ECE6616: Cognitive Radio Networks

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Homework 1

1. **Performance of Primary Signal Detection**

   Consider the following binary hypothesis test on whether or not a primary signal s(n) exists in a particular channel. This test is performed on the received signal x(t), which is modeled as follows:

   H_0: x(t) = n(t) no primary signal

   H_1: x(t) = s(t) + n(t) there is a primary signal

   where s(t) is the primary signal and n(t) is the ambient noise. Let us model n(t) and s(t) as random variables with normal distributions $N(u_0, \sigma_0^2)$ and $N(u_1, \sigma_1^2)$, respectively.

   a. Show that the false alarm probability P_f is given by: $\Pr(x(t) > \lambda \mid H_0) = Q((\lambda - u_0)/\sigma_0^2)$ where Q is the standard Gaussian tail function and λ is the detection threshold.
   b. Derive the probability of detection P_d in terms of P_f.
   c. Draw ROC curves (P_d vs P_f) using Matab for the following cases: 1) $u_0 = u_1$, $\sigma_0 = \sigma_1$; 2) $2u_0 = u_1$, $\sigma_0 = \sigma_1$; 3) $2u_0 = u_1$, $\sigma_0 = 2\sigma_1$; and 4) $2u_0 = u_1$, $2\sigma_0 = \sigma_1$. The Q(•) and Q−1(•) functions are available in MATLAB as qfunc.m and qfuncinv.m.
   d. Show the impact of noise uncertainty on the probability of detection.

2. **Energy Detection**

   Consider the energy detection scheme presented in class for spectrum sensing. Assume there is no fading and time-bandwitdh (TW) product m is 5:

   a. Provide ROC curve, i.e., P_d vs P_f, for energy detection for SNR= 10, 15, 20, 25 dB
   b. Provide a complimentary ROC curve for probability of miss detection ($1-P_d$) vs P_f for SNR= 10, 15, 20, 25 dB.

2. **Cyclostationary Feature Detection**

   Consider a noise-free received primary signal $x(t) = s(t) = a(t)\cos(2\pi f_c t + \phi_0)$ being processed by a cyclostationary feature detector:

   a. If the autocorrelation of a(t) is $R_a(\tau)$, derive $R_x(\tau)$ in terms of $R_a(\tau)$.
   b. Derive the cyclic autocorrelation function $R_x^\alpha(\tau)$ in terms of $R_a(\tau)$ by using Fourier Series.
   c. Derive the spectral correlation function $S_x^\alpha(f)$ in terms of $S_a(f)$ (the PSD of a(t)).
   d. Assume $R_a(\tau) = A\,\mathrm{sinc}^2(A\tau)$, where $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$ and A is a constant, draw $|S_x^\alpha(f)|$ in a three-dimensional plot. Mark the magnitudes and (α,f)-coordinates of all components.
   e. What are the cyclic features of the primary signal? What is the minimum detection threshold?
   f. If the received signal x(t) = s(t) + n(t) where $n(t) \sim N(0,1)$, redraw the spectral correlation function $|S_x^\alpha(f)|$. The noise and the primary signal are independent.
   g. What is the difference between the $|S_x^\alpha(f)|$ obtained in (d) and the $|S_x^\alpha(f)|$ in (f)? Can the cyclostationary feature detector detect the primary signal with additive noise? Why?

3. Spectrum Sensing Design

Design a spectrum sensing algorithm to detect a weak signal x(t) in white Gaussian noise n(t)~$N(u, \sigma^2)$. Let us assume that the bandwidth of the receiver is 5 MHz and the sampling rate is 80 MHz. The detection false alarm should not exceed 0.1 and the probability of detection is at least 0.9. The sensing algorithm should be designed for detecting the following two cases:

a. The target signal $x(t)= A \sin(2\pi f_c t)$ (A is known) where $f_c = 2$ MHz.

b. The carrier frequency of the signal is within +/- 100 KHz accuracy and A is unknown.

The design should explain the detection algorithm, how many samples are needed to meet the sensing performance requirements in terms of P_d and P_f, the computational complexity of your algorithm, and show how this complexity can be reduced.

4. Compressed Sensing

You are trying to detect a wireless microphone signal in a wide range of spectrum (e.g. 1 GHz). Assume that you have an access to the received signal samples acquired at high sampling rate that satisfies the Nyquist limits for scanning 1 GHz (e.g. 2 GHz sampling rate). Based on the wireless microphone simulation guidelines in IEEE 802.22 standard document:
https://mentor.ieee.org/802.22/.../22-07-0124-00-0000-wireless-microphone-signal-simulation.doc
you can model the wireless microphone signal as an FM signal s(t):

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi\Delta\int_0^t m(\tau)d\tau\right]$$

where A_C is the carrier amplitude , f_c is the carrier frequency, Δ is the frequency sensitivity (Deviation factor), and $m(\tau)$ is the modulating signal. There are three models of wireless microphone signals that can be used:

	Model 1 Silent	Model 2 Soft Speaker	Model 3 Loud Speaker
the modulating signal $m(\tau)$	32 KHz	3.9 KHz	13.4 KHz
FM deviation factor [kHz] Δ	± 5	± 15	± 32.6

Pick one wireless microphone signal model at an arbitrary carrier frequency $f_c < 1$ GHz. Then,

a. Use the energy detector to show how many samples are required to detect the wireless microphone signal with 0.1 probability of false alarm and 0.9 probability of detection.

b. Use the compressive sensing techniques to detect the wireless microphone signals. You can use the Matlab code in
http://www.convexoptimization.com/wikimization/index.php/Candes.m
to apply compressed sensing with signal recovery using optimization technique.