signal $x(t)= A\cos(2\pi f_c t)$

$x^2(t)= A^2\cos(2\pi f_c t)=(A^2\cos(2\pi 2 f_c t)+A^2)/2$, by cosine conversion identity

After integration -> $Y=(A^2\sin(4\pi f_c T)+A^2 T)/2$, which is a scalar

Accordingly, threshold can be applied at the receiver

$$Y \underset{H_1}{\overset{H_o}{\lessgtr}} \lambda$$

→ Decide H_0 or H_1

Energy Detection

If the power of the random Gaussian noise is known, then the energy detector is optimal

Energy Detection: Non-Fading Environment

Probability of Detection P_d and Probability of False Alarm P_f are:

$$P_d = P\{Y > \lambda \mid H_1\} = Q_m(\sqrt{2\gamma}, \sqrt{\lambda})$$

$$P_f = P\{Y > \lambda \mid H_0\} = \frac{\Gamma(m, \lambda/2)}{\Gamma(m)}$$

where γ is the SNR

m = TW is the (observation/sensing) time bandwidth product

$\Gamma(\cdot)$ and $\Gamma(\cdot, \cdot)$ are complete and incomplete gamma functions

$Q_m(\ )$ is the generalized Marcum Q-function

λ is the threshold value

Energy Detection: Non-Fading Environment

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Energy Detection: Fading Environment

- The amplitude gain h of the channel varies due to the shadowing/fading → variation of SNR
- P_f is the same as that of non-fading case (independent of SNR, γ)
- P_d gives the probability of the detection conditioned on instantaneous SNR as (from $P_d = P\{Y > \lambda \mid H_1)$ →

$$P_d = \int_x Q_m(\sqrt{2\gamma}, \sqrt{\lambda}) f_\gamma(x) dx$$

where $f_\gamma(x)$ is the probability distribution function of SNR under fading.

Advantages of Energy Detection

- Easy to implement

Problems of Energy Detection

- Increased sensing time (i.e., sensing speed is low)

- Noise
  - We assumed that noise variance is precisely known to the receiver and the threshold λ is set accordingly

  - However, noise is an aggregation of various sources and varies continuously

Problems of Energy Detection

Performance is susceptible to uncertainty in noise power
→ SNR problem !!!

(Pilot tone from PU helps to improve the accuracy of the energy detector)

Problems of Energy Detection

- Cannot differentiate signal types but can only determine the presence of the signal.

- Cannot differentiate modulated signals, noise and interference.

(accordingly benefits of detection and interference cancellation techniques cannot be utilized).

Problems of Energy Detection

→ Energy detector is prone to the false detection triggered the unintended signals

Cyclostationary Feature Detection

W.A. Gardner, "Signal interception: a unifying theoretical framework for feature detection," IEEE Trans. on Communications, vol. 36, no. 8, Aug. 1988.

- **Cyclostationarity**
  - Statistical parameters (e.g., mean and autocorrelation) of a cyclostationary random process vary periodically with time
  - Characteristic properties of cyclostationarity are cyclic features
- **Cyclostationary feature detection**
  - Exploit the periodicity of carriers, pulse trains, repeating spreading codes, etc. (cyclic features) in PU signals for detection

Cyclostationary Signals

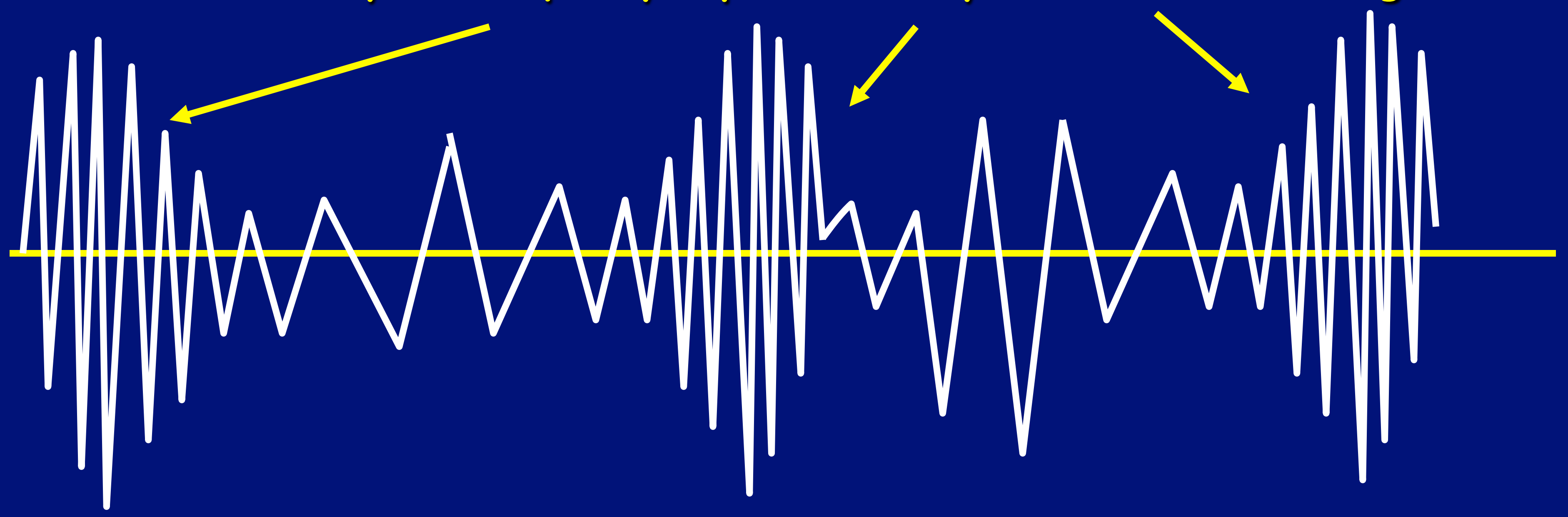
D. Cabric, S. M. Mishra, and R. W. Brodersen,
"Implementation issues in spectrum sensing for cognitive radios,"
in Proc. 38th Asilomar Conf. on Signals, Systems and Computers, Nov. 2004.

- Modulated signals are cyclostationary
  - They are coupled with several sources of periodicities such as sine wave carriers, pulse trains, repeating spreading, hopping sequences, or cyclic prefixes
- These introduced periodicities cause spectral redundancy
- Spectral redundancy can be measured by the correlation between spectral components of cyclostationary signals

Example of Cyclostationary Signals

Sine-based Cyclostationary Detection

Primary Tx frequency repeats over symbol durations at regular intervals T



Problem: Can these cyclic regularities be detected at the CR user?

Detection of Cyclostationary Signals

- This periodicity appearing in transmitted signal of the PUs can be used by CR users to detect PUs
- Cyclic features in cyclostationary signals can be captured by cyclic frequencies α's in the spectral correlation function (SCF)
- Questions:
  - What are the cyclic frequencies in PU signals?
  - How to find the spectral correlation function (SCF)?

Cyclic Frequencies of Primary Signals

Type of Signal	Cyclic Frequencies
Analog Television	Cyclic frequencies at multiples of the TV-signal horizontal line-scan rate (15.75 kHz in USA, 15.625 kHz in Europe)
AM signal: $x(t) = a(t)\cos(2\pi f_0 t + \varphi_0)$	$\pm 2 f_0$
PM/FM signal: $x(t) = \cos(2\pi f_0 t + \varphi(t))$	$\pm 2 f_0$
Amplitude-Shift Keying: $x(t) = [\sum_{n=-\infty}^{\infty} a_n p(t - nT_0 - t_0)]\cos(2\pi f_0 t + \varphi_0)$	$k / T_0 (k \neq 0)$ $\pm 2 f_0 + k / T_0, k = 0, \pm 1, \pm 2, \ldots$
Phase-Shift Keying: $x(t) = \cos[2\pi f_0 t + \sum_{n=-\infty}^{\infty} a_n p(t - nT_0 - t_0)]$.	$k / T_0 (k \neq 0)$ $\pm 2 f_0 + k / T_0, k = 0, \pm 1, \pm 2, \ldots$

Cyclic Frequencies for Primary Detection

Since we have knowledge of the cyclic frequencies of signals like TV and wireless microphones,

we only need to compute

the spectral correlation function (SCF)

at very limited number of discrete cyclic frequencies (α's)

Finding SCFs: Cyclic Autocorrelation Approach

W.A. Gardner, "Signal interception: a unifying theoretical framework for feature detection," IEEE Trans. on Communications, vol. 36, no. 8, Aug. 1988.

- Power spectral density is the Fourier transform of the autocorrelation
- Since the autocorrelation function $R_x(\tau)$ of received signal x(t) is periodic, it can be represented by a Fourier Series

$$R_x(\tau) = E[x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})] = R_x(t+\frac{\tau}{2}, t-\frac{\tau}{2}) = \sum_{\alpha} R_x^{\alpha}(\tau)e^{j2\pi\alpha t}$$

where $R_x^{\alpha}(\tau)$ is the Fourier Series coefficients called cyclic autocorrelation function with spectral components at cyclic frequencies α's

- The density of spectral correlation (spectral correlation function (SCF)) is the Fourier Transform of cyclic autocorrelation function $R_x^{\alpha}(\tau)$

Finding SCFs: Spectral Correlation Approach

- Spectral correlation function (SCF), $S_x^\alpha(f)$, is the density of correlation between spectral components at $(f+\alpha/2)$ and $(f-\alpha/2)$

$$S_x^\alpha(f) = \lim_{T\to\infty}\lim_{\Delta t\to\infty}\frac{1}{T\Delta t}\int_{-\Delta t/2}^{+\Delta t/2} X_T(t, f+\frac{\alpha}{2})X_T(t, f-\frac{\alpha}{2})dt$$

where X(t,f) is the spectral components of received signal x(t) at frequency f with bandwidth 1/T:

$$X_T(t,f) = \int_{t-T/2}^{t+T/2} x(u)e^{-j2\pi fu}du$$

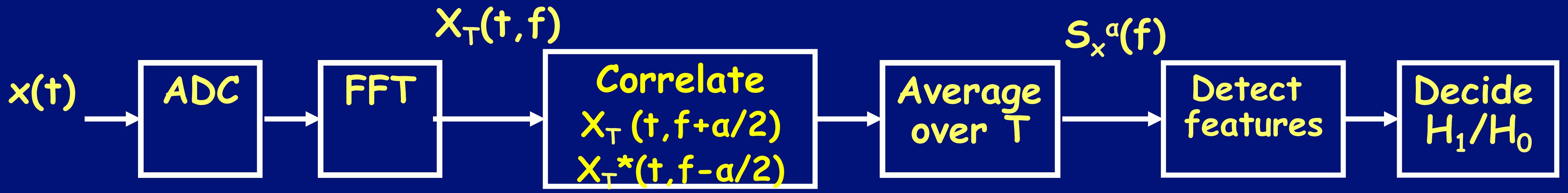
- For $\alpha=0$, SCF reduces to power spectral density (PSD)
- $S_x^\alpha(f) = S_x(\alpha, f)$ is two-dimensional transform of $R_x(t;\tau)$ where $R_x(t;\tau) = R_x(\tau)$ is the autocorrelation function of x(t)

Steps of Cyclostationary Feature Detection

- Obtain the spectral components $X_T(t,f)$ of received signal x(t)
- Find the correlation between spectral components at frequencies $(f+\alpha/2)$ and $(f-\alpha/2)$ for cyclic frequency α to obtain the SCF: $S_x^\alpha(f)$
- Measure magnitude of SCF at specific cyclic frequencies α's (detect features)
- Declare primary signal s(t) present if spectral components are detected at α's (i.e. $|S_x^\alpha(f)|$ > threshold λ)

Implementation of Cyclostationary Feature Detector

D. Cabric, S. M. Mishra, and R. W. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in Proc. Asilomar Conf. on Signals, Systems and Computers, Nov. 2004.

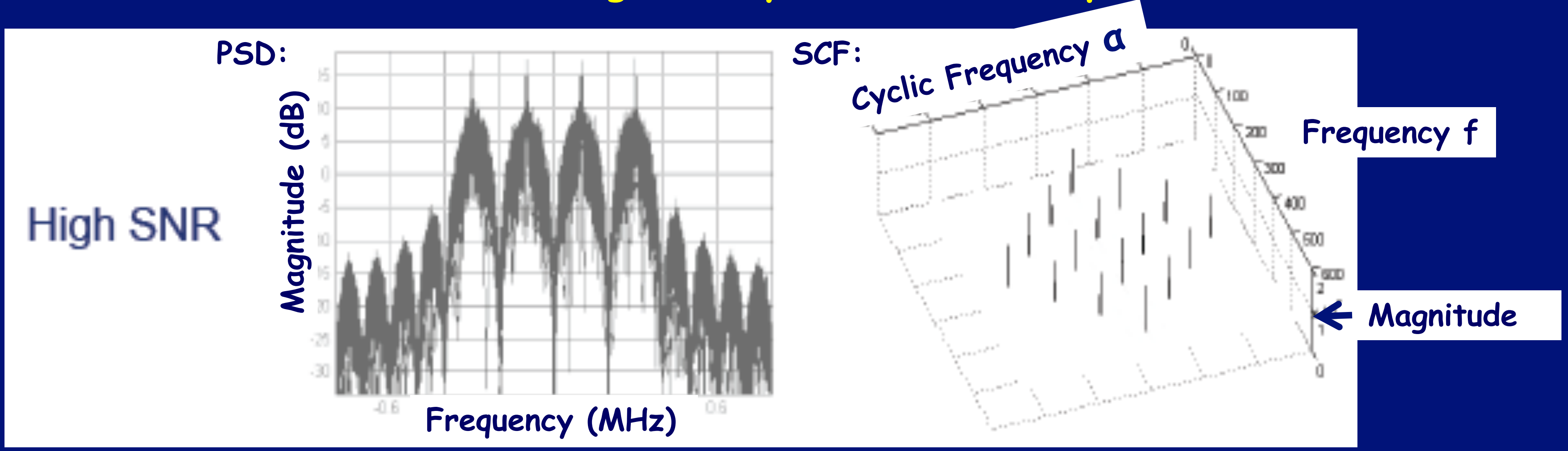


$x(t)$:	Received signal
$X_T(t,f)$:	Spectral components of $x(t)$ at frequency f with BW $1/T$
α:	Cyclic frequency
$S_x^\alpha(f)$:	Spectral correlation function (SCF)

If the magnitude of $S_x^\alpha(f)$ at cyclic freq α is greater than the threshold (high spectral correlation), the primary signal is detected

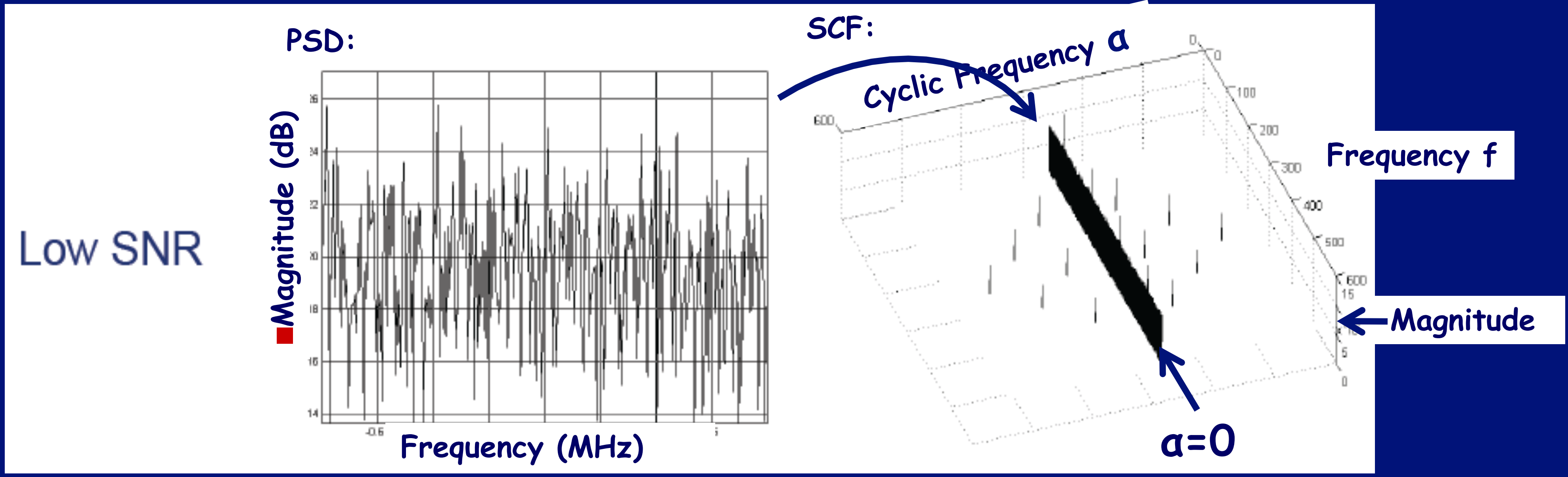
Cyclostationary Feature Detection and Energy Detection in High SNR

D. Cabric, S. M. Mishra, and R. W. Brodersen,
"Implementation issues in spectrum sensing for cognitive radio,"
in Proc. Asilomar Conf. on Signals, Systems, and Computers, Nov. 2004.



- Primary signal power can be easily detected from the PSD obtained by energy detection
- Cyclic features are also clear in the SCF obtained by cyclostationary feature detection

Advantage of Cyclostationary Feature Detection over Energy Detection in low SNR



- Noise uncertainty limits the effectiveness of Energy detection (primary signal cannot be identified in the PSD)
- Cyclic features still clearly visible for α≠0 where noise has no components

Advantages of Cyclostationary Feature Detection

- Advantages over Energy Detection

➔ Discriminate against noise due to its robustness to the uncertainty in noise power

➔ Better detector performance even in low SNR regions

Advantages of Cyclostationary Feature Detection

- **Signal Classification Capability**
  - Different signals have different cyclic frequencies and exhibit distinct spectral characteristics
  - Detected features from an unknown signal can be used as inputs to signal classification

- **Flexibility of Operations**

➔Can be used as an energy detector in $\alpha = 0$ mode

Disadvantages of Cyclostationary Feature Detection

- Computationally complex and requires significantly long observation (sensing) time
- A priori knowledge of target signal characteristics needed
  - ➔Cycle frequency should be known a priori
  - ➔Cannot be applied to detecting unknown signals

Example
Steps of Cyclostationary Feature Detection

a) Consider a received primary signal, AM modulated with frequency f_c

$$x(t) = a(t)\cos(2\pi f_c t + \varphi_0)$$

The autocorrelation of a(t) is

$$R_a(\tau) = E\left[a\left(t+\frac{\tau}{2}\right)a^*\left(t-\frac{\tau}{2}\right)\right]$$

Find the autocorrelation of x(t) in terms of $R_a(\tau)$

$$R_x(\tau) = E\left[x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)\right] = \frac{1}{2}R_a(\tau)\,[\cos(2\pi f_c\tau) + \cos(2\pi(2f_c)t + 2\,\varphi_0)] \quad (1)$$

Example
Steps of Cyclostationary Feature Detection

b) The autocorrelation $R_x(\tau)$ can be represented by Fourier Series as

$$R_x(\tau) = R_x\left(t+\frac{\tau}{2}, t-\frac{\tau}{2}\right) = \sum_{\alpha=-\infty}^{+\infty} R_x^{\alpha}(\tau)\, e^{j2\pi\alpha t} \quad (*)$$

(1) can be manipulated to look like a Fourier series

$$R_x(\tau) = \frac{1}{2}R_a(\tau)\cos(2\pi f_c\tau)\, e^{j2\pi(0)t} + \frac{1}{4}R_a(\tau)\left(e^{j(-2\varphi_0)}e^{j2\pi(-2f_c)t} + e^{j2\varphi_0}e^{j2\pi(2f_c)t}\right) \quad (**)$$

Comparing (*) and (**), we find $R^{\alpha}_{x}(\tau)$

$$R_x^{\alpha}(\tau) = \begin{cases} \frac{1}{2}R_a(\tau)\cos(2\pi f_c\tau), & \text{at } \alpha = 0 \\ \frac{1}{4}R_a(\tau)e^{\pm j2\varphi_0}, & \text{at } \alpha = \pm 2f_c \\ 0, & \text{otherwise} \end{cases}$$

Example
Steps of Cyclostationary Feature Detection

c) The spectral correlation $S^{\alpha}_{x}(f)$ in terms of $S_a(f)$ is found by F.T.

$$S_x^{\alpha}(f) = F\{R_x^{\alpha}(\tau)\} = \begin{cases} \frac{1}{4}\left(S_a(f - f_c) + S_a(f + f_c)\right), & \alpha = 0 \\ \frac{1}{4} S_a(f) e^{\pm j2\varphi_0}, & \alpha = \pm 2f_c \\ 0, & \text{otherwise} \end{cases}$$

Assume $R_a(\tau)$ = $\text{sinc}^2(a\tau)$, then $S_a(f)$ is the triangular function defined as

$$\Lambda(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & \text{otherwise} \end{cases}.$$

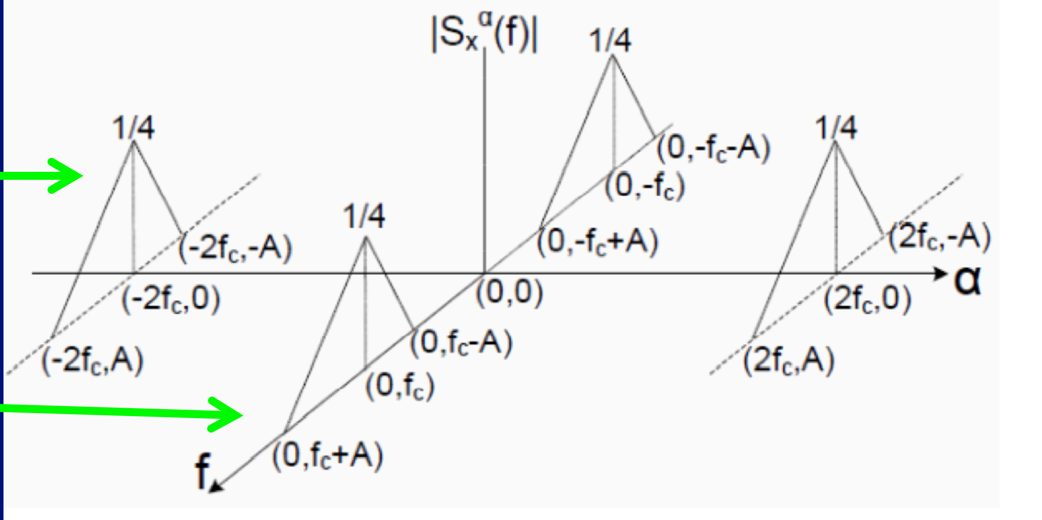
Example
Steps of Cyclostationary Feature Detection

c) The spectral correlation $S^{\alpha}_{x}(f)$ becomes

$$S_x^{\alpha}(f)=\begin{cases}\frac{1}{4}\Lambda\left(\frac{f-f_c}{A}\right)+\frac{1}{4}\Lambda\left(\frac{f+f_c}{A}\right), & \alpha=0\\ \frac{1}{4}\Lambda\left(\frac{f}{A}\right)e^{\pm j2\varphi_0}, & \alpha=\pm 2f_c\\ 0, & \text{otherwise}\end{cases} \Rightarrow |S_x^{\alpha}(f)|=\begin{cases}\frac{1}{4}\Lambda\left(\frac{f-f_c}{A}\right)+\frac{1}{4}\Lambda\left(\frac{f+f_c}{A}\right), & \alpha=0\\ \frac{1}{4}\Lambda\left(\frac{f}{A}\right), & \alpha=\pm 2f_c\\ 0, & \text{otherwise}\end{cases}$$

The cyclic features at cyclic frequencies $\alpha=\pm 2f_c$

The cyclic features at cyclic frequencies $\alpha=0$



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Spectrum Sensing Techniques: PU Detection Techniques

COHERENT DETECTION

primary signal can be coherently detected by comparing the received signal with a priori knowledge of primary signals

NON-COHERENT DETECTION

no a priori knowledge is required for detection

Spectrum Sensing Techniques: PU Detection Techniques

- COHERENT DETECTION
  - Matched Filter Detection
- NON-COHERENT DETECTION
  - Energy Detection
  - Cyclostationary Feature Detection
  - Covariance Matrix-based Detection
  - Wavelet Detection
  - Compressed Sensing

Spectrum Sensing Techniques: PU Detection Techniques

- NARROWBAND SENSING
  - Matched Filter Detection
  - Energy Detection
  - Cyclostationary Feature Detection
  - Covariance Matrix-based Detection
- WIDEBAND SENSING
  - Wavelet Detection
  - Compressed Sensing

Classification of Spectrum Sensing Techniques

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COVARIANCE-MATRIX BASED DETECTION

S. J. Shellhammer, S. Shankar, N. Tandra, and J. Tomcik.
"Performance of power detector sensors of DTV signals in IEEE 802.22 WRANs."
Proc. of the First ACM Int. Workshop on Technology and Policy for Accessing Spectrum (TAPAS), 2006

- Energy detector is vulnerable to noise uncertainty because the detection depends on the knowledge of the noise power

- There are several sources of noise uncertainty:
  - RF component non-linearity
  - Non-uniform and time variant thermal noise
  - Interference noise

COVARIANCE-MATRIX BASED DETECTION

- In practice → difficult to estimate the noise power accurately
- Small errors in noise power prediction cause a considerable loss in energy detection performance

Minimum SNR values that can be detected at different noise uncertainty

Noise Uncertainty	0.1 dB	0.5 dB	1 dB
Minimum SNR	-13.37 dB	-6.4 dB	-3.3 dB

Approximately: 1 dB noise uncertainty costs 10 dB loss in detection performance

COVARIANCE-MATRIX BASED DETECTION

Y. Zeng and Y.-C. Liang.
"Covariance based signal detections for cognitive radios",
Proc. *IEEE Int. Symp. New Frontiers Dynamic Spectrum Access (DySpan), Dublin, Ireland, 2007*

- To solve the noise uncertainty problem
- Based on covariance matrix of signals received at CR user
- PRINCIPLE based on:

  Difference between the statistical covariance matrices of the signal and noise.

COVARIANCE-MATRIX BASED DETECTION

- Revisit the HYPOTHESIS formula:

$$Y = \begin{cases} \frac{1}{N}\sum_{i=1}^{N} n[i]^2 & H_0 \\ \frac{1}{N}\sum_{i=1}^{N} \left(x[i]+n[i]\right)^2 & H_1 \end{cases}$$

where N is the number of samples

n[i] is the i-th sample of the Gaussian noise n(t)

x[i] is the i-th sample of the received signal x(t)

COVARIANCE BASED DETECTION

- Let $\underline{x}[k]$, $\underline{y}[k]$ and $\underline{n}[k]$ be the vectors of signal components in the previous equation

- Each vector consists of the L latest outputs at time k which are expressed:

$$\mathbf{x}[k] = \left[x[k], x[k-1], \ldots, x[k-L+1]\right]^T$$

$$\mathbf{y}[k] = \left[y[k], y[k-1], \ldots, y[k-L+1]\right]^T$$

$$\mathbf{n}[k] = \left[n[k], n[k-1], \ldots, n[k-L+1]\right]^T$$

COVARIANCE BASED DETECTION

- Then the statistical covariance matrices of $\mathbf{x}[k]$ and $\mathbf{y}[k]$ show the following relation:

$$\mathbf{R}_y = \mathbf{R}_x + \sigma_n^2 \mathbf{I}$$

where

σ_n^2 is the variance of the noise

I is the identity matrix and

$\mathbf{R}_x, \mathbf{R}_y$ are covariance matrices of $\mathbf{x}[k]$ and $\mathbf{y}[k]$

$$\mathbf{R}_x = E\left[\mathbf{x}[k]\mathbf{x}^T[k]\right]$$

$$\mathbf{R}_y = E\left[\mathbf{y}[k]\mathbf{y}^T[k]\right]$$

COVARIANCE BASED DETECTION

- From this we can derive the maximum λ_{max} and minimum λ_{min} eigenvalues of $\mathbf{R}_y$

$$\lambda_{max} = \rho_{max} + \sigma_n^2$$
$$\lambda_{min} = \rho_{min} + \sigma_n^2$$

with the maximum ρ_{max} and minimum ρ_{min} eigenvalues of $\mathbf{R}_x$

COVARIANCE BASED DETECTION

These statistical relations lead to the following observations:

- $\rho_{\max} = \rho_{\min}$ if and only if $R_x = \delta I$.
  However, this case is not likely to happen if signal x[n] is present
- If there is no signal, i.e., $R_x = 0, \lambda_{\max} = \lambda_{\min}$
- Otherwise, i.e., $R_x \neq \delta I$ and $R_x \neq 0, \lambda_{\max} \lambda_{\min} > 1$

Thus, the detector can use the ratio $\lambda_{\max} / \lambda_{\min}$ to determine the presence of the signal

EIGENVALUE BASED DETECTION

- In practice, a finite number of samples is available, and hence the sample covariance matrix can be used for detection instead of statistical covariance matrix

EXTENSION:

- Using oversampling and Eigen decomposition of the covariance matrix, the detection method can be extended to Eigenvalue-based detection

PRINCIPLE based on:

Difference between the eigenvalues of the covariance matrices of the signal and noise.

EIGENVALUE BASED DETECTION

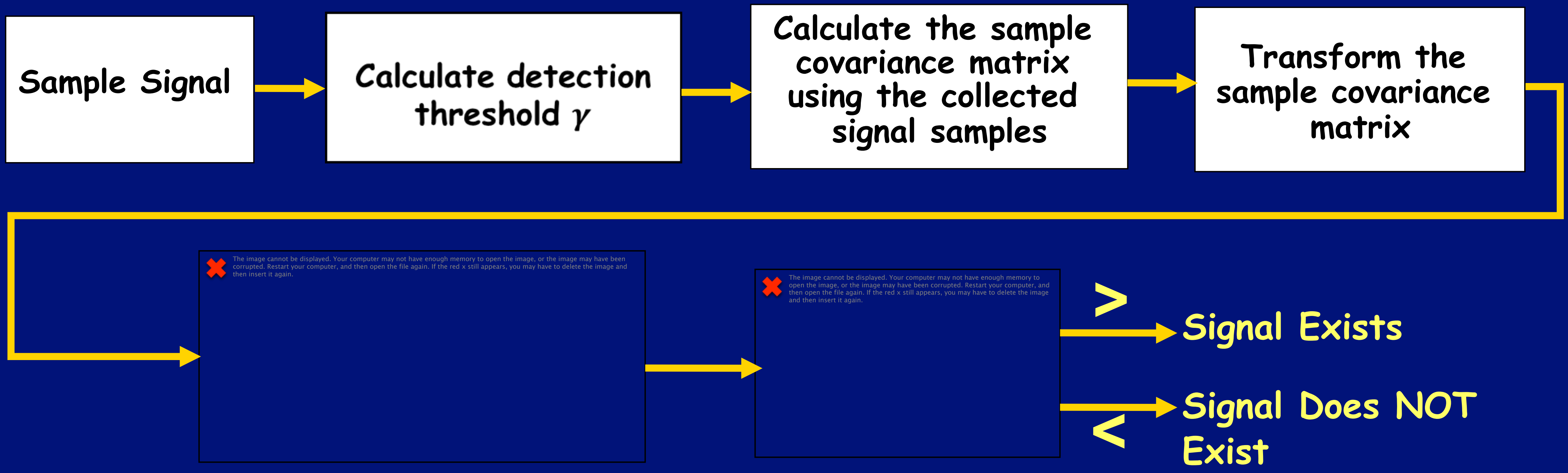
Y. Zeng and Y.-C. Liang.
"Eigenvalue-based spectrum sensing algorithms for cognitive radio",
IEEE Transactions on Communications, June 2009.

Based on the sample covariance matrix two detection methods exist:

1. Maximum-Minimum Eigenvalue (MME) Detection

2. Energy with Minimum Eigenvalue (EME) Detection

EIGEN-VALUE BASED DETECTION MAX-MIN EIGENVALUE BASED (MME) (Informal)



EIGEN-VALUE BASED DETECTION
MAX-MIN EIGENVALUE BASED (MME) (Formal)

- **Step 1:** Compute the sample covariance matrix of the received signal

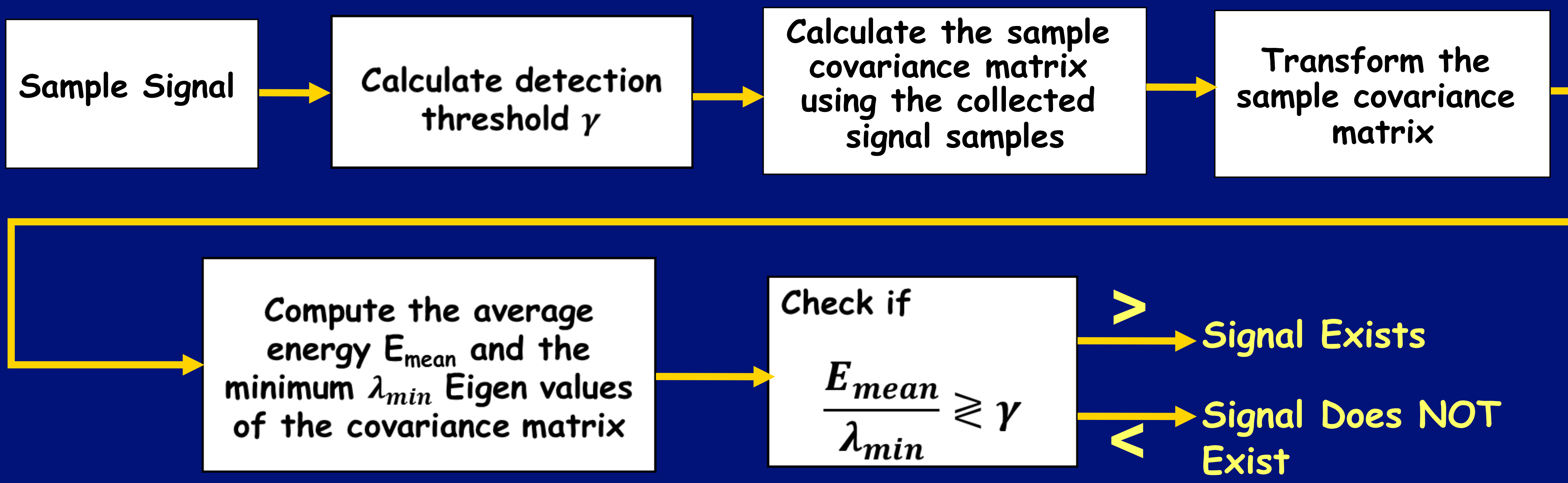
$$R_x(N_S) \overset{def}{=} \frac{1}{N_S} \sum_{n=L-1}^{L-2+N_S} \hat{x}(n)\hat{x}^*(n),$$

where N_S is the number of collected samples; (* complex conjuguate)

- **Step 2:** Obtain the maximum and minimum eigenvalue of the matrix $R_x(N_S)$, that is, $\lambda_{\max}$ and $\lambda_{\min}$

- **Step 3:** Decision: if $\lambda_{\max} / \lambda_{\min} > \gamma_1$, signal exists ("yes" decision); otherwise, signal does not exist ("no" decision), where $\gamma_1 > 1$ is a threshold

EIGEN-VALUE BASED DETECTION ENERGY WITH MINIMUM EIGENVALUE (EME) (Informal)



EIGEN-VALUE BASED DETECTION ENERGY WITH MINIMUM EIGENVALUE (EME) (Formal)

- **Step 1:** The same as that in Algorithm 1
- **Step 2:** Compute the average power of the received signal $T(N_S)$ and the minimum eigenvalue $\lambda_{\min}$ of the matrix $\mathrm{R}_x(N_S)$
- **Step 3:** Decision: if $T(N_S)/\lambda_{\min} > \gamma_2$, signal exists ("yes" decision); otherwise, signal does not exist ("no" decision), where $\gamma_2 > 1$ is a threshold

ADVANTAGES OF COVARIANCE MATRIX BASED DETECTION

- No signal information is needed
- Robust to multipath propagation compared to coherent detection
- No synchronization is needed
- No noise uncertainty problem

DISADVANTAGES OF COVARIANCE MATRIX BASED DETECTION

- Relatively high computational costs
- It cannot differentiate signal types but can only determine the presence of the signal.
- It cannot differentiate modulated signals, noise and interference.

(accordingly benefits of detection and interference cancellation techniques cannot be utilized).

Example
Covariance Matrix-based Detection

- Given a received PU signal $x(n) = \beta e^{j\hat{\omega}n} + v(n)$, $n=0,1,\ldots,N_s-1$
  where $\hat{\omega}$ is the normalized frequency in rad/sample
- and $v(n)$ is zero-mean with $E[v(n)v^*(n-k)]=\sigma^2$, for $k=0$,
- and $E[v(n)v^*(n-k)]=0$ for $k\neq 0$.

a) Find the mean of x(n), E[x(n)]

Since E[v(n)]=0

$$E\left[\beta e^{j\hat{\omega}n} + v(n)\right] = E\left[\beta e^{j\hat{\omega}n}\right] + E[v(n)] = \frac{\beta}{N_s}\sum_{n=0}^{N_s-1} e^{j\hat{\omega}n}$$

Example:
Covariance Matrix-based Detection

b) Find the autocorrelation function r(k)

$$r(k) = E[x(n)x^*(n-k)] = E\left[\left(\beta e^{j\hat{\omega}n} + v(n)\right)\left(\beta e^{-j\hat{\omega}(n-k)} + v^*(n-k)\right)\right]$$

$$= |\beta|^2 e^{j\hat{\omega}k} + E[v(n)v^*(n-k)] = |\beta|^2 e^{j\hat{\omega}k} + \begin{cases} \sigma^2, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

$$= \begin{cases} |\beta|^2 + \sigma^2, & k = 0 \\ |\beta|^2 e^{j\hat{\omega}k}, & k \neq 0 \end{cases}$$

Example
Covariance Matrix-based Detection

c) Find the autocorrelation matrix R_x for $N_s = 3$.

$$R_x = \begin{bmatrix} |\beta|^2 + \sigma^2 & |\beta|^2 e^{j\hat{\omega}} & |\beta|^2 e^{j2\hat{\omega}} \\ |\beta|^2 e^{j\hat{\omega}} & |\beta|^2 + \sigma^2 & |\beta|^2 e^{j\hat{\omega}} \\ |\beta|^2 e^{j2\hat{\omega}} & |\beta|^2 e^{j\hat{\omega}} & |\beta|^2 + \sigma^2 \end{bmatrix}$$

Example
Covariance Matrix-based Detection

d)

Given $\sigma^2 = 5, \beta = 7, \omega = 10{,}000\pi$ rad/sec, sampling frequency f_s = 5k samples/sec, and $\hat{\omega} = \omega/f_s$, find the MME and EME using Matlab.

- **MME**: maximum-minimum eigen value
- **EME**: energy with minimum eigenvalue

$$R_x = \begin{bmatrix} 54 & 49 & 49 \\ 49 & 54 & 49 \\ 49 & 49 & 54 \end{bmatrix} \Rightarrow \mathrm{Eig}(R_x) = 5, 5, 152$$

$$\mathrm{MME} = \lambda_{max}/\lambda_{min} = 152/5 = 30.4$$

$$\mathrm{EME} = \mathrm{Tr}(R_x)/\lambda_{min} = 162/5 = 32.4$$

Classification of Spectrum Sensing Techniques

- Transmitter Detection
  - Matched Filter Detection
  - Energy Detection
  - Cyclostationary Feature Detection
  - Covariance Matrix-based Detection
  - Wavelet Detection
  - Compressed Sensing
- Receiver Detection
- Interference Temperature Management

WAVELET DETECTION

What is Wavelet Transform?

- A multi-resolution analysis method where an input signal is decomposed into different frequency components

- Each component is studied with resolutions matched to its scales

- Wavelet transforms use irregularly shaped wavelets as basic functions

- They offer better tools to represent sharp changes and local features

WAVELET DETECTION PROs

- It enables efficient joint analysis in time and frequency domain.

→ Represent functions with discontinuities and sharp peaks.

→ Capture local irregularities and discontinuities in the spectrum

WAVELET DETECTION

Z. Tian and G. Giannakis,
"A Wavelet Approach to Wideband Spectrum Sensing for Cognitive Radios",
Proc. of Int. Conf on CR Oriented Wireless Networks and Communications (CROWNCOM), June 2006.
Y. Hur, J. Park, K. Kim, J. Lee, K. Lim, C. Lee, H. Kim, and J. Laskar,
"A cognitive radio (CR) testbed system employing a wideband multiresolution spectrum sensing (MRSS) technique," in Proc. IEEE Veh.Technol. Conf., Montreal, Sept. 2006

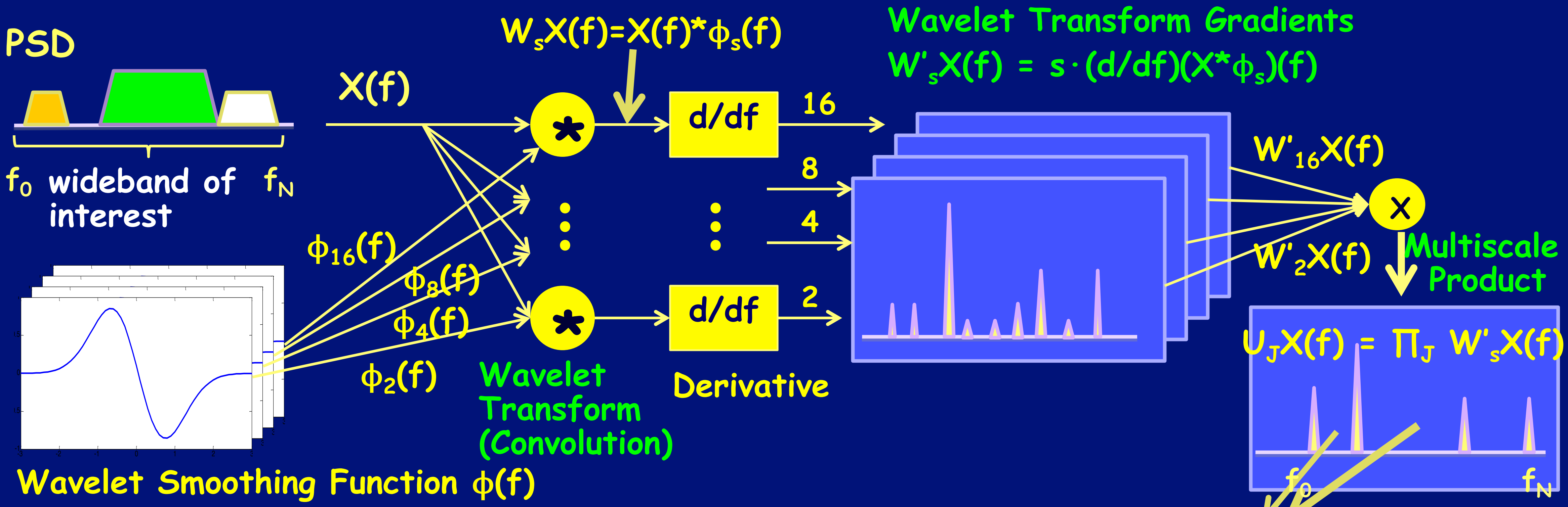
- Wavelet transform has been applied to spectrum sensing
  - In digital domain: Spectral Edge Detection Technique
  - In analog domain: Multiresolution Spectrum Sensing (MRSS) Technique

WAVELET DETECTION: DIGITAL DOMAIN

Z. Tian and G. Giannakis,
"A Wavelet Approach to Wideband Spectrum Sensing for Cognitive Radios",
Proc. of Int. Conf on CR Oriented Wireless Networks and Communications (CROWNCOM), June 2006

- Wavelets are used for detecting spectral edges in the wideband channel.
- Spectral edges correspond to transitions between an occupied and an empty band
- Powers within two spectral edges are estimated to check its occupancy.

WAVELET DETECTION: Multiscale Wavelet Products



Wavelet Smoothing Function $\phi(f)$ with a scale factor s:

$\phi_s(f) = (1/s)\cdot\phi(f/s)$, $s=2^j$, $j=1,2,\ldots$

Examine the power spectral densities at bands of interest to check their availability

Spectrum Sensing via Multiscale Wavelet Products

X(f): power spectral density (PSD) of the wideband of interest

1. Select wavelet smooth function $\phi(f)$ (such as Gaussian) and scale factor s
   - Dilation of $\phi(f)$: $\phi_s(f) = (1/s)\cdot\phi(f/s)$, $s=2^j$, $j=1,2,\ldots, J$
2. Perform continuous waveform transform (CWT) of X(f) with wavelets $\phi_s(f)$
   - $W_sX(f) = X(f)*\phi_s(f)$, $s=2^j$, where * denotes convolution
3. Find the first derivative of X(f) smoothed by scaled wavelet $\phi_s(f)$
   - Wavelet transform gradients: $W'_sX(f) = s\cdot(d/df)(X*\phi_s)(f)$
4. Construct multiscale product of J CWT gradients
   - $U_JX(f) = \Pi_J W'_{s=2^j}X(f)$
5. Identify boundaries $\{f_n\}$ of bands $\{B_n\}$ by picking local maxima of $U_JX(f)$
   - $f_n = \text{maxima}_f \{|U_JX(f)|\}$, f in (f_0, f_N)
6. Estimate the PSD in the bands of interest to check their availability

ADVANTAGES OF WAVELET DETECTION

- Effective for Wideband Signal
- Implementation cost is low
- Flexible

DISADVANTAGES OF WAVELET DETECTION

- Not useful for spread spectrum signals
- High computation costs

MULTI-RESOLUTION SPECTRUM SENSING (MRSS): ANALOG DOMAIN

Y. Hur, J. Park, K. Kim, J. Lee, K. Lim, C. Lee, H. Kim, and J. Laskar, "A Cognitive Radio (CR) Testbed System Employing a Wideband Multiresolution Spectrum Sensing (MRSS) Technique," in Proc. IEEE Veh.Technol. Conf., Montreal, Sept. 2006.

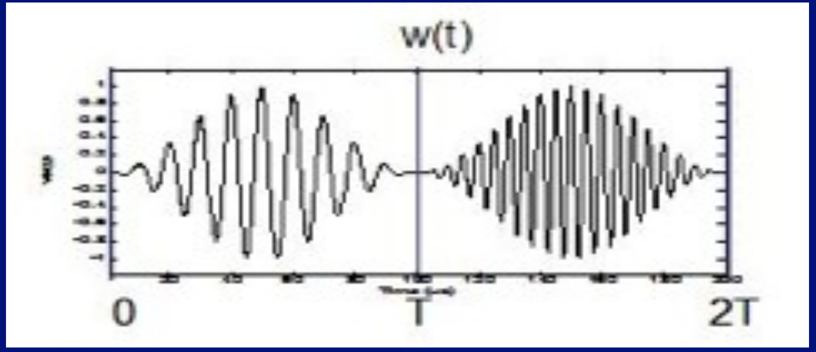
- Wavelet transform is applied to the input signal in the analog domain
- Analog implementation yields low power consumption and enables real-time operation.
- Multi-resolution spectrum sensing is achieved

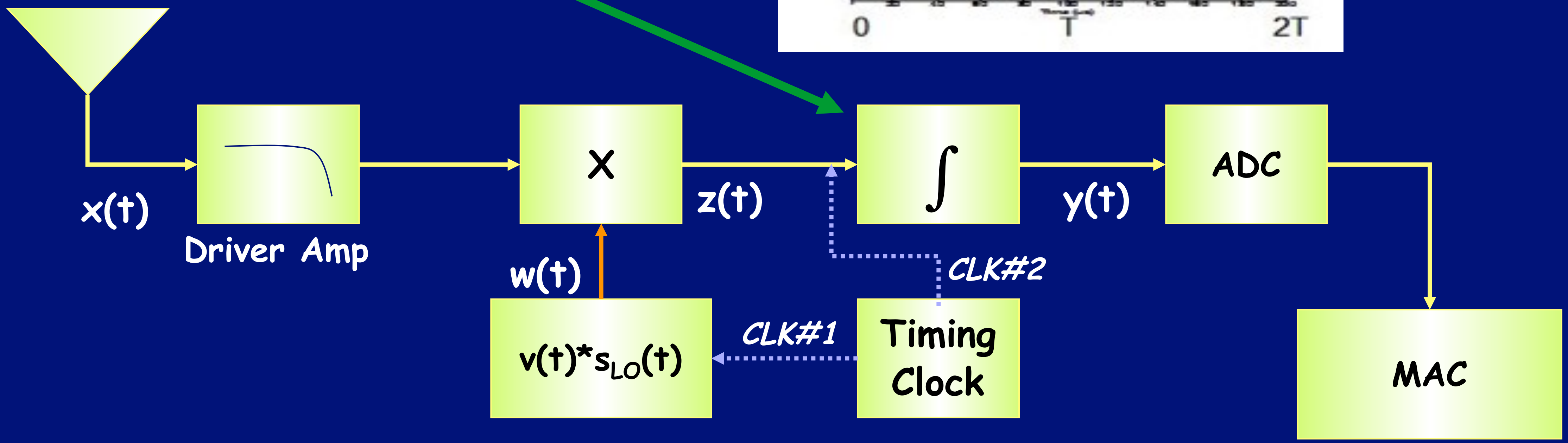
- Bandwidth, resolution and center frequency can be controlled by wavelet function

MULTI-RESOLUTION SPECTRUM SENSING (MRSS)
BLOCK DIAGRAM

MRSS is energy detector.
According to this diagram accumulated energy is calculated.

WAVELET





Wavelet Generator:
use Hann window function v(t) * signal s created by Local Oscillator

MRSS BUILDING BLOCKS

- Analog wavelet waveform generator
  - Wavelet pulse w(t) is generated and modulated with I and Q sinusoidal carrier with the given frequency
  - Hann window with 5 MHz BW is selected as the wavelet.

- Analog multiplier, multiply w(t) with x(t)

- Local Oscillator
  - By sweeping the local oscillator (LO) frequency spectrum range with a certain interval, the signal power and the frequency values are detected over the spectrum range of interest

MRSS BUILDING BLOCKS

- **Analog integrator**
  - to compute the correlation with the wavelet waveform w(t) with the given spectral width, i.e. the spectral sensing resolution.
  - Resulting correlation of the received signal x with I and Q components of the wavelet waveforms w are input to ADC
- **Low speed ADC to digitize the calculated analog correlation values**
  - Digitized values are recorded

MRSS OPERATION

After ADC

- $Y(t) > \lambda$ → PU
- $Y(t) < \lambda$ → no PU

MRSS OPERATION

- Narrow wavelet pulse + large tuning step size of LO

→ fast and coarse spectrum sensing.

- Wide wavelet pulse + small tuning step size of LO

→ fine spectrum sensing.

ADVANTAGES OF MRSS

- Full analog signal processing
  - Drastically reduce power consumption
  - Faster recognition

- Flexible sensing resolution and speed

- Wideband operation

DISADVANTAGES OF MRSS

- Need to generate wavelet waveform in analog domain
- Much more complex RF circuitry

Classification of Spectrum Sensing Techniques

- Transmitter Detection
  - Matched Filter Detection
  - Energy Detection
  - Cyclostationary Feature Detection
  - Covariance Matrix-based Detection
  - Wavelet Detection
  - Compressed Sensing
- Receiver Detection
- Interference Temperature Management

Traditional Signal Acquisition Approach

Typical Signal Acquisition Approach

Sample a signal very densely (at lease twice the highest frequency), and then compress the information for storage or transmission

- Image Acquisition

This 18.1 Mega-Pixels digital camera senses 18.1e+6 samples to construct an image.

The image is then compressed using JPEG to an average size smaller than 3MB – a compression ratio of ~12.

Compressive Sensing?

A natural question to ask is

Could the two processes (sensing & compression) be combined?

Move the burden from sampling to reconstruction

The answer is YES!

This is what Compressed Sensing (CS) is about.

What is Compressive Sensing (CS) About?

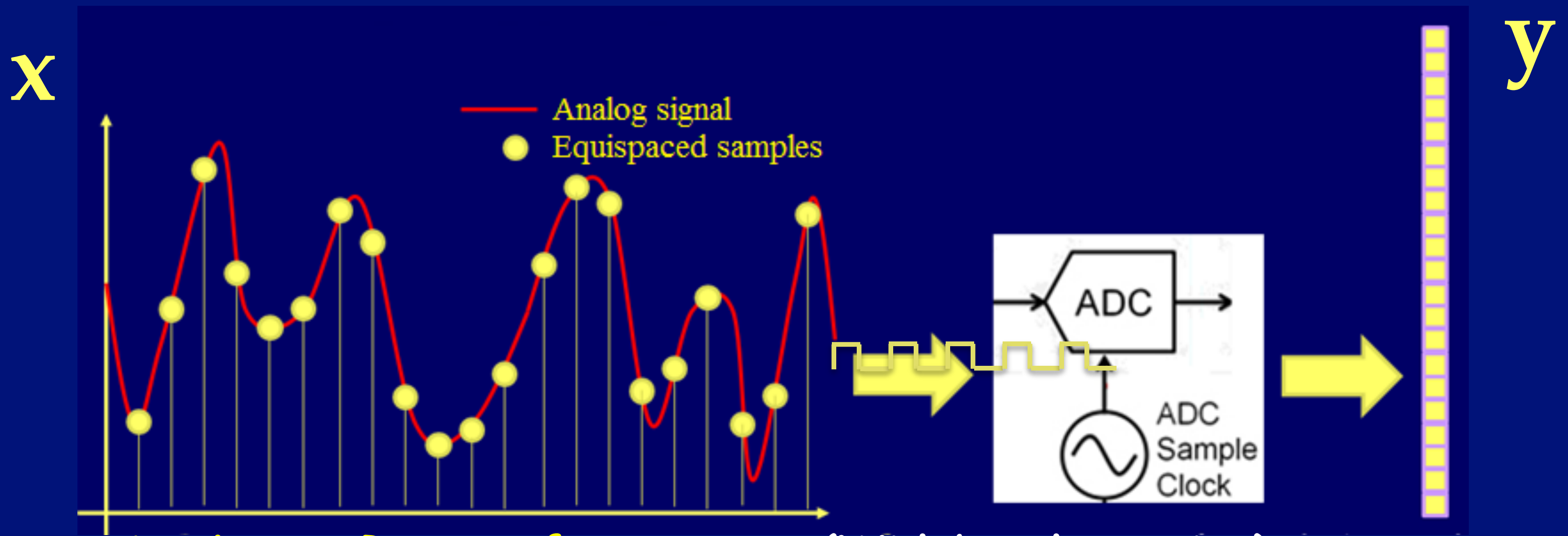
- An emerging field of research
- Beat Nyquist sampling theorem
- Explore sparsity & redundancy of signals
- Construct the combination of sensing & compression

COMPRESSED SENSING

- Conventional spectrum sensing techniques operate on signals sampled at Nyquist rate.
- Wideband signal detection requires very high sampling rate devices and computing capabilities.
- Novel approach to sample and recover signals sampled at a sampling rate below Nyquist rate.

WHY?
Conventional Sampling Technique

Example: To sense 1 GHz of spectrum at 1 KHz resolution, we need 1 Million samples acquired at 2 GHz sampling rate

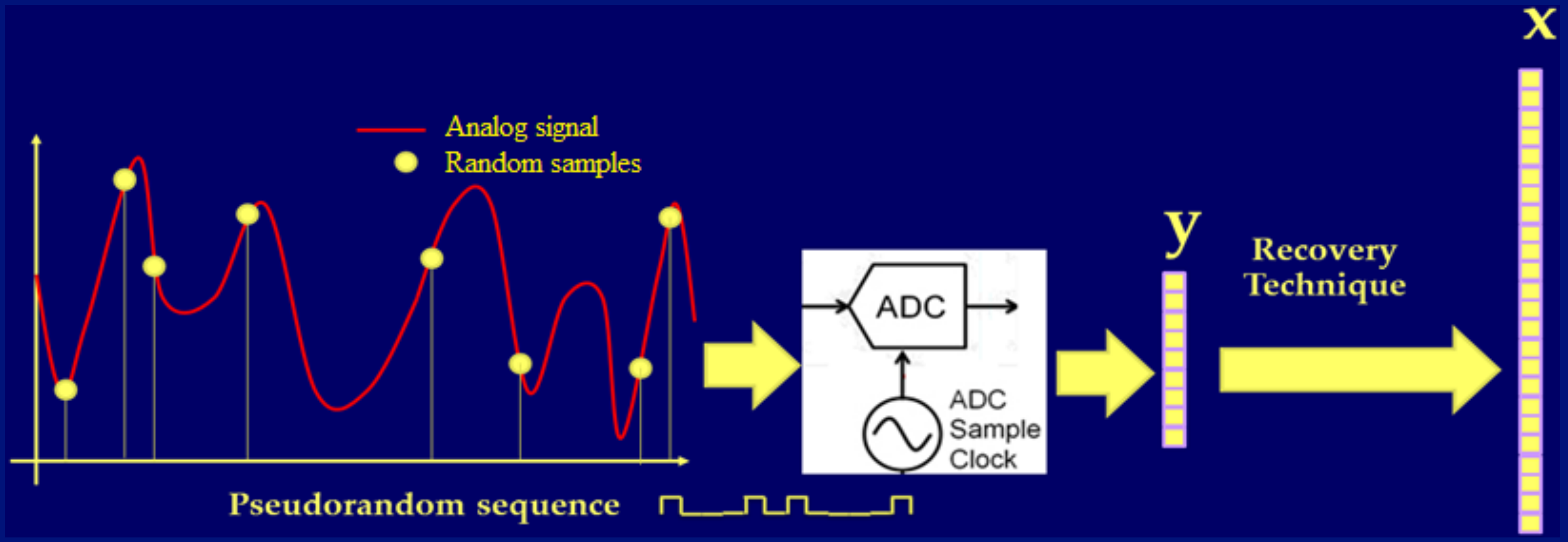


For Sensing Large Range of Spectrum (Wideband Sensing)

This sampling method is expensive:

- High Sampling Rate Requirement (expensive ADC circuit)
- Large Number of Samples (High Computational Cost)

COMPRESSED SAMPLING



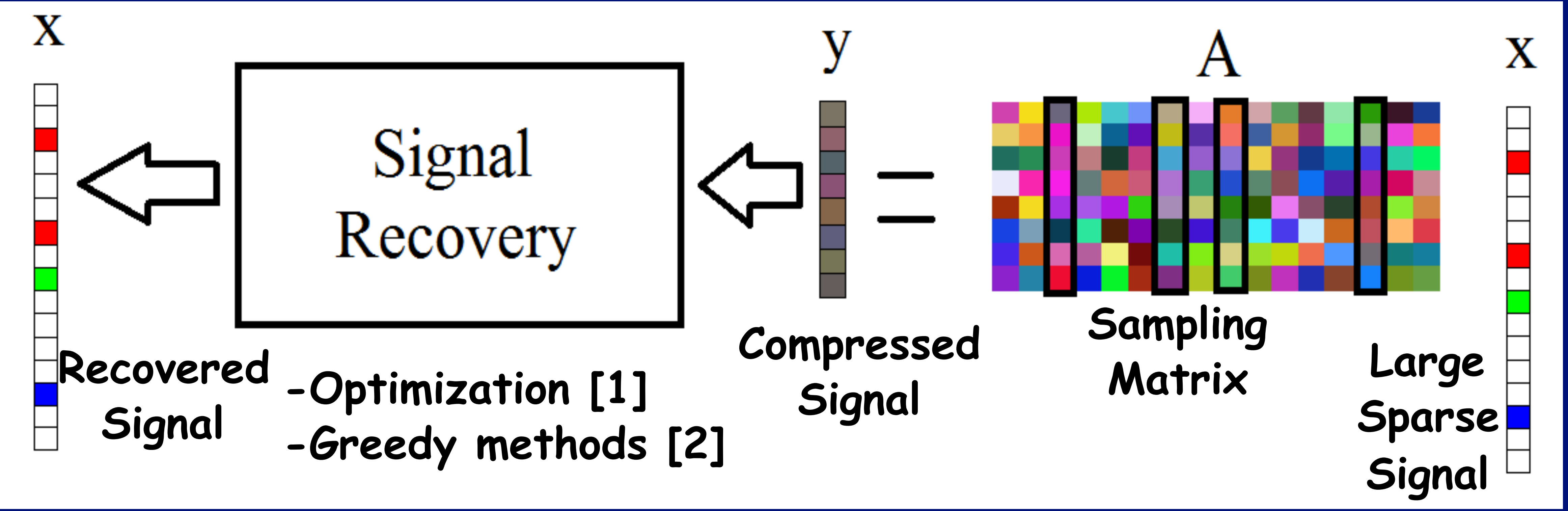
COMPRESSED SENSING

Z. Tian "Compressed Wideband Sensing in Cooperative Cognitive Radio Networks," *IEEE Globecom, 2008.*

Y. Tachwali, J. Barnes, F. Basma, H. Refai, "The Feasibility of a Fast Fourier Sampling Technique for Wireless Microphone Detection in IEEE 802.22 Air Interface," IEEE *INFOCOM, 2010*.

Main Idea

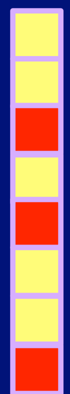
Multiply A and x



Randomly generate A

EXAMPLE: COMPRESSED SENSING

Received Signal x(t)



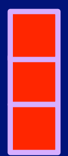
X

Sampling Matrix (A)

0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1

=

Compressed Samples y(t)

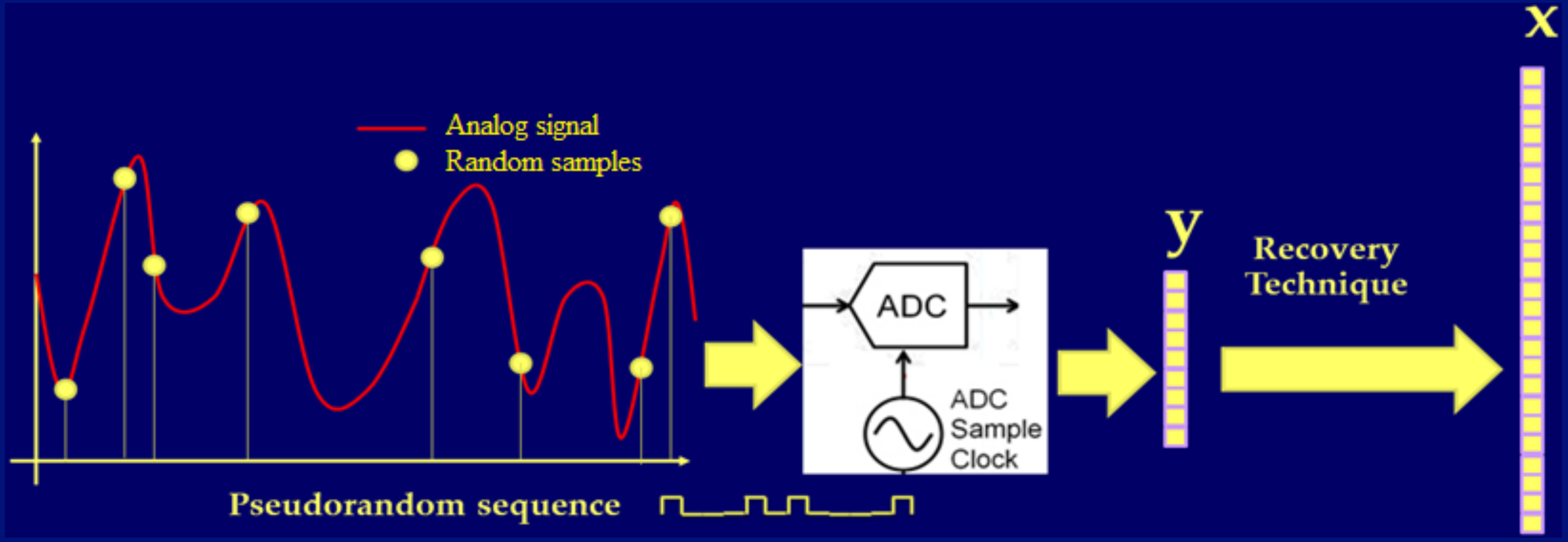


How to recover x(t) from few samples y(t)?

- Ax=y is underdetermined system (e.g., 8 unknowns and 3 eqns)
- However, we know one property of the solution x(t) → X(f) is sparse
- Sparsity can be measured by

$||X(f)||_0$ (number of nonzero elements in X(f)) OR $||X(f)||_1$ (sum of X(f) elements)

COMPRESSED SAMPLING

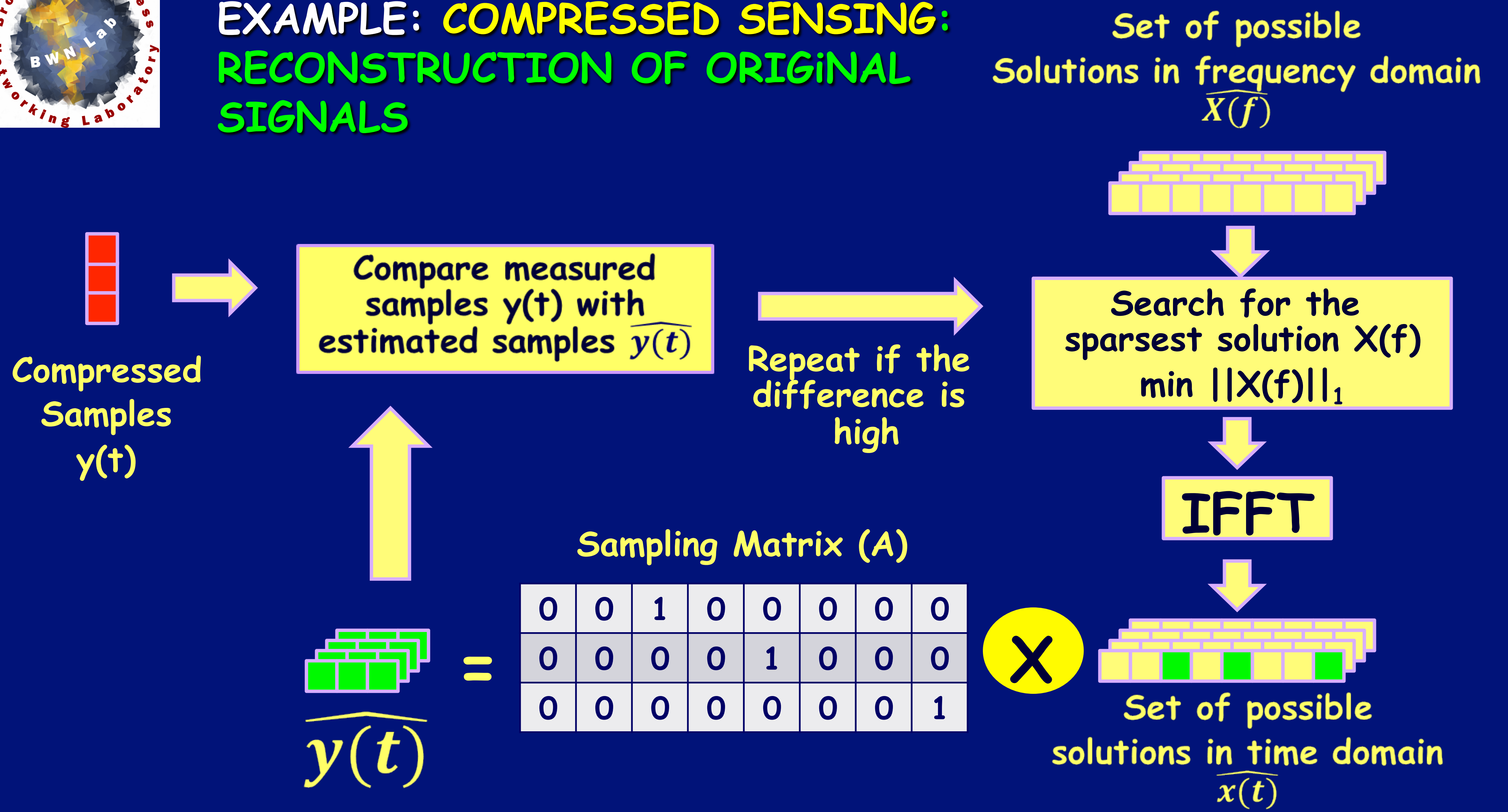


Recovery technique constructs the X^(f) from the compressed samples y so that X^(f) is as close as to the original sparse spectrum X(f)

$$\hat{X}(f) = \arg\min_{X(f)} \|X(f)\|_1 \text{ s.t. } A \bullet x(t) = y(t) : A \text{ is sampling matrix}$$

$||X(f)||_1$ (sum of X(f) elements)

EXAMPLE: COMPRESSED SENSING: RECONSTRUCTION OF ORIGINAL SIGNALS



0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1

CONDITIONS FOR SUCCESSFUL COMPRESSED SENSING

- Sparsity:

Rate of information is small compared to the signal size in its representation domain.

CONDITIONS FOR SUCCESSFUL COMPRESSED SENSING

Explanation:

Representation Domain = Frequency Domain and
Measurement Domain = Time Domain.

In order for compressed sensing scheme to work, the signal in representation domain X(f) has to be sparse,
→ it has few important samples compared to signal size X(f).

In CR network:
if wireless spectrum is underutilized → the spectrum is sparse.

CONDITIONS FOR SUCCESSFUL COMPRESSED SENSING

- Incoherency:

  Representation domain and measurement domain of the signal should be incoherent

CONDITIONS FOR SUCCESSFUL COMPRESSED SENSING

Explanation:

Representation Domain = Frequency Domain
Measurement Domain = Time Domain.

Time and frequency domains are incoherent, →
a sudden change 'spike' in the domain spreads to the other one.

In Spectrum Sensing:
Signal is sampled in time domain and is examined in frequency domain
→ representation domain = time, measurement domain = frequency

Example
Compressed Sensing

- Detect a wireless signal in a wide range of spectrum (e.g., 1GHz). Assume the received signal samples satisfy the Nyquist limits (e.g., f_s=2 GHz sampling rate).

  The wireless signal is modeled as an FM signal s(t):

$$s(t) = \cos\left[2\pi f_C t + 2\pi\Delta \int_0^t m(\tau)d\tau\right]$$

Where f_c=200MHz: carrier freq.;

Δ=5kHz: FM deviation factor;

$m(\tau)=\sin(2\pi f_m \tau / f_s)$ the modulating signal;

f_m=32kHz: signal freq.

Example
Compressed Sensing

- The compressed received signal y(t) is obtained by sub-Nyquist sampling, e.g., number of samples m=256, instead of using a Fourier matrix F with $N=2*10^9$ frequency components
  - Considering the given s(t) and assuming SNR=20dB, we create a random vector for y(t) in Matlab with m=256 samples

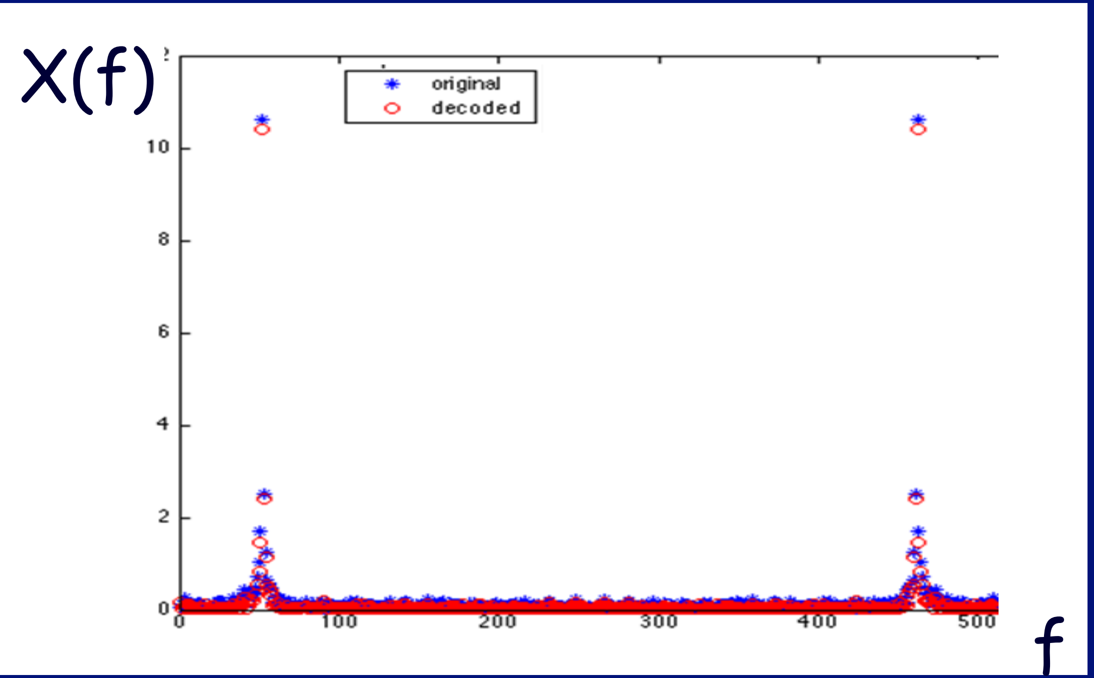
- The sampling matrix A needs to be pre-determined based on spectrum utilization characteristics. However, here we consider an arbitrary random sampling matrix A for m=256 samples:
  - Matrix A is generated from Fourier Matrix F by randomly selecting m=256 frequencies with equal probability, i.e.,
    - A=[exp(-j2πkt/N)/sqrt(N)], where m=256 values of k are selected from the interval k=[0,…,N-1] with equal probability

Example
Compressed Sensing

Then, we solve y(t)=Ax(t) for minimum $||X(f)||_1$, which must be solved by using iterative optimization techniques

- Initially we predict x(t)=0, and hence, X(f) is also 0.
- Apply the numeric solver algorithm to minimize $||X(f)||_1$ subject to y(t)=A.x(t)

(The problem is formulated in time domain, at each iteration x(t) can be obtained by IFFT of X(f))



ADVANTAGES OF COMPRESSED SENSING

- Effective wideband spectrum sensing
- Low implementation cost
- Reduced overhead in cooperative sensing (less measurements exchange)

DISADVANTAGES OF COMPRESSED SENSING

- Sparsity of wireless spectrum cannot be guaranteed in CR network
- Probabilistic signal recovery
- Sensitivity to the near-far problem

Classification of Spectrum Sensing Techniques

- Spectrum Sensing
  - Transmitter Detection
    - Matched Filter Detection
    - Energy Detection
    - Cyclostationary Feature Detection
    - Covariance Matrix-based Detection
    - Wavelet Detection
    - Compressed Sensing
  - Receiver Detection
  - Interference Temperature Management

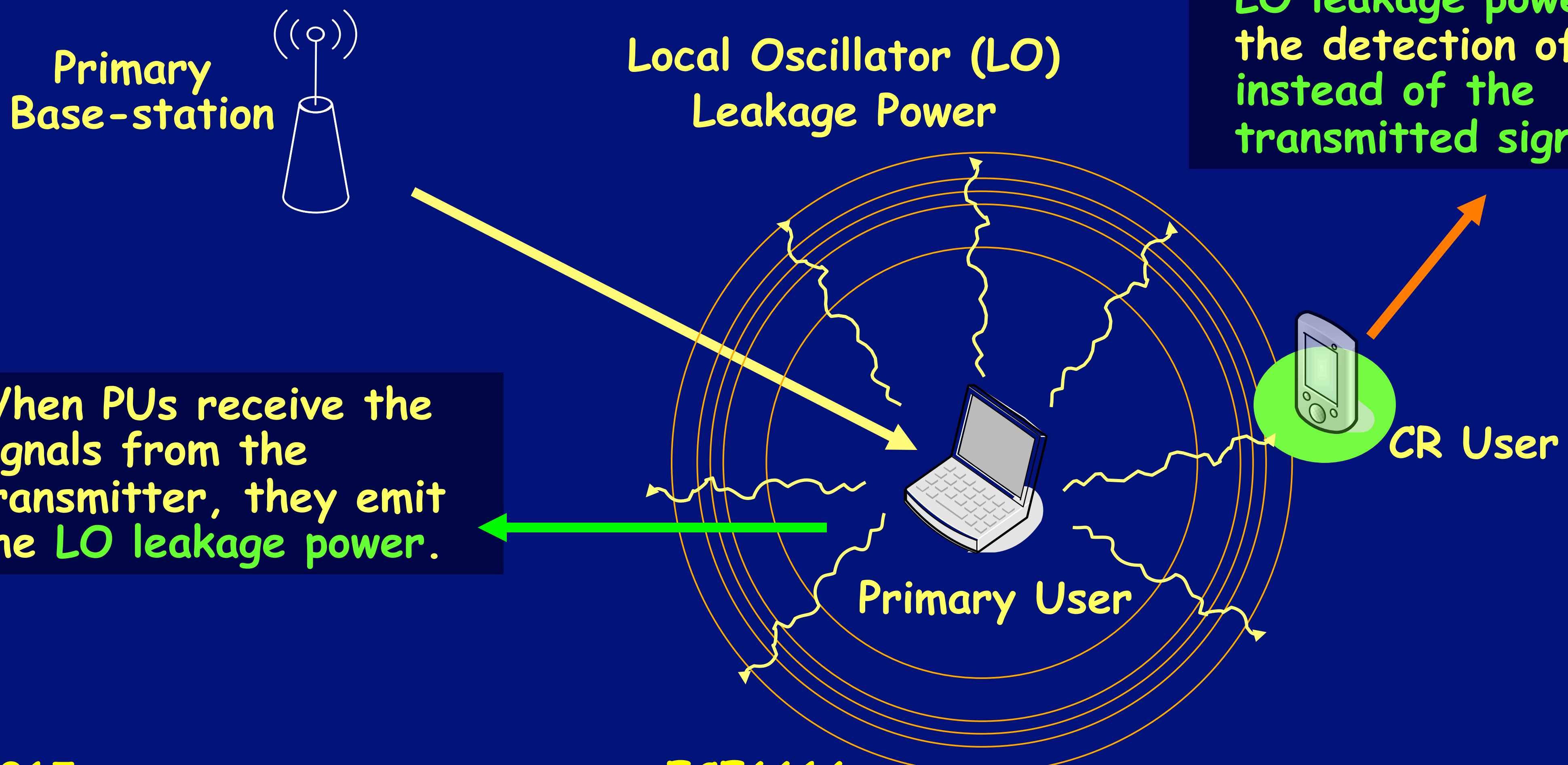
Why Receiver Detection?

- Eliminates the assumption that a PU is passive
- Detection of the Exact PU Channel
- High probability of finding a free Spectrum EVEN IN HIGH DENSITY of PU Receivers

Primary Receiver Detection

B. Wild and K. Ramchandran, "Detecting Primary Receivers for Cognitive Radio Applications" in Proc. IEEE DySPAN, Nov. 2005.

CR users detect the LO leakage power for the detection of PUs instead of the transmitted signals



When PUs receive the signals from the transmitter, they emit the LO leakage power.

How can the LO Leakage Power be detected?

- Same methods as before, i.e.,

  (Matched filter detection, Energy detection or Cyclostationary feature detection)

Primary (User) Receiver Detection

- Primary (User) receiver detection can solve the receiver uncertainty problem in the transmitter detection
- However, since the LO leakage signal is typically weak, implementation of a reliable detector is not trivial.
- Currently this method is only feasible in the detection of the TV receivers.

Pitfalls with Receiver Detection

- Need a highly sensitive "energy detector"
- Require Additional Power margin to account for "bad channel" seen by the sensor
- Near-Far Problem
  - Two nearby devices use the same frequency tone
  - CR gets confused about the location of the PU

Classification of Spectrum Sensing Techniques

- Spectrum Sensing
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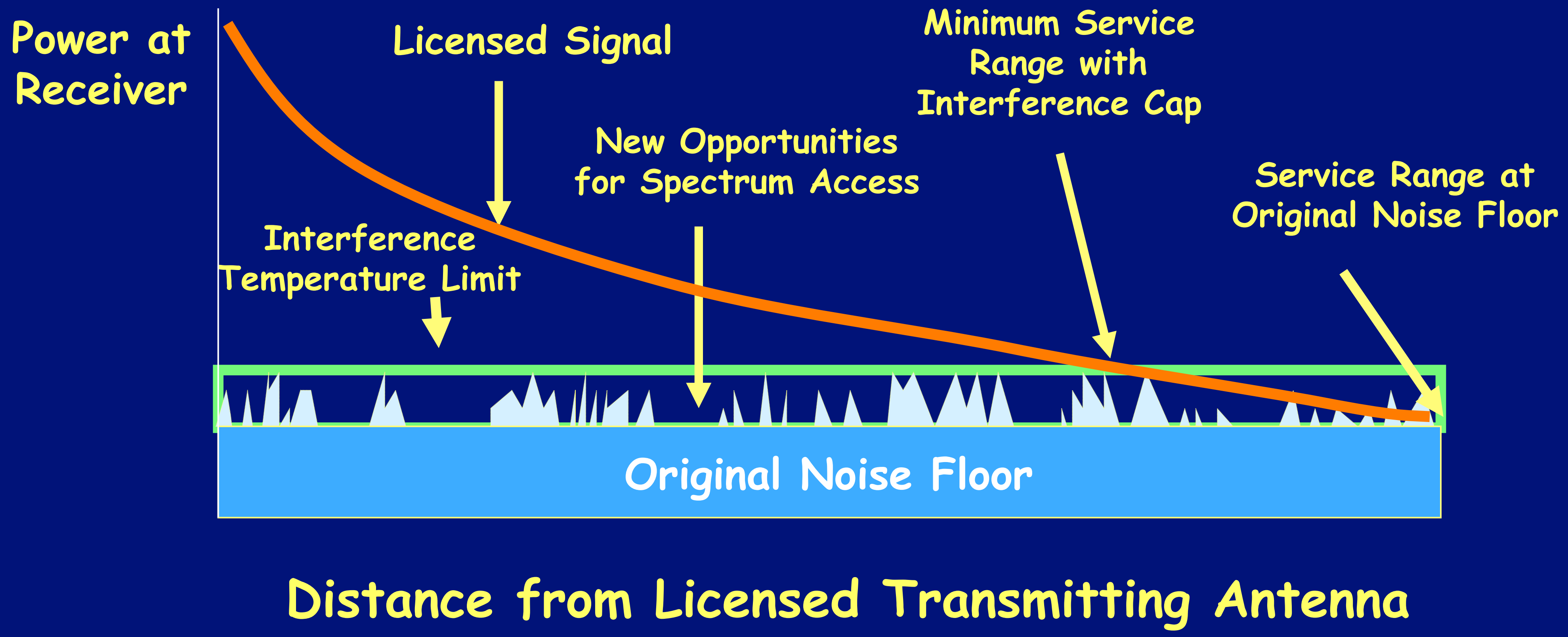
Interference Temperature Model



Interference Temperature (IT) Definition

T. C. Clancy, "Formalizing the Interference Temperature Model" in Wiley Journal on Wireless Communications and Mobile Computing, 2006.

ITM is a model that controls the spectrum use.
Either temperature or power based.

- T_I is the Interference temp in Kelvin
- P_I is the Average Interference Power (Watts) centered at f_c
- Covering Bandwidth B (Hz)
- Center Frequency f_c (Hz)
- Boltzmann constant, k=1.38*10{-23} Joules per Kelvin degree

$$T_I(f_c, B) = \frac{P_I(f_c, B)}{kB}$$

Interference Temperature Model #1 (IDEAL)

- T_L is the Interference Temp. limit set by FCC (location specific)
- P is transmit power of the CR over a particular band
- This freq. band contains signals from n PUs
- The signal from user i has bandwidth B_i centered at freq. f_i
- M_i is a multiplicative attenuation factor between 0 and 1 representing fading and path loss between CR user and the i-th PU
- Then, the transmission of a CR must ensure the following interference temperature limit for the PUs:

$$T_I(f_i, B_i) + \frac{M_i P}{k B_i} \leq T_L(f_i) \qquad for \qquad i = 1, \ldots, n$$

Problems with Model #1

- No practical way for a CR to measure or estimate the interference temperature.
  (CR users cannot distinguish between actual signals from the PU and noise/interferences).

- Interference temperature limit should be location dependent of the PUs which is not easy to determine.

Interference Temperature Model #2
(GENERALIZED MODEL)

Model 1 has problem that is not easy to determine unique M_i

$$T_I(f_c, B) + \frac{MP}{kB} \leq T_L(f_c)$$

- Use a single constant M fixed by FCC
- Scenario: 1 CR + n PUs
- Constraint MUST be satisfied for the entire bandwidth

Overall Problems

- Increasing the interference temperature limit will affect primary network's capacity and coverage.