# ECE 6616: Cognitive Radio Networks 

## Spring 2015

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Homework 3
Due: 11:59pm, Sunday, April 19, 2015

## 1. Sensing Parameter Optimization

A CR user is going to transmit on the following spectrum band.

| Bandwidth <br> $(\mathrm{MHz})$ | PU arrival <br> rate | PU <br> departure rate | PU receiver <br> sensitivity <br> $(\mathrm{dBm})$ | Noise power <br> $(\mathrm{dBm})$ | Interference <br> constraint $(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 2 | 1 | -127 | -124 | 1 |

- PU receiver sensitivity: received signal power at the cell edge of the primary network.
- Interference constraint means the maximum interference fraction that primary network can tolerate during its transmission.

The CR user is assumed to perform MAP-based Energy detection in AWGN channel. The detection threshold is determined as $P_{o n}-P_{d}(\lambda)=P_{f}(\lambda)$. The lengths of ON and OFF periods in each band are exponentially distributed, respectively. The CR user senses the spectrum bands in every sensing cycle, and cannot transmit during the sensing operation.
a) Assume that the transmission time T is fixed to 10 msec . Calculate the optimal sensing time to satisfy the interference constraint.
b) Assume that the detection error probability, $P_{o n}\left(1-\bar{P}_{d}\right)$ is fixed to $0.5 \%$ (i.e., sensing time is fixed). Calculate the optimal transmission time to satisfy the interference constraint.
c) Calculate optimal sensing and transmission times to maximize the sensing efficiency subject to interference constraint. Compare this sensing efficiency with those in a) \& b).
(You may need MATLAB programming for finding optimal solution. At least you should show the entire procedures to determine the optimal sensing parameters, and choose any sub-optimal solutions with better sensing efficiency than the solutions of a) and b).)

Hint: See Section III in the following paper
W.-Y. Lee, and I. F. Akyildiz, "Optimal spectrum sensing framework for cognitive radio networks," IEEE Trans. Wireless Communications, Oct. 2008.
> Correction: Eq. (16)

$$
\bar{P}_{f}<\frac{\frac{T_{P}}{P_{o f f}}-P_{o n}\left(1-e^{-\mu T}\right)}{e^{-\mu T}}=e^{\mu T}\left(\frac{T_{P}}{P_{o f f}}-P_{o n}\right)+P_{o n}=\bar{P}_{f}(T)
$$

Hint: $\mathrm{Q}^{-1}(\mathrm{x})=\sqrt{2} \operatorname{erf}^{-1}(1-2 x)$

- Calculation of inverse error function $\rightarrow$ Use MATLAB function 'erfinv’


## 2. Spectrum Selection \& Sensing Scheduling

The following spectrum bands are allowed for the secondary access.

|  | Band | Bendwidth <br> $(\mathrm{MHz})$ | Spectral <br> efficiency <br> $(\mathrm{bit} / \mathrm{sec} / \mathrm{Hz})$ | Idle <br> Prob. | Sensing parameters  <br>   |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A |  | 2 | 0.5 | 0.1 | 0.05 |
| B | 5 | 5 | 0.333 | 0.2 | 0.04 |
| C | 0.1 | 10 | 0.667 | 0.05 | 0.02 |
| D | 0.5 | 1 | 0.2 | 0.01 | 0.01 |
| E | 1 | 0.5 | 0.375 | 0.08 | 0.02 |

a) A CR user has RF frontends dedicated for spectrum sensing, which can sense one spectrum band at a time.
i) How many RF frontends are required for sensing all spectrum bands above?
ii) Assume that the CR user has only one RF frontend for spectrum band. Which spectrum bands should be selected so as to maximize the expected transmission capacity of the CR user?
b) A single sensing RF frontend has just started sensing spectrum bands determined in a)-ii from the band with the highest opportunistic capacity. Based on the LCFS principle, schedule the sensing operation of the RF frontend for the next 0.15 sec .
(Example answer)


Hint: See Section IV in the following paper
W.-Y. Lee, and I. F. Akyildiz, "Optimal spectrum sensing framework for cognitive radio networks," IEEE Trans. Wireless Communications, Oct 2008.

## 3. Game Theory for Spectrum Sharing (15 points)

Two CR users wish to share the spectrum for U.S. TV channels 14,15 , and 16 . Each user may select a single channel, or two consecutive channels to operate on. The users will communicate using QPSK modulation, with a 1 MHz guard band between each channel (unless the user operates consecutive channels, then the middle guard band is not used). The users are 2 km apart and transmit with a power of 100 mW . The noise floor is -115 dBm .
a) Formulate a game and find the Nash Equilibrium and Pareto Optimal outcomes of this game (if they exist). State any assumptions that you make.
b) Discuss how this scenario might scale to a larger number of CR users and channels.

## 4. Auction-based Spectrum Sharing ( 20 points)

A Spectrum Broker (SB) allocates 100 channels among 10 CR users based on an auction scheme for spectrum sharing in a CR network. Each user should submit to the SB a demand curve that outlines the price they are willing to pay for a given number of channels. The demand curve of each CR user follows a unique exponential distribution given by:

$$
D(n)=\alpha *(\max P-\min P) *\left(1-e^{-\frac{\beta * n}{\max P-\min P}}\right)+\min P
$$

where $\alpha$ and $\beta$ are random variables with uniform distribution on $[0,1], \max P$ and $\min P$ are the maximum and the minimum individual channel prices, which are set to $\$ 25$ and $\$ 100$, respectively, and $n$ is the number of channels. Each user also submits their desired number of channels given by:

$$
C=\frac{2 * \gamma * \text { numChannels }}{\text { numUsers }}
$$

where $\gamma$ is a random variable with uniform distribution on $[0,1]$, numChannels and numUsers are the number of channels and users, respectively. The SB uses the following spectrum sharing policies for channel allocations: i) If there are enough channels for everyone, allocate them all at the average market price for each user's desired number of channels. ii) If not, the highest bidders get theirs first for the price they bid. iii) All channels allocated to each user must be contiguous.
a) Model the Spectrum Broker (SB) described above using Matlab (recommended) or the language of your choice.
b) Plot both users' demand curve and average market demand versus the number of channels in the same figure.
c) Plot channel allocations (user numbers versus channel numbers) for two cases: i) channel supply is greater than total channel demand and ii) channel supply is less than total channel demand. You can observe these two cases by running the experiment multiple times.
d) Find the total revenue for the SB and the average cost per channel for those two cases in c).
e) How can you increase the total revenue for the SB by changing the spectrum sharing policies? Show this improvement with the implementation of your new policies.

## 5. Spectrum Mobility ( 20 points)

Consider a CR user moving from $\mathbf{a}$ to $\mathbf{b}$ (for 12 seconds) while transmitting and receiving data. There are two options for supporting the mobility of the CR user: 1) Option 1: traveling across three smaller cells, and 2) Option 2: traveling across two larger cells, as illustrated in the below figure.


Each option has the following conditions:

- The Option 2 has a twice larger coverage than Option 1.
- Each cell in Options 1 and 2 exploits a single and the same spectrum band.
- The spectrum band has three independent primary user (PU) activity regions, as presented in the above figure.
- The lengths of ON and OFF periods in each PU activity region are exponentially distributed with the rate $0.04(\alpha)$ and $0.04(\beta)$, respectively.
- The CR user sense the spectrum band in every 1 sec and the sensing time is small enough to be ignored.
Note that the CR user cannot transmit during the cell switching. Also the CR user can sense the spectrum correctly without any detection error or false alarm.
Initially, all PU activity regions are assumed to be idle. When the CR user joins the new cell, the initial idle probability of each PU activity region is expected to be 0.5.
a) Calculate the idle probabilities of the spectrum band in the cell (i) of Option 1 and cell (I) of Option 2 at $r^{\text {th }}$ sensing period?
Hint: See Eq. (18) in the following paper
W.-Y. Lee, and I. F. Akyildiz, "Spectrum-aware mobility management in cognitive radio cellular networks," IEEE Trans. Mobile Computing, April 2012.
b) In conventional mobile communication systems that do not consider the PU activity region, Option 1 shows better performance in terms of handoff delay. Is this still valid in CR networks? Choose the best option for the mobile CR user by comparing the link availabilities of each option. (Here the link availability is defined as the ratio of the available time for transmission to an entire travel time)


## 6. Cognitive Radio MACs (15 points)

a) Explain how the following network characteristics impact the performance of the CR MAC protocols: i) network topology, ii) the number of CR nodes, and iii) CR coverage area.
b) In CSMA/CS-based MAC,
i) Why use 4-way handshaking mechanism? What happens if two-way handshaking is used instead?
ii) Can using multi-radio transceivers improve the performance?
c) List two drawbacks of time-slotted MAC for 802.22 and suggest a way to improve each drawback.
d) Explain whether or not C-MAC can
i) Avoid hidden terminal problem and
ii) Adapt to the PU activity.
e) Can POMDP-based decentralized cognitive MAC be used in highly dynamic cognitive radio network? Why?

## 7. Routing (15 points)

Source $S$ wishes to send a packet to destination $D$, and there are two alternatives for possible paths. The first path is along nodes S-A-B-D, say Path 1 , and the second path is along nodes S-X-Y-Z-D, say, Path 2. There are two possible choices of spectrum chosen at each node. The default channel is the
licensed 600 MHz channel, and there is a backup 2.4 GHz unlicensed ISM band channel that can be used when the licensed channel is occupied by a primary user (PU). The values $p_{1}=0.4, p_{2}=0.3, p_{3}=0.2$ indicate the probability with which the PUs cause channel switches into the ISM band at the node locations, as indicated in the figure.


Whenever a channel switch occurs, there is switching time penalty of $\mathrm{T}_{\mathrm{sw}}=1 \mathrm{~ms}$. The transmission time of a packet in the 600 MHz and ISM band are $\mathrm{T}_{600}=3 \mathrm{~ms}$ and $\mathrm{T}_{\text {ISM }}=5 \mathrm{~ms}$, respectively. Note that transmission is always first attempted on the licensed channel (i.e., the licensed channel is the default choice) at each node.
a) What are the best case and worst-case end-to-end latencies for each of the two paths?
b) Which path would be chosen, considering the average end-to-end latency for the two paths?

